Basic Algebra

Brown • Smith • Dolciani

Teacher's Edition

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Basic Algebra

TEACHER'S EDITION

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The authors of BASIC ALGEBRA wish to express appreciation to Professor Frank Ebos, Associate Professor of Mathematics, Faculty of Education, University of Toronto, for his valuable contribution to this Teacher's Edition.

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Guide to Individualized Assignments T25
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Lesson Commentary T34
A discussion of each lesson, with teaching suggestions, chalkboard examples, and extensions.

Student Text
A full-sized facsimile of each student page with answers to all exercises annotated on the page.

Introduction

This textbook has been designed for those students who would normally have difficulty with a standard first-year algebra course. It provides the fundamentals of a standard course, with lessons that are easy to read and an abundance of carefully chosen exercises. A colorful design and interesting special features help make algebra an enjoyable subject.

We invite you to browse through the next few pages of this Teacher's Edition, which illustrate many of the features of BASIC ALGEBRA with reduced sample pages from the text.

Organization of the Text

The text is organized into six units, each divided into two chapters. The chapters consist of short lessons that are accompanied by a generous number of classroom and written exercises.

7 • Writing Algebraic Expressions

Before you can use algebra to solve real-life problems you will need practice in using variables to translate sentences into equations.



The Sox won 15 more games than they lost.
Let x be the number of games lost.
Write an expression to show the number of games won.

The number won is 15 more than the number lost.

The number won $=$ 15 $+$ x

So, the number won is $15 + x$.

Here are some more examples.

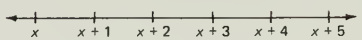
EXAMPLE 1 Joanna has twice as much money as Barb.
Let x be Barb's amount of money.
Then ___?___ is Joanna's amount.

Joanna's amount is twice Barb's amount.

Joanna's amount $=$ 2 $\cdot$ x

So, Joanna's amount $= 2x$.

EXAMPLE 2 Let x be a whole number.
Then the next two whole numbers are ___?___ and ___?___.



The next two whole numbers are $x + 1$ and $x + 2$.

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SELF-TEST

Multiply.

1. $(m - 13)(m + 13)$
2. $(3 + 4x)(3 - 4x)$
3. $(5a - 7)(5a + 7)$

Factor.

4. $a^2 - 64$
5. $m^2 - 36n^2$
6. $16x^2 - 9y^2$
7. $3x^2 + 15x + 18$
8. $2y^2 - 4y - 30$
9. $2m^2 - 8n^2$

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Self-Tests occur after a cluster of related lessons, providing frequent review. Answers to the Self-Tests are supplied at the back of the text.

Reviewing the Chapter

Solve. *(See pp. 34–45.)*

1. $a - 5 = 13$
2. $x + 9 = 24$
3. $3b = 24$
4. $5m = 35$
5. $2x - 7 = 9$
6. $3y + 4 = 19$
7. $\frac{b}{2} = 7$
8. $\frac{x}{3} = 13$
9. $\frac{m}{4} = 7$

Each chapter is followed by Reviewing the Chapter, a two-page review keyed to the lessons in the chapter. Chapter Tests that follow exactly the same format are provided in this Teacher's Edition, beginning on page T11.

Reviewing Arithmetic Skills

Use your estimating skills. Estimate in your head.

Sample $39 + 142$ — *Answer:* about 180

$40 + 140 = 180$

1. $22 + 59$
2. $47 + 11$
3. $86 + 21$
4. $79 + 32$
5. $41 + 51$
6. $99 + 16$
7. $81 + 39$
8. $64 + 89$

Each unit contains a Reviewing Arithmetic Skills and a Cumulative Review. Five additional cumulative reviews begin on page 438.

Cumulative Review

Simplify.

1. $7b - 4b + 3$
2. $x(8x)(-3y)$
3. $5(a + 6) - 11$
4. $9(3n^2 + 2m) - 18m$
5. $6(s - 4t) - 2s$
6. $7n - 11n$
7. $-3a^2 + 5a^2$
8. $x - (7 - 3x)$
9. $-6a - 2(4 + 3a)$
10. $\frac{12t}{t}$
11. $17 \cdot \frac{0}{b}$
12. $(-m)^2 + m$

Tell which of the numbers shown in color are solutions.

13. $x - 13 < 10$ — 25, 23, 21
14. $a^2 + 2 > 10$ — 1, 3, 5
15. $5b - 6 > 11$ — 1, 3, 5
16. $3(t + 1) < 15$ — 2, 4, 6

Solve for x.

17. $x + 15 = 22$
18. $x - 9 = 11$
19. $9x = 117$

Diagnostic Tests in Arithmetic precede the text and are helpful in pinpointing each student's computational strengths and weaknesses.

Diagnostic Tests in Arithmetic*

1. Whole Numbers—Addition

1. $\begin{array}{r} 6 \\ +9 \\ \hline \end{array}$	2. $\begin{array}{r} 2 \\ 2 \\ +3 \\ \hline \end{array}$	3. $\begin{array}{r} 4 \\ 6 \\ +7 \\ \hline \end{array}$	4. $\begin{array}{r} 55 \\ +25 \\ \hline \end{array}$	5. $\begin{array}{r} 68 \\ +15 \\ \hline \end{array}$
6. $\begin{array}{r} 575 \\ +371 \\ \hline \end{array}$	7. $\begin{array}{r} 778 \\ +177 \\ \hline \end{array}$	8. $\begin{array}{r} 814 \\ 857 \\ +311 \\ \hline \end{array}$	9. $\begin{array}{r} 7456 \\ 3701 \\ 6101 \\ +5592 \\ \hline \end{array}$	

2. Whole Numbers—Subtraction

1. 7 2. 11 3. 97 4. 769 5. 684

Each lesson is short, presenting manageable bits of material. Reading is minimized. Ideas are often displayed graphically. Many illustrative examples with complete solutions are provided in both the text and the exercises. Color is used effectively in the text and the diagrams to clarify and emphasize important concepts.

7 · Multiplying at Sight

When you multiplied binomials by using the vertical form, you may have noticed a pattern. The pattern is pointed out at the right.

$$\begin{array}{r} x + 2 \\ x + 3 \\ \hline x^2 + 2x \\ \quad + 3x + 6 \\ \hline x^2 + 5x + 6 \end{array}$$

product of the first terms

product of the last terms

sum of $2 \cdot x$ and $3 \cdot x$

You don't have to write out each step every time you multiply two binomials. You can multiply at sight if you follow three steps.

Step 1: $(x + 3)(x + 2) = x^2 \ldots$ ← Multiply the *first* terms.

Step 2: $(x + 3)(x + 2) = x^2 + 5x \ldots$ ← Multiply the *outer* terms. Multiply the *inner* terms. Add the two products.

Step 3: $(x + 3)(x + 2) = x^2 + 5x + 6$ ← Multiply the *last* terms.

An easy way to remember this is to think of it as the **FOIL method.**

FOIL stands for First, Outer, Inner, Last!

Here's how the FOIL method works.

EXAMPLE 1 Multiply $(x + 4)(x + 5)$.

Step 1: $(x + 4)(x + 5) = x^2 \ldots$ ← F

Step 2: $(x + 4)(x + 5) = x^2 + 9x \ldots$ ← O + I

Step 3: $(x + 4)(x + 5) = x^2 + 9x + 20$ ← L

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Classroom Practice

Factor.

1. $2a^2 + 14a + 20$
2. $2x^2 - 14x + 24$
3. $3y^2 + 3y - 18$
4. $4x^2 + 4x - 80$
5. $3x^2 - 27$
6. $9a^2 - 4$
7. $x^3 - 16x$
8. $y^3 - x^2y$
9. $2r^2s - 8st^2$

Written Exercises

Factor.

A
1. $3y^2 + 18y + 24$
2. $4x^2 + 24x - 64$
3. $2y^2 - 16y + 30$
4. $2b^2 - 4b - 48$
5. $5a^2 - 20a - 60$
6. $3x^2 + 30x + 27$
7. $4x^2 + 8x - 60$
8. $3y^2 - 12y - 63$
9. $2x^2 - 16x + 32$
10. $5a^2 - 5b^2$
11. $8x^2 - 32$
12. $x^2y - y^3$
13. $4x^2 - 36$
14. $3a^2 - 27b^2$
15. $6x^2 - 24y^2$

B
16. $-x^2 - 2xy - y^2$
17. $-x^2 + 4x - 3$
18. $2a^3 - 8a$
19. $-162 + 2x^2$
20. $-48 + 3x^2$
21. $4a - 4a^2 + 8$
22. $-3x^2 + 30x - 75$
23. $-200b^2 + 2a^2$
24. $-2x^2 + 24x - 72$

There is an abundance of classroom and written exercises. Written Exercises are graded by the letters A, B, and C in order of increasing difficulty. Thirty pages of Extra Practice Exercises begin on page 406. Annotations at the end of selected lessons key the Extra Practice Exercises to the text.

Written Exercises

A
1. Bill and Mike are 56 km apart.
Each boy travels toward the other.
Bill travels at 20 km/h.
Mike travels at 8 km/h.
After how long do they meet?

	rate × time = Distance		
Bill	20	t	$20t$
Mike	?	?	?
			56

2. At noon, a train leaves Omaha to go to Mobile at 100 km/h.
At the same time a train leaves Mobile for Omaha at 120 km/h.
The distance between the cities is 1760 km.
At what time do the trains pass each other?



	rate × time = Distance		
Omaha—Mobile	100	t	?
Mobile—Omaha	?	?	?
			?

Sections of word problems occur throughout the text, providing many practical applications. The problems, often accompanied by helpful charts and diagrams, are stated in simple sentences, one sentence on each line, to make reading easier.

Special Features

Consumer Corner, an optional feature appearing at the end of each chapter, helps students learn practical skills for everyday living. Career Notebook, occurring in each unit, illustrates many jobs that need a background in mathematics. Puzzles & Things adds a light touch throughout the book. Cartoons introduce and illustrate many algebraic ideas.

CONSUMER CORNER



Writing a Check

It isn't safe to carry much cash with you. It may be lost or stolen. Many people put their money in a checking account. Then they can pay their bills by check or they can cash a check for themselves when they need money.

CAREER NOTEBOOK

Marketing

Many people aren't aware of all that is involved in putting a product on the market. Considerations must be given to consumer needs, competitive products, advertising, packaging, and profitability.



Packaging designers use artistic talent and marketing skills to design packages that protect and promote their contents.



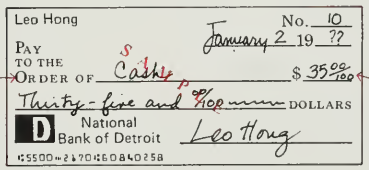
Product designers work to improve the appearance and functional design of products.



Market researchers analyze data to forecast sales and recommend product design, packaging, and advertising.

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Suppose you wish to get some cash for yourself. You can just go to your bank and make out a check for *Cash*.


Leo Hong
No. 10
January 2 19 ??
PAY TO THE ORDER OF Cash $ 35.00
Thirty-five and no/100 DOLLARS
National Bank of Detroit
Leo Hong
⑈5500⑈ ⑆170460840258

Write $\frac{00}{100}$ when there are no cents.

Be careful! It might be possible for somebody else to cash the check if you lose it.

Now why not try writing

1. Write a check for $1
2. Write a check for $ broke.
3. Write a check for bicycle.
4. Write a check for $7 a car.
5. Write a check for $1
6. Write a check for $ rock concert.
7. Write a check for $2
8. Write a check for $1

Puzzles & Things

Believe it or not, there's a tree that grows its leaves according to a mathematical formula. The tree is *Schumanniophyton problematicum*. Its formula is $L = 12a + 4$. L represents the number of leaves produced, while a represents the age of the tree. Smart tree!

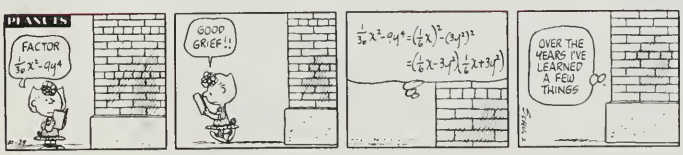


EXAMPLE 3 Factor $x^2 - 4y^2$.

Step 1: $x^2 - 4y^2 = (x \quad)(x \quad)$

Step 2: $x^2 - 4y^2 = (x + 2y)(x - 2y)$

Check: $(x + 2y)(x - 2y) = x^2 - 4y^2$



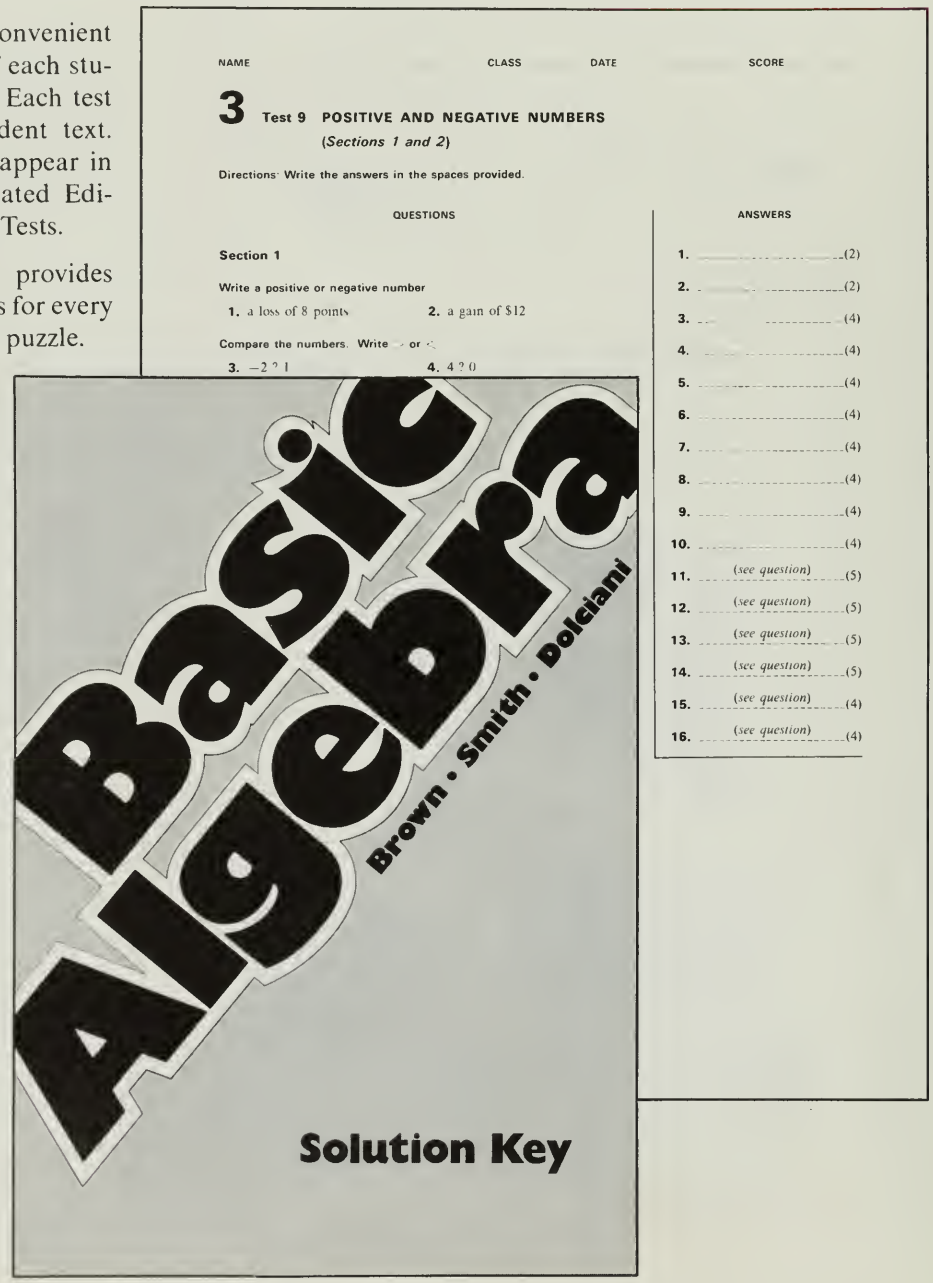
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Supplementary Materials

This Teacher's Edition contains lesson-by-lesson commentary on the student text, with teaching suggestions, chalkboard examples, and extensions. Chapter tests and answers, an individualized assignment guide, and a table of related references are also provided. Answers to classroom and written exercises appear on full-sized facsimiles of the student pages.

Progress Tests are a convenient way to keep track of each student's performance. Each test is keyed to the student text. Answers to all tests appear in the Teacher's Annotated Edition of the Progress Tests.

The Solution Key provides step-by-step solutions for every written exercise and puzzle.



Chapter Tests

Chapter 1

If $x = 6$, find the value of the expression.

1. $x + 7$ **2.** $3x$ **3.** $x \div 3$ **4.** $5(x - 2)$

Find the value of the expression.

5. $3 \cdot 7 - 2$ **6.** $(5 + 6)(7 - 5)$ **7.** $(9 - 3) \div 3 - 2$

Simplify.

8. $3x + 7x + x$ **9.** $5a - a + 3a$ **10.** $3x + 4y + x - 3y$

Find the value of the expression.

11. 5^2 **12.** 2^5 **13.** 3^3 **14.** 2 to the fourth power

Simplify.

15. $3 \cdot (5x)$ **16.** $4 \cdot 6a$ **17.** $8 \cdot x \cdot x \cdot 2 \cdot x$

18. $(9x) \cdot 7$ **19.** $(3a)(4a)$ **20.** $c(3c)(5c)$

State the expression without parentheses.

21. $3(x + 4)$ **22.** $5(3a - 7)$ **23.** $6(3a - 2b)$ **24.** $4(5 - 4m)$

Let $x = 8$. Find the value if possible. If not, write *impossible*.

25. $3(x - 8)$ **26.** $\dfrac{x + 8}{x - 8}$ **27.** $\dfrac{x - 8}{x}$ **28.** $\dfrac{8}{x}$

Simplify.

29. $3x + 5 + 4x - 5$ **30.** $7a - 6b + 6b - 3a$ **31.** $12a + 6(5 - 2a)$

Tell which of the numbers are solutions.

32. $3x - 7 = 11$ **4, 6, or 8?** **33.** $3 + 2x = 9$ **1, 2, or 3?**

34. $x + 8 > 20$ **11, 12, 13** **35.** $3y + 8 < 25$ **4, 5, 6**

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Chapter 2

Solve.

1. $x - 7 = 15$

2. $y + 4 = 13$

3. $8x = 96$

4. $\frac{a}{4} = 5$

5. $x - 10 = 11$

6. $6y = 42$

7. $7y + 6 = 20$

8. $3a + 5 = 29$

9. $12x - 7 = 53$

Write an equation. Solve for the variable.

10. 8 less than twice a number is 6.

Complete.

11. Barb has \$5 more than Jack.
Jack has \$3 more than Nora.
Let n = Nora's amount of money.
Then __?__ = Jack's amount of money, and
__?__ = Barb's amount of money.

Solve.

12. Roger has twice as much money as Elaine.
Together they have \$84.
How much does each have?

13. There are 850 students in Franklin High School.
There are 30 more girls than boys.
How many girls are there?

14. The difference between $8y$ and $2y$ is 42.
What is y?

Solve and check.

15. $12x = 36 + 8x$

16. $3m = 40 - 7m$

17. $x + 24 = 3x$

18. $2(y + 1) = 3 + y$

19. $8(x - 3) = 1 + 3x$

20. $5(m - 4) = 4 + m$

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Chapter 3

Compare the numbers. Write $>$ or $<$.

1. $7 \; ? \; 8$ **2.** $-5 \; ? \; -4$ **3.** $1 \; ? \; -3$ **4.** $-7 \; ? \; 0$

Graph the solution of the equation.

5. $x + 1 = 2$

Graph the solutions of the inequality.

6. $x < 2$

Add.

7. $(-3) + (-4)$ **8.** $(-3) + 1$ **9.** $2 + (-4)$ **10.** $(-2) + 5$

Subtract.

11. $6 - (-1)$ **12.** $-9 - (-8)$ **13.** $-5 - 4$ **14.** $6 - 10$

Multiply.

15. $(-2)(-7)$ **16.** $(x - 3)x$ **17.** $(x + y)(-3)$ **18.** $-5(-m + n)$

Divide.

19. $\frac{56}{8}$ **20.** $\frac{-16}{4}$ **21.** $\frac{-33}{-11}$ **22.** $\frac{25}{-5}$

Simplify.

23. $-4x + 3x$ **24.** $-a - 3 - 6a$ **25.** $-3y + 2y + 1$

26. $-m^2 + 11m^2$ **27.** $-(a)^2$ **28.** $-(-3x - 2)$

29. $-(-x)^2$ **30.** $y - (4x - 3y)$ **31.** $-m - (-2m + n)$

Solve for x.

32. $-10 + x = 5$ **33.** $5x - 4x = -10$ **34.** $4x + 3 = 3x$

35. $5x = 8x - 15$ **36.** $5x = -35$ **37.** $6x - 8x = -12$

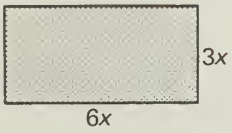
38. $-8x = 40$ **39.** $4x - 7x = 12$ **40.** $4x + 10 = 6x$

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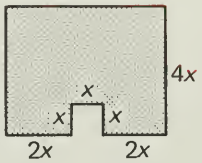
Chapter 4

Find the perimeter *and* the area of the shaded figure.

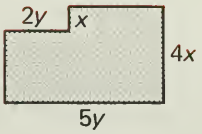
1.



2.



3.



4. The length of a rectangle is 8 cm more than the width.
The perimeter is 40 cm.
Find the length and width.

5. One can of paint will cover 5 square meters of area.
How many cans of paint are needed to paint a floor that is 10 meters by 10 meters?

6. Each edge of a cube is $2x$ units long.
What is the volume of the cube?

7. Joy buys 2 bats and 3 balls.
Bats cost $\$x$ each.
Balls cost $\$y$ each.
The total cost is __?__.

8. Bert walks for 2 hours at x km/h.
He rides for 5 hours at y km/h.
His total distance is __?__.

9. Two cars are 510 km apart.
They travel toward each other.
One travels at 80 km/h.
The other travels at 90 km/h.
How long will it take them to meet?

Solve for x.

10. **a.** $2x + 1 = 7$ **b.** $2x + 1 = a$ **c.** $2x + y = a$

11. Solve $A = bh$ for b.

12. Solve $\frac{t}{s} = r$ for t.

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Chapter 5

Add.

1. $\begin{array}{r} a + 6 \\ \underline{4a + 1} \end{array}$ **2.** $\begin{array}{r} 6n - 5 \\ \underline{n + 4} \end{array}$ **3.** $\begin{array}{r} 1 + n^2 \\ \underline{4 + 5n^2} \end{array}$ **4.** $\begin{array}{r} a^2 + b^2 \\ \underline{3a^2 - b^2} \end{array}$

5. $(5x + 3) + (6x + 1)$ **6.** $(3x - y) + (7x + 2y)$ **7.** $(n + 5) + (3n - 2)$

Subtract.

8. $\begin{array}{r} 5n + 1 \\ \underline{-(2n - 5)} \end{array}$ **9.** $\begin{array}{r} 7x - 3 \\ \underline{-(3x + 4)} \end{array}$ **10.** $\begin{array}{r} x^2 + y^2 \\ \underline{-(x^2 - y^2)} \end{array}$ **11.** $\begin{array}{r} mn + 4 \\ \underline{-(4mn - 1)} \end{array}$

12. $(a + 1) - (5a - 2)$ **13.** $(x + 2y) - (x + 8y)$ **14.** $(x^2 + 1) - (5x^2 + 1)$

Simplify.

15. $(n^2)^3$ **16.** $(-2x)^2$ **17.** $(xy^2)^4$ **18.** $(-3n^2)^3$

Multiply.

19. $x^2 \cdot x^3$ **20.** $(-6n^2)(2n^2)$ **21.** $(7a^2)(ab)$

22. $6(y + 8)$ **23.** $-2(4 - 2m)$ **24.** $a(2a + b)$

25. $(x - 2)(x - 4)$ **26.** $(2n - 3)(n + 3)$ **27.** $(x + y)(3x + 2y)$

28. $(n + 3)(n + 6)$ **29.** $(x - 4)(x - 8)$ **30.** $(3y + 1)(2y - 1)$

Express as a trinomial square.

31. $(x + 2)^2$ **32.** $(n - 3)^2$ **33.** $(2a + b)^2$ **34.** $(3x - 2y)^2$

Divide.

35. $\dfrac{5a^2}{a}$ **36.** $\dfrac{-25n^4}{5n^2}$ **37.** $\dfrac{16ab^3}{-8ab}$ **38.** $\dfrac{-36m^3n^2}{-9m^2n^2}$

39. $\dfrac{14a + 7b}{7}$ **40.** $\dfrac{2a - a^2}{a}$ **41.** $\dfrac{5m^3 - 10m^2 + 15m}{5m}$

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Chapter 6

Find the greatest common factor of each pair.

1. 10, 12 **2.** 14, 98 **3.** $8x^2$, $40x$ **4.** $12y$, 18

5. 16, 72 **6.** 48, 56 **7.** $3x$, $12x^2$ **8.** $5a$, $25a^3$

Factor.

9. $2x^2 + 12$ **10.** $x^2y + xy^2$ **11.** $x^2 + 8x + 16$

12. $y^2 + 14y + 49$ **13.** $n^2 + 20n + 64$ **14.** $n^2 - 10n + 25$

15. $y^2 - 7y + 12$ **16.** $x^2 - 9x + 14$ **17.** $x^2 + 5x - 14$

18. $y^2 - 5y - 14$ **19.** $m^2 - m - 20$ **20.** $n^2 - 4n - 12$

Multiply.

21. $(x + 5)(x - 5)$ **22.** $(n - 3)(n + 3)$ **23.** $(3x + 1)(3x - 1)$

24. $(1 + 4a)(1 - 4a)$ **25.** $(5x - 2)(5x + 2)$ **26.** $(8n + 3)(8n - 3)$

Factor.

27. $x^2 - y^2$ **28.** $x^2 - 9y^2$ **29.** $16 - n^2$

30. $n^2 - 1$ **31.** $4x^2 - 9$ **32.** $x^3 - 4x$

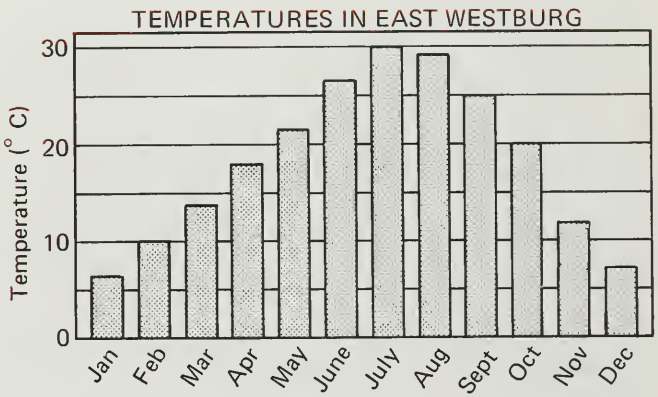
33. $2x^2 + 8x + 8$ **34.** $4n^2 - 24n + 20$ **35.** $3x^2 - 12$

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Chapter 7

The graph at the right shows the average monthly temperatures for East Westburg.

1. Which month has the highest average temperature?
2. About what is the average temperature in March?
3. About how much warmer is it in May than in November?



Complete.

4. $y = -5x$

x	y
0	?
1	?
−1	?

5. $x + y = 7$

x	y
0	?
1	?
−1	?

6. $y = -x + 4$

x	y
0	?
1	?
−1	?

7. $y = 3x + 4$

x	y
0	?
1	?
−1	?

Draw a graph of the equation.

8. $y = x - 4$ **9.** $y = -3x + 1$ **10.** $y + x = 4$

Give the slope of the line when the graph of the equation is drawn.

11. $y = 5x - 6$ **12.** $y = -2x + 1$ **13.** $y = \frac{1}{2}x - 3$

The Athletic Club earns 75¢ for each cake it sells.

14. The amount of money the club earns is a function of ___?___.

15. Complete the table.

n = number sold	10	15	20	50	?
A = Amount earned	?	?	?	?	90

16. An equation relating n and A in Exercise 15 is: $A =$ ___?___.

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Chapter 8

Solve by the graphing method.

1. $y = x + 2$
 $y = 3x$

2. $y - x = 2$
 $y + x = 0$

3. $y = 3x + 1$
 $y - x = 2$

Find the slope of the line whose equation is given.

4. $y = 8x$

5. $y + 4x = 0$

6. $7x + y = -3$

7. $y - \frac{1}{4}x = 0$

Tell whether the equations share *one* solution pair, *no* solution pair, or *all* solution pairs.

8. $y + x = -5$
 $y = x + 5$

9. $y - 7x = 3$
 $y = 7x - 6$

10. $y = -3x + 4$
 $y + 3 = 4$

Solve.

11. $y = -2x$
 $x + y = 3$

12. $y = 3x - 2$
 $2x + y = 13$

13. $3x + y = 4$
 $x - y = 8$

14. $-x + 2y = 7$
 $x - 3y = 3$

15. $4x + 5y = 11$
 $x - 5y = 9$

16. $x + 3y = 26$
 $x - 4y = 5$

17. $4x + y = 21$
 $x + y = 6$

18. $x - 4y = 1$
 $3x + 2y = 31$

19. $2x + 3y = 0$
 $3x + y = 7$

Solve. Use two variables and two equations.

20. The sum of two numbers is 16.
 Their difference is 12.
 Find the numbers.

21. One pizza and three colas cost \$3.20.
 Three pizzas and six colas cost \$8.70.
 How much does each item cost?

22. Two adult tickets and 2 student tickets cost \$2.70.
 Three adult tickets and 4 student tickets cost \$4.65.
 How much does each kind of ticket cost?

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Chapter 9

Simplify.

1. $\frac{5a}{10}$ **2.** $\frac{25m^2}{5m}$ **3.** $\frac{3-n}{5n-15}$ **4.** $\frac{(x+2)^2}{4-x^2}$

5. $\frac{6}{30} \cdot \frac{25}{3}$ **6.** $\frac{x}{5} \cdot \frac{35}{x^2}$ **7.** $\frac{3}{5x-10} \cdot \frac{x-2}{5}$ **8.** $\frac{m^2-4}{9m} \cdot \frac{3m}{m-2}$

9. $\frac{1}{7} \div \frac{3}{14}$ **10.** $\frac{x}{12} \div \frac{x}{3}$ **11.** $\frac{4}{x^2y^2} \div \frac{16}{xy}$ **12.** $\frac{m+1}{6} \div \frac{2m+2}{3}$

13. $\frac{1}{4} + \frac{5}{4}$ **14.** $\frac{6x}{3} - \frac{2x}{3}$ **15.** $\frac{x+2}{3x} - \frac{1}{3x}$ **16.** $\frac{7}{x+y} + \frac{x}{x+y}$

17. $\frac{1}{4} + \frac{1}{3}$ **18.** $\frac{n}{5} - \frac{n}{2}$ **19.** $\frac{3}{a} + \frac{a}{2}$ **20.** $\frac{1}{3b} - \frac{5}{9b^2}$

Solve.

21. $\frac{x}{5} = \frac{1}{2}$ **22.** $\frac{n}{2} = \frac{n+3}{4}$ **23.** $\frac{x-1}{x+1} = \frac{2}{5}$

24. $\frac{x}{5} - \frac{x}{3} = 20$ **25.** $\frac{3}{4n} - \frac{1}{n} = \frac{1}{4}$ **26.** $\frac{4x}{3} + \frac{x}{2} = 11$

27. 3 dozen eggs cost \$1.56.
How much do 2 dozen cost?

28. Amanda and Tom are clearing snow from the sidewalk.
Amanda could do the job in 3 hours alone.
Tom could do the job in 4 hours alone.
How long will it take them to do the job together?

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Chapter 10

Add or subtract.

1. $0.52 + 0.98$ **2.** $16 - 3.19$ **3.** $5.04 - 0.813$ **4.** $9 + 8.5 + 0.62$

Multiply.

5. 3.2×5.01 **6.** 6×7.55 **7.** 10×3.75 **8.** 0.03×7.89

Divide. Round the answer to one decimal place.

9. $0.2\overline{)8.43}$ **10.** $6.1\overline{)75.2}$ **11.** $0.01\overline{)6.4}$ **12.** $0.06\overline{)0.75}$

Write as a decimal. Use three dots with a repeating decimal.

13. $\frac{5}{8}$ **14.** $\frac{1}{6}$ **15.** $\frac{4}{5}$ **16.** $\frac{3}{8}$ **17.** $\frac{2}{9}$

Write as a percent.

18. 0.08 **19.** 0.035 **20.** $\frac{1}{3}$ **21.** 0.6 **22.** $\frac{3}{4}$

Write as a decimal.

23. 55% **24.** $1\frac{1}{2}\%$ **25.** 3.5% **26.** 2.25% **27.** $8\frac{1}{3}\%$

Solve.

28. $0.6x + 0.9x = 45$ **29.** $0.8a = 64$ **30.** $0.4(x + 4) + 0.6x = 0$

31. Two storms left a total of 32.9 cm of snow on the ground. One storm left 7.7 cm more than the other. How much snow fell in each storm?

Write as a fraction.

32. 70% **33.** 15% **34.** 85% **35.** $33\frac{1}{3}\%$ **36.** 1%

Write an equation and solve.

37. 50% of 84 is what number?

38. What percent of 50 is 20?

39. 40% of what number is 16?

40. What percent of 12 is 4?

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Chapter 11

Find the value.

1. $\sqrt{49}$ 2. $\sqrt{36}$ 3. $-\sqrt{4}$ 4. $\sqrt{13^2}$ 5. $\sqrt{121}$

Use the table on page 350 to find the value. Round to one decimal place.

6. $\sqrt{90}$ 7. $\sqrt{15}$ 8. $\sqrt{10}$ 9. $\sqrt{5}$ 10. $\sqrt{7}$

Is the number rational or irrational?

11. 4 12. $\frac{1}{3}$ 13. $3\frac{1}{2}$ 14. $\sqrt{5}$ 15. $\sqrt{16}$

Find the value in simplest radical form.

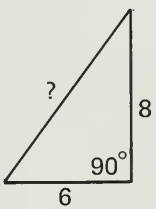
16. $\sqrt{12}$ 17. $\sqrt{54}$ 18. $\sqrt{90}$ 19. $\sqrt{300}$

Find the solutions to the equation.

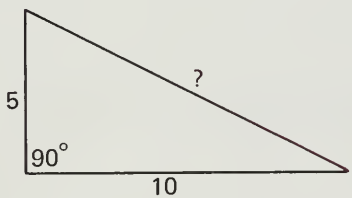
20. $n^2 = 36$ 21. $\frac{x}{5} = \frac{5}{x}$ 22. $x^2 - 4 = 32$

Find the length of the third side of the triangle.

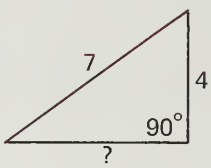
23.



24.



25.



Find the square root. Results should be in simplest form.

26. $\sqrt{\frac{3}{36}}$ 27. $\sqrt{\frac{36}{81}}$ 28. $\sqrt{\frac{n^2}{9}}$ 29. $\sqrt{\frac{4x^2}{25}}$

Simplify.

30. $5 \cdot 2\sqrt{5}$ 31. $3\sqrt{6} \cdot 2\sqrt{6}$ 32. $\sqrt{2} \cdot \sqrt{8}$ 33. $2\sqrt{n} \cdot 5\sqrt{n}$

34. $\frac{\sqrt{25}}{\sqrt{5}}$ 35. $\sqrt{\frac{3}{8}}$ 36. $-\sqrt{\frac{x^2}{3x}}$ 37. $\frac{36x}{\sqrt{18}}$

Express in simplest form.

38. $\sqrt{6} - 4\sqrt{6}$ 39. $5\sqrt{2} + 2\sqrt{4}$ 40. $\sqrt{6} + 4\sqrt{24}$

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Chapter 12

Solve.

1. $x(x + 4) = 0$
2. $5m(m - 6) = 0$
3. $(a - 5)(a - 4) = 0$
4. $(x + 3)(x - 9) = 0$
5. $x^2 + 11x = 0$
6. $6m - m^2 = 0$
7. $y^2 + 7y + 12 = 0$
8. $x^2 - 8x + 16 = 0$
9. $m^2 = -2m$
10. $x^2 - x = 12$
11. $n^2 = -4n + 5$
12. $y^2 = 15y - 44$
13. $x^2 = 25$
14. $3m^2 = 12$
15. $y^2 = 24$
16. $n^2 + 2 = 18$
17. $(m - 1)^2 = 36$
18. $(x + 9)^2 = 100$
19. $(2m + 3)^2 = 25$
20. $(3x - 4)^2 = 64$
21. $x^2 - x - 3 = 0$
22. $x^2 - 3x - 4 = 0$
23. $3x^2 - 15 = 0$
24. $2x^2 = 1 - 3x$

25. The length of a rectangle is 6 cm more than the width. The area is 81 cm^2. Find the width.

26. Make a table of values for the equation $y = x^2 - 3$. For what values of x does $y = 6$?

27. Make a table of values for the equation $y = x^2 - 3x + 4$. For what values of x does $y = 2$?

Draw the graph of the equation.

28. $y = x^2 - 5$
29. $y = 3x^2 + 2$
30. $y = x^2 + x - 3$

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Answers to Chapter Tests

Chapter 1

1. 13 **2.** 18 **3.** 2 **4.** 20 **5.** 19 **6.** 22
7. 0 **8.** $11x$ **9.** $7a$ **10.** $4x + y$ **11.** 25
12. 32 **13.** 27 **14.** 16 **15.** $15x$ **16.** $24a$
17. $16x^3$ **18.** $63x$ **19.** $12a^2$ **20.** $15c^3$
21. $3x + 12$ **22.** $15a - 35$ **23.** $18a - 12b$
24. $20 - 16m$ **25.** 0 **26.** impossible
27. 0 **28.** 1 **29.** $7x$ **30.** $4a$ **31.** 30
32. 6 **33.** 3 **34.** 13 **35.** 4 and 5

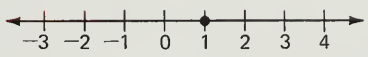
Chapter 2

1. 22 **2.** 9 **3.** 12 **4.** 20 **5.** 21 **6.** 7
7. 2 **8.** 8 **9.** 5 **10.** $2n - 8 = 6$; $n = 7$
11. $n + 3$; $n + 8$ **12.** Elaine: \$28; Roger: \$56 **13.** 440 **14.** 7 **15.** 9 **16.** 4 **17.** 12
18. 1 **19.** 5 **20.** 6

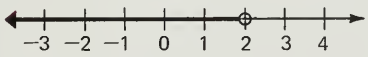
Chapter 3

1. $<$ **2.** $<$ **3.** $>$ **4.** $<$

5.



6.



7. -7 **8.** -2 **9.** -2 **10.** 3 **11.** 7
12. -1 **13.** -9 **14.** -4 **15.** 14
16. $x^2 - 3x$ **17.** $-3x - 3y$ **18.** $5m - 5n$
19. 7 **20.** -4 **21.** 3 **22.** -5 **23.** $-x$
24. $-7a - 3$ **25.** $-y + 1$ **26.** $10m^2$
27. $-a^2$ **28.** $3x + 2$ **29.** $-x^2$
30. $5y - 4x$ **31.** $m - n$ **32.** 15 **33.** -10
34. -3 **35.** 5 **36.** -7 **37.** 6 **38.** -5
39. -4 **40.** 5

Chapter 4

1. $P = 18x$; $A = 18x^2$ **2.** $P = 18x$; $A = 11x^2$ **3.** $P = 10y + 8x$; $A = 18xy$
4. length: 14 cm; width: 6 cm **5.** 20
6. $8x^3$ **7.** $(2x + 3y)$ dollars
8. $(2x + 5y)$ km/h **9.** 3 hours **10. a.** 3
b. $\frac{a-1}{2}$ **c.** $\frac{a-y}{2}$ **11.** $b = \frac{A}{h}$ **12.** $t = sr$

Chapter 5

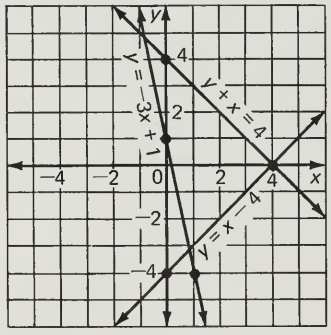
1. $5a + 7$ **2.** $7n - 1$ **3.** $5 + 6n^2$ **4.** $4a^2$
5. $11x + 4$ **6.** $10x + y$ **7.** $4n + 3$
8. $3n + 6$ **9.** $4x - 7$ **10.** $2y^2$
11. $-3mn + 5$ **12.** $-4a + 3$ **13.** $-6y$
14. $-4x^2$ **15.** n^6 **16.** $4x^2$ **17.** x^4y^8
18. $-27n^6$ **19.** x^5 **20.** $-12n^4$ **21.** $7a^3b$
22. $6y + 48$ **23.** $-8 + 4m$ **24.** $2a^2 + ab$
25. $x^2 - 6x + 8$ **26.** $2n^2 + 3n - 9$
27. $3x^2 + 5xy + 2y^2$ **28.** $n^2 + 9n + 18$
29. $x^2 - 12x + 32$ **30.** $6y^2 - y - 1$
31. $x^2 + 4x + 4$ **32.** $n^2 - 6n + 9$
33. $4a^2 + 4ab + b^2$ **34.** $9x^2 - 12xy + 4y^2$
35. $5a$ **36.** $-5n^2$ **37.** $-2b^2$ **38.** $4m$
39. $2a + b$ **40.** $2 - a$ **41.** $m^2 - 2m + 3$

Chapter 6

1. 2 **2.** 14 **3.** $8x$ **4.** 6 **5.** 8 **6.** 8 **7.** $3x$
8. $5a$ **9.** $2(x^2 + 6)$ **10.** $xy(x + y)$
11. $(x + 4)^2$ **12.** $(y + 7)^2$
13. $(n + 16)(n + 4)$ **14.** $(n - 5)^2$
15. $(y - 3)(y - 4)$ **16.** $(x - 7)(x - 2)$
17. $(x + 7)(x - 2)$ **18.** $(y - 7)(y + 2)$
19. $(m - 5)(m + 4)$ **20.** $(n - 6)(n + 2)$
21. $x^2 - 25$ **22.** $n^2 - 9$ **23.** $9x^2 - 1$
24. $1 - 16a^2$ **25.** $25x^2 - 4$ **26.** $64n^2 - 9$
27. $(x - y)(x + y)$ **28.** $(x - 3y)(x + 3y)$
29. $(4 - n)(4 + n)$ **30.** $(n - 1)(n + 1)$
31. $(2x - 3)(2x + 3)$ **32.** $x(x - 2)(x + 2)$
33. $2(x + 2)^2$ **34.** $4(n - 5)(n - 1)$
35. $3(x + 2)(x - 2)$

Chapter 7

1. July **2.** about 13° C **3.** about 10° **4.** 0; -5; 5 **5.** 7; 6; 8 **6.** 4; 3; 5 **7.** 4; 7; 1
8.–10.



11. 5 **12.** -2 **13.** $\frac{1}{2}$ **14.** the number of cakes it sells **15.** \$7.50; \$11.25; \$15; \$37.50; 120 **16.** $A = 75n¢$

Chapter 8

1. (1, 3) **2.** (−1, 1) **3.** $(\frac{1}{2}, 2\frac{1}{2})$

4. 8 **5.** −4 **6.** −7 **7.** $\frac{1}{4}$ **8.** one **9.** no

10. all **11.** (−3, 6) **12.** (3, 7) **13.** (3, −5)
14. (−27, −10) **15.** (4, −1) **16.** (17, 3)
17. (5, 1) **18.** (9, 2) **19.** (3, −2) **20.** 14, 2
21. pizza: \$2.30; cola: \$.30 **22.** adult: \$.75; student: \$.60

Chapter 9

1. $\frac{a}{2}$ **2.** $5m$ **3.** $-\frac{1}{5}$ **4.** $\frac{x+2}{2-x}$ **5.** $\frac{5}{3}$ or $1\frac{2}{3}$ **6.** $\frac{7}{x}$ **7.** $\frac{3}{25}$ **8.** $\frac{m+2}{3}$ **9.** $\frac{2}{3}$ **10.** $\frac{1}{4}$

11. $\frac{1}{4xy}$ **12.** $\frac{1}{4}$ **13.** $\frac{3}{2}$ or $1\frac{1}{2}$ **14.** $\frac{4x}{3}$

15. $\frac{x+1}{3x}$ **16.** $\frac{7+x}{x+y}$ **17.** $\frac{7}{12}$ **18.** $-\frac{3n}{10}$

19. $\frac{6+a^2}{2a}$ **20.** $\frac{3b-5}{9b^2}$ **21.** $\frac{5}{2}$ or $2\frac{1}{2}$

22. 3 **23.** $\frac{7}{3}$ or $2\frac{1}{3}$ **24.** −150 **25.** −1

26. 6 **27.** \$1.04 **28.** $\frac{12}{7}$ or $1\frac{5}{7}$ hours

Chapter 10

1. 1.5 **2.** 12.81 **3.** 4.227 **4.** 18.12
5. 16.032 **6.** 45.3 **7.** 37.5 **8.** 0.2367
9. 42.2 **10.** 12.3 **11.** 640.0 **12.** 12.5
13. 0.625 **14.** 0.166 . . . **15.** 0.8 **16.** 0.375
17. 0.22 . . . **18.** 8% **19.** 3.5% **20.** $33\frac{1}{3}\%$
21. 60% **22.** 75% **23.** 0.55 **24.** 0.015
25. 0.035 **26.** 0.0225 **27.** 0.08333 . . .
28. 30 **29.** 80 **30.** −1.6 **31.** 20.3 cm, 12.6 cm **32.** $\frac{7}{10}$ **33.** $\frac{3}{20}$ **34.** $\frac{17}{20}$ **35.** $\frac{1}{3}$

36. $\frac{1}{100}$ **37.** $0.5(84) = n$; 42
38. $50n = 20$; 40% **39.** $0.4n = 16$; 40
40. $12n = 4$; $33\frac{1}{3}\%$

Chapter 11

1. 7 **2.** 6 **3.** −2 **4.** 13 **5.** 11 **6.** 9.5
7. 3.9 **8.** 3.2 **9.** 2.2 **10.** 2.6 **11.** rational
12. rational **13.** rational **14.** irrational
15. rational **16.** $2\sqrt{3}$ **17.** $3\sqrt{6}$
18. $3\sqrt{10}$ **19.** $10\sqrt{3}$ **20.** ± 6 **21.** ± 5
22. ± 6 **23.** 25 **24.** $5\sqrt{5}$ **25.** $\sqrt{13}$

26. $\frac{\sqrt{3}}{6}$ **27.** $\frac{2}{3}$ **28.** $\frac{n}{3}$ **29.** $\frac{2x}{5}$ **30.** $10\sqrt{5}$

31. 36 **32.** 4 **33.** $10n$ **34.** $\sqrt{5}$ **35.** $\frac{\sqrt{6}}{4}$

36. $\frac{-\sqrt{3x}}{3}$ **37.** $6x\sqrt{2}$ **38.** $-3\sqrt{6}$

39. $5\sqrt{2} + 4$ **40.** $9\sqrt{6}$

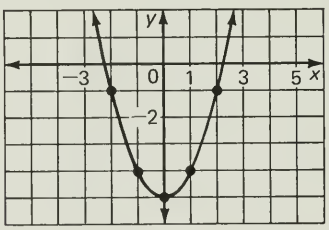
Chapter 12

1. 0 or −4 **2.** 0 or 6 **3.** 5 or 4
4. −3 or 9 **5.** 0 or −11 **6.** 0 or 6
7. −3 or −4 **8.** 4 **9.** 0 or −2
10. 4 or −3 **11.** −5 or 1 **12.** 11 or 4
13. ± 5 **14.** ± 2 **15.** $\pm 2\sqrt{6}$ **16.** ± 4
17. 7 or −5 **18.** 1 or −19 **19.** 1 or −4
20. 4 or $-\frac{4}{3}$ **21.** $\frac{1 \pm \sqrt{13}}{2}$ **22.** 4 or −1

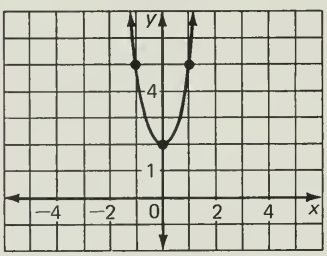
23. $\pm\sqrt{5}$ **24.** $\frac{-3 \pm \sqrt{17}}{4}$

25. width: $\frac{-3 + 3\sqrt{2}}{2}$ cm **26.** ± 3 **27.** 2, 1

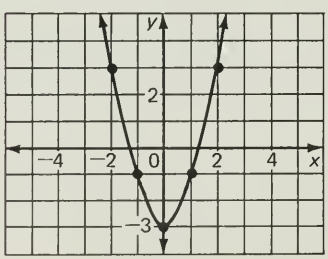
28.



29.



30.



Guide to Individualized Assignments

The chart which follows summarizes the basic assignments for an average course and a comprehensive course. The assignments for the average course include those concepts which are prerequisites for the lessons to follow. The assignments for the comprehensive course include all the concepts presented in this text, at times providing somewhat less drill than the assignments for the average course.

Summary Time Schedule for the Assignments

Chapter	1	2	3	4	5	6	7	8	9	10	11	12	Total
Average Course	14	16	13	10	13	15	10	11	18	14	13	13	160
Comprehensive Course	13	15	12	10	14	17	9	11	19	14	13	13	160

Lesson	AVERAGE COURSE		COMPREHENSIVE COURSE	
	Pages	Assignment	Pages	Assignment
	Chapter 1		***Chapter 1***	
1	4,5	all	4,5	all
2	7	all	7	all
3	9	all	9	all
4	11	all	11	all
5	13	all	13	all
6	15	all	15	all
7	17	1–30	17	all
8	19	all	19	all
9	21	all; Self-Test	21	all; Self-Test
10	23	1–14	23	odd or even
11	25	all; Self-Test	25	all; Self-Test

Lesson	AVERAGE COURSE		COMPREHENSIVE COURSE	
	Pages	Assignment	Pages	Assignment
12	26 27	Reviewing Arithmetic Skills Career Notebook	27 28,29 30,31	Career Notebook Consumer Corner Reviewing the Chapter
13	30,31	Reviewing the Chapter	T11	Chapter 1 Test
14	T11	Chapter 1 Test		
	Chapter 2		**Chapter 2**	
15	35	1–32	35	odd or even
16	37, top	1–16	37, top	odd or even
17	37, btm	all	37, btm	all
18	39	all	39	all
19	43–45	1–54 even	43–45	odd or even; Self-Test
20	43–45	1–54 odd; Self-Test	47	all
21	47	1–6	48,49	all; Self-Test
22	48,49	1–20; Self-Test	51–53	odd or even; Puzzles & Things
23	51–53	1–16	55–57	odd or even; Self-Test
24	55–57	1–12; Self-Test	59	odd or even
25	59	1–36	61	odd or even
26	61	1–29	62,63	odd or even; Self-Test
27	62,63	1–6; Self-Test	64,65 66,67	Consumer Corner Reviewing the Chapter
28	66,67	Reviewing the Chapter	T12	Chapter 2 Test
29	T12	Chapter 2 Test	68	Cumulative Review
30	68	Cumulative Review	**Chapter 3**	
	Chapter 3		73 75	odd or even odd or even; Self-Test
31	73	1–19	77	all
			80,81	odd or even; Puzzles & Things

Lesson	AVERAGE COURSE Pages	Assignment	COMPREHENSIVE COURSE Pages	Assignment
32	75	1–30; Self-Test	83	odd or even; Self-Test
33	77	all	85	odd or even
34	80,81	odd or even	87	odd or even
35	83	1–27; Self-Test	89	all
36	85	1–43	91	all; Self-Test
37	87	1–40	92,93	odd or even
38	89	all	95 96,97 98,99	Career Notebook Consumer Corner Reviewing the Chapter
39	91	all; Self-Test	T13	Chapter 3 Test
40	94	Reviewing Arithmetic Skills	438	Cumulative Review
41	95 98,99	Career Notebook Reviewing the Chapter		
42	T13	Chapter 3 Test		
43	438	Cumulative Review		
	Chapter 4		**Chapter 4**	
44	103	all	103	all
45	106,107	1–22	106,107	odd or even
46	110,111	1–10; Self Test	110,111	odd or even
47	113	1–12	113	all
48	116,117	1–6	116,117 119–121	odd or even odd or even; Self-Test
49	119–121	1–6; Self-Test	124,125	odd or even; Self-Test
50	124,125	1–34; Self-Test	127	all
51	130,131	Reviewing the Chapter	128,129 130,131	Consumer Corner Reviewing the Chapter
52	T14	Chapter 4 Test	T14	Chapter 4 Test
53	132	Cumulative Review	132	Cumulative Review
			Chapter 5	
			137	all
			139	all; Self-Test
			141	all; Puzzles & Things

Lesson	AVERAGE COURSE Pages	Assignment	COMPREHENSIVE COURSE Pages	Assignment
	Chapter 5			
54	137	all	143	odd or even
55	139	1–25; Self-Test	145	odd or even; Self-Test
56	141	all	147	odd or even
57	143	1–27	150,151	odd or even; Puzzles & Things
58	145	1–22; Self-Test	153	all; Self-Test
59	147	1–24	155	odd or even; Puzzles & Things
60	150,151	1–35	157	all; Puzzles & Things
61	153	all; Self-Test	159	odd or even; Self-Test
62	155	1–24	161	odd or even; Puzzles & Things
63	159	1–21; Self-Test	163 164,165 166,167	Career Notebook Consumer Corner Reviewing the Chapter
64	162 163	Reviewing Arithmetic Skills Career Notebook	T15	Chapter 5 Test
65	166,167	Reviewing the Chapter		*Chapter 6*
66	T15	Chapter 5 Test	171	all
	Chapter 6		173	odd or even
67	171	all	175	all; Self-Test
68	173	1–22	178,179	odd
69	178,179	1–42 odd	178,179	even; Puzzles & Things
70	178,179	1–42 even	181	all
71	181	all	183	all
72	183	all	186	odd
73	186	odd	186	even
74	186	even	187	all; Self-Test

Lesson	AVERAGE COURSE		COMPREHENSIVE COURSE	
	Pages	Assignment	Pages	Assignment
75	187	all; Self-Test	189	odd or even
76	189	1–21	191	odd or even
77	191	1–24	193	odd or even; Self-Test
78	193	1–15; Self-Test	195	odd or even; Puzzles & Things
79	198,199	Reviewing the Chapter	196,197 198,199	Consumer Corner Reviewing the Chapter
80	T16	Chapter 6 Test	T16	Chapter 6 Test
81	200 or 440	Cumulative Review	200 or 440	Cumulative Review
	Chapter 7		***Chapter 7***	
82	206,207	all	206,207	all
83	209–211	all	209–211	all
84	214,215	1–27; Self-Test	214,215	odd or even; Self-Test
85	218,219	1–38	218,219	odd or even
86	221	1–21 odd or even	221	odd or even
87	223	1–20	223	odd or even
88	226,227	1–11; Self-Test	226,227	all; Self-Test
89	228 229	Reviewing Arithmetic Skills Career Notebook	229 230,231 232,233	Career Notebook Consumer Corner Reviewing the Chapter
90	232,233	Reviewing the Chapter	T17	Chapter 7 Test
			Chapter 8	
91	T17	Chapter 7 Test	238,239	odd or even; Puzzles & Things
	Chapter 8			
92	238,239	1–19	242,243	odd or even; Self-Test
93	242,243	1–28; Self-Test	245	odd or even
94	245	1–16	247–249	odd or even; Puzzles & Things

Lesson	AVERAGE COURSE Pages	Assignment	COMPREHENSIVE COURSE Pages	Assignment
95	247,248	1–32	252,253	odd or even; Puzzles & Things
96	249	odd or even; Self-Test	255	odd or even
97	252,253	1–8	256,257	odd or even; Self-Test
98	255	1–16	259	odd or even
99	256,257	1–6; Self-Test	260,261 262,263	Consumer Corner Reviewing the Chapter
100	262,263	Reviewing the Chapter	T18	Chapter 8 Test
101	T18	Chapter 8 Test	264	Cumulative Review
102	264	Cumulative Review		
	Chapter 9		***Chapter 9***	
103	269	1–30	269	odd or even
104	271	1–24	271	odd or even
105	273	1–32; Self-Test	273	odd or even; Self-Test
106	275	1–12	275	odd or even
107	277	1–9	277	odd or even
108	279	1–27	279	odd or even
109	281–283	1–14; Self-Test	281–283	odd or even; Self-Test
110	285	1–29	285	odd or even
111	287	1–22	287	odd or even
112	289	all; Self-Test	289	all; Self-Test
113	291	all	291	all; Puzzles & Things
114	293	1–28; Self-Test	293	odd or even; Self-Test
115	297	1–15	295	all; Puzzles & Things
			297	odd or even; Puzzles & Things
116	299–301	1–8; Self-Test	299–301	odd or even; Self-Test
117	304 305	Reviewing Arithmetic Skills Career Notebook	303	all; Puzzles & Things

Lesson	AVERAGE COURSE		COMPREHENSIVE COURSE	
	Pages	Assignment	Pages	Assignment
118	308,309	Reviewing the Chapter	305 306,307 308,309	Career Notebook Consumer Corner Reviewing the Chapter
119	T19	Chapter 9 Test	T19	Chapter 9 Test
120	442	Cumulative Review	442	Cumulative Review
	Chapter 10		***Chapter 10***	
121	313	all	313	all
122	315	all	315	all
123	317	all	317	all
124	319	1–22	319	odd or even
125	321	all; Self-Test	321	all; Self-Test
126	324,325	all	324,325	all
127	327	all	327	all
128	330,331	all; Self-Test	330,331	all; Self-Test
129	333	all	333	all
130	335	all	335	all
131	337–339	all; Self-Test	337–339	all; Self-Test
132	342,343	Reviewing the Chapter	340,341 342,343	Consumer Corner Reviewing the Chapter
133	T20	Chapter 10 Test	T20	Chapter 10 Test
134	344	Cumulative Review	344	Cumulative Review
	Chapter 11		***Chapter 11***	
135	349	1–28	349	odd or even; Puzzles & Things
136	351	1–40	351	odd or even
137	353	1–30	353	odd or
138	355	1–20; Self-Test	355	odd or even; Self-Test
139	357	all	357	all; Puzzles & Things
140	360,361	1–11; Self-Test	360,361	all; Self-Test

Lesson	AVERAGE COURSE		COMPREHENSIVE COURSE	
	Pages	Assignment	Pages	Assignment
141	363	all	363	all; Puzzles & Things
142	365	1–44	365	odd or even
143	367	1–20	367	odd or even
144	369	1–21; Self-Test	369	odd or even; Self-Test
145	372 373	Reviewing Arithmetic Skills Career Notebook	371	odd or even
146	376,377	Reviewing the Chapter	373 374,375	Career Notebook Consumer Corner
			376,377	Reviewing the Chapter
147	T21	Chapter 11 Test	T21	Chapter 11 Test
	Chapter 12		***Chapter 12***	
148	381	1–24	381	odd or even
149	383	all	383	all
150	385	1–24; Self-Test	385	odd or even; Self-Test
151	387	1–32	387	odd or even
152	389	1–21; Self-Test	389	odd or even; Self-Test
153	392	1–30	392	odd or even
154	393	odd	393	odd
155	393	even	393	even; Puzzles & Things
156	395	1–10	395	odd or even
157	398,399	1–24; Self-Test	398,399	odd or even; Self-Test
158	402,403	Reviewing the Chapter	400,401 402,403	Consumer Corner Reviewing the Chapter
159	T22	Chapter 12 Test	T22	Chapter 12 Test
160	404 or 444	Cumulative Review	404 or 444	Cumulative Review

Table of Related References

Chapter	Progress Tests	For use with Sections
Chapter 1	Test 1 Test 2 Test 3	1–4 5–9 10,11
Chapter 2	Test 4 Test 5 Test 6 Test 7 Test 8	1–3 4 5–6 7–8 9–11
Chapter 3	Test 9 Test 10 Test 11	1,2 3–5 6–9
Cumulative Review/Test	Test 12	
Chapter 4	Test 13 Test 14 Test 15	1–3 4–6 7
Chapter 5	Test 16 Test 17 Test 18 Test 19	1,2 3–5 6–8 9–11
Chapter 6	Test 20 Test 21 Test 22	1–3 4–8 8–10
Cumulative Review/Test	Test 23	

Chapter	Progress Tests	For use with Sections
Chapter 7	Test 24 Test 25	1–3 4–7
Chapter 8	Test 26 Test 27 Test 28	1,2 3,4 5–7
Chapter 9	Test 29 Test 30 Test 31 Test 32 Test 33	1–3 4–7 8–10 11,12 14,15
Cumulative Review/Test	Test 34	
Chapter 10	Test 35 Test 36 Test 37	1–5 6–8 9–11
Chapter 11	Test 38 Test 39 Test 40	1–4 5,6 7–10
Chapter 12	Test 41 Test 42 Test 43	1–3 4,5 6–10
Cumulative Review/Test	Test 44	

Lesson Commentary

Chapter 1. Working with Variables

Early in the program the function concept is introduced by using a calculating machine that accepts an input number and produces an output number. It is also through this medium that the students learn that variables "stand for" numbers. Simplifying expressions is an important skill, also developed in this chapter. It provides a foundation for the students' work with algebra in subsequent chapters.

Objectives for Pages 2–11

1. To find the value of expressions with variables.
2. To recognize and combine like terms.
3. To use the commutative and associative properties to simplify expressions.

1 • USING LETTERS FOR NUMBERS (pp. 2–5)

Teaching These Pages

The skill of substitution, introduced in this section, is essential in subsequent work. It is especially important when students verify that a number is a root of an equation.

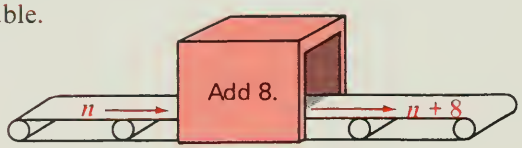
To introduce the notion of input and output, have the students arrange themselves in groups of four, then assign an operation to each student and a number to each group.

Student 1	Add 5 to the number. Pass the result on to Student 2.
Student 2	Add 3 to the number received and pass the result on.
Student 3	Subtract 2; pass the result on.
Student 4	Subtract 4; pass the result on.

If the students have been trained in the precise language of modern mathematics, which distinguishes a number from its representation (numeral), you might point out that this book uses "number" for both.

Chalkboard Examples

1. Complete the table.



INPUT	OUTPUT	
n	$n + 8$	
3	?	11
7	?	15
12	?	20

If $k = 6$, find the value of the expression.

2. $k + 3 = 9$ **3.** $12 - k = 6$ **4.** $5k = 30$ **5.** $k \div 3 = 2$

If $a = 4$ and $b = 5$, find the value of the expression.

6. $a + b = 9$ **7.** $5b = 25$ **8.** $2b - a = 6$ **9.** $3a + 2b = 22$

Suggested Extension

A path is hidden from *Start* to *Finish*. To find the path, you must find a pattern by finding the value of each expression if $a = 4$ and $b = 6$. You may move along the path horizontally or vertically. (The answers along the path are shown in red.)

Start ↓

$2b + 2a = 20$	$b - a = 2$	$b \div 2 = 3$	$a + 0 = 4$
$12 - b = 6$	$\frac{ab}{6} = 4$	$2b - 3a = 0$	$2b - a - 6 = 2$
$3a - 4 = 8$	$2b - 2a = 4$	$2b - a = 8$	$b + b - b = 6$
$a + b = 10$	$3b - 6 = 12$	$2a + b = 14$	$b \times b = 36$

↓ *Finish*

Each value along the path increases by 2.

2 • PARENTHESES, ORDER OF OPERATIONS (pp. 6-7)

Teaching These Pages

Have the students calculate the value of these expressions and make note of the various answers they obtain.

$$8 + 2 \times 3 \qquad 2 \times 8 + 3 \qquad 8 - 2 \times 3$$

The variety of answers introduces the need for the students to choose an order in which operations will be performed.

Insert parentheses in the above examples to illustrate the use of parentheses.

$$(8 + 2) \times 3 = 30 \qquad (2 \times 8) + 3 = 19$$
$$8 + (2 \times 3) = 14 \qquad 2 \times (8 + 3) = 22$$

Summarize that the order of operations is a convention mathematicians have agreed upon to avoid confusion.

Chalkboard Examples

Find the value of the expression.

1. $12 + 8 \times 2 = 28$ **2.** $(12 + 8) \times 2 = 40$ **3.** $16 + 8 \div 4 + 4 = 22$

4. $(16 + 8) \div (4 + 4) = 3$ **5.** $16 + 8 \div (4 + 4) = 17$ **6.** $(16 + 8) \div 4 + 4 = 10$

If $a = 6$, find the value of the expression.

7. $3(a + 2) = 24$ **8.** $3a - 6 = 12$ **9.** $3a + 2 = 20$ **10.** $3a - 4 = 14$

Suggested Extension

Use the symbols $+$, $-$, $\times$, $\div$, parentheses, and all the digits of the date 1492 to continue the pattern as far as you can.

$1 = 4 - 9 \div (2 + 3)$ $5 = (9 + 1) \div (4 \div 2)$
$2 = 9 - (1 + 2 + 4)$ $6 = 9 - 4 \div 2 - 1$
$3 = 9 - 4 - 2 \times 1$ $7 = 4 + 9 \div (2 + 1)$
$4 = (9 - 1) \div (4 \div 2)$ $8 = 9 + 1 - 4 \div 2$

Other dates can be used in a similar way in later sections as a review of this section.

3 • COEFFICIENTS AND TERMS (pp. 8-9)

Teaching These Pages

Point out that a shorthand is often used in mathematics.

$4 + 4 + 4 + 4 + 4$ is written as 5×4.
$n + n + n + n + n$ is written as $5 \times n$.

$5 \times n$ is then written more compactly as $5n$.

Often students will try to simplify $2x + 3y$ as $5xy$. The students can use the substitution work of Section 1 to identify this error. Let $x = 1$ and $y = 2$.

$$2x + 3y = 2 + 6 = 8$$
$$5xy = 5(1)(2) = 10$$

Chalkboard Examples

Simplify. Point out that $1x$ is written as x.

1. $2x + x = 3x$ **2.** $4y - 2y = 2y$

3. $a + 2b + b = a + 3b$ **4.** $x + 3x + 2y + 4y = 4x + 6y$

5. $4x - 3x + 6y - 5y = x + y$ **6.** $2a - 3b + 2c = 2a - 3b + 2c$

4 • PROPERTIES OF ADDITION (pp. 10–11)

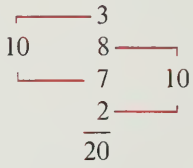
Teaching These Pages

To reinforce the meaning of *commutative,* show the students some operations that are not commutative.

$$4 - 2 \neq 2 - 4 \qquad 4 \div 2 \neq 2 \div 4$$

In a similar way, the meaning of *associative* could be reinforced by showing that division is an operation that is not associative.

Use numerical examples to point out that the commutative and associative properties are helpful in developing mental arithmetic.



Chalkboard Examples

Some students can check their work by substituting a value for the variable and finding the value of the expression.

Simplify.

1. $3a + 2 + 4a + 6 = 7a + 8$

2. $4b + 6 - 2b - 2 = 2b + 4$

3. $5x + 2x - 4y + x = 8x - 4y$

4. $12m + 10n + 10m - 2n = 22m + 8n$

Objectives for Pages 12–21

1. To find the value of expressions with exponents.

2. To use exponents to simplify expressions.

3. To use the commutative and associative properties to simplify expressions.

4. To use the distributive property to simplify expressions.

5. To use the properties of 0 and 1 to simplify expressions.

5 • EXPONENTS AND FACTORS (pp. 12–13)

Teaching These Pages

Point out that the use of exponents is another shorthand used in mathematics. Exponents are often used to write large numbers in a concise form. For example, one billion is written as 10^9, one trillion as 10^{12}, and the distance from the earth to the sun as 150×10^6 km.

Chalkboard Examples

Find the value of the expression.

1. $2^3 = 8$ **2.** $3^2 = 9$ **3.** $10^3 = 1000$ **4.** $3^4 = 81$ **5.** $5^1 = 5$

Simplify.

6. $y \cdot y \cdot y = y^3$ **7.** $a \cdot a \cdot a \cdot b = a^3b$ **8.** $8 \cdot x \cdot x \cdot x \cdot y \cdot y = 8x^3y^2$

Suggested Extensions

1. Which is the larger value: 2^5 or 5^2? 3^2 or 2^3? 2^4 or 4^2? 2^5; 3^2; $2^4 = 4^2$

The students should continue the above theme by picking other pairs of numbers and recording their results.

2. How many squares can you find in this figure? 5



How many squares in a 3-unit by 3-unit square? The students should find a pattern in order to predict the number of squares in 4- by 4-unit squares, 5- by 5-unit squares, etc. The number of squares in a 2- by 2-unit square is $2^2 + 1^2$, in a 3- by 3-unit square is $3^2 + 2^2 + 1^2$, etc.

6 • PROPERTIES OF MULTIPLICATION (pp. 14–15)

Teaching These Pages

This section extends the commutative and associative properties to multiplication. Again, these properties can be applied to mental arithmetic. To motivate the class, illustrate how quickly one can multiply by 50.

$$50 \cdot 24 = \left(\frac{1}{2} \cdot 100\right) \cdot 24 = \frac{1}{2} \cdot (100 \cdot 24) = \frac{1}{2} \cdot 2400 = 1200$$

Point out the meaning of *evaluate*.

Chalkboard Examples

1. Evaluate $2 \cdot 16 \cdot 5$. Show how you use the properties.

$$\begin{aligned} 2 \cdot 16 \cdot 5 &= 2 \cdot 5 \cdot 16 && \text{commutative property} \\ &= (2 \cdot 5) \cdot 16 && \text{associative property} \\ &= 10 \cdot 16 \\ &= 160 \end{aligned}$$

Simplify.

2. $3 \cdot 5y = 15y$ **3.** $2y \cdot 3y = 6y^2$ **4.** $(3k)(4k) = 12k^2$

5. $(4s)(5s) = 20s^2$ **6.** $5 \cdot a \cdot a \cdot 6 = 30a^2$ **7.** $3 \cdot k \cdot 2 \cdot k = 6k^2$

Suggested Extension

Explain each step of the method in Teaching These Pages to multiply by 50. Have the students use that method to compute these products mentally.

1. $50 \cdot 12 = 600$ **2.** $24 \cdot 50 = 1200$ **3.** $52 \cdot 50 = 2600$

4. $50 \cdot 76 = 3800$ **5.** $38 \cdot 50 = 1900$ **6.** $50 \cdot 98 = 4900$

Provide a sheet of questions based on the above skill. Have the students try the questions over a period of time to see the improvement in their mental computation skills.

7 • THE DISTRIBUTIVE PROPERTY (pp. 16-17)

Teaching These Pages

To introduce this topic, illustrate how quickly certain problems can be solved mentally by using the distributive property.

$$3 \cdot 99 = 3(100 - 1) = 300 - 3 = 297$$

The class may be motivated by randomly picking products that involve 99 and seeing how quickly the product can be found.

Chalkboard Examples

Evaluate each expression in two different ways.

1. $5(2 + 8) = 50$ **2.** $4(97 + 3) = 400$ **3.** $12(46 + 54) = 1200$

State the expression without parentheses.

4. $3(x + 2) = 3x + 6$ **5.** $4(y - 6) = 4y - 24$ **6.** $7(a - 2b) = 7a - 14b$

7. $8(3x - 2) = 24x - 16$ **8.** $6(2a + 4b) = 12a + 24b$ **9.** $5(x - 11y) = 5x - 55y$

Use the distributive property. Then combine like terms.

10. $3(2x + 3) - 6 = 6x + 3$ **11.** $4(a + b) - 4a = 4b$ **12.** $4(m + n) - 2n = 4m + 2n$

Suggested Extension

Explain each step of the method in Teaching These Pages to multiply by 99. Have the students use that method to compute these products mentally.

1. $99 \cdot 15 = 1485$ **2.** $22 \cdot 99 = 2178$ **3.** $40 \cdot 99 = 3960$

4. $99 \cdot 65 = 6435$ **5.** $99 \cdot 75 = 7425$ **6.** $99 \cdot 99 = 9801$

Provide a sheet of questions based on the above skill. Have the students try the questions over a period of time to see the improvement in their mental computation skills.

8 • PROPERTIES OF 0 (pp. 18–19)

Teaching These Pages

A "mind-reading trick" can be used in motivating the class to study zero. Have the students perform the following steps.

Steps	*Results*
1. Choose a whole number.	x
2. Double the number.	$2x$
3. Add 6.	$2x + 6$
4. Divide by 2.	$x + 3$
5. Subtract your original number.	3
6. Subtract 3.	0

Ask the class if they all obtained 0 for an answer. If any students have made mechanical errors, retrace the steps in order to locate the errors as well as review the work for the rest of the class.

Chalkboard Examples

Find the value if possible. (NP indicates not possible.)

1. $6 + 0 = 6$ **2.** $0 \cdot 9 = 0$ **3.** $\frac{0}{12} = 0$ **4.** $\frac{10 \cdot 0}{5} = 0$

5. $45 + 0 = 45$ **6.** $0^2 = 0$ **7.** $\frac{12}{0}$ NP **8.** $\frac{0}{0}$ NP

Let $x = 4$. Find the value of the expression.

9. $x + 4 = 8$ **10.** $6(x - 4) = 0$ **11.** $\frac{2x - 8}{12} = 0$ **12.** $5(3x - 11) = 5$

Simplify.

13. $9 + 2a - 2a + 9 = 18$ **14.** $8 + 2x - 2x - 8 = 0$ **15.** $3(m + 4) - 12 = 3m$

16. $(2x + x - 3x) \div 0$ NP **17.** $4a + 9 - 4a + 8 = 17$ **18.** $5(3 + 2x) - 15 = 10x$

Suggested Extension

1. Can you explain why the steps of the "mind-reading trick" given above always result in zero?

Students who can answer Question 1 can be encouraged to make up their own mind-reading tricks.

9 • PROPERTIES OF 1 (pp. 20–21)

Teaching These Pages

A "mind-reading trick" can be used to introduce this section. Have the students perform the following steps.

Steps	*Results*
1. Choose a whole number.	x
2. Add the next whole number to it.	$2x + 1$
3. Add 9.	$2x + 10$
4. Divide by 2.	$x + 5$
5. Subtract your original number.	5
6. Subtract 4.	1

All students should obtain the answer 1. If any students have made mechanical errors, retrace the steps.

Objectives for Pages 22–25

1. To check whether a given number is a solution of an equation.

2. To check whether a given number is a solution of an inequality.

10 • INTRODUCTION TO EQUATIONS (pp. 22–23)

Teaching These Pages

To introduce this lesson, use some recent baseball standings. Ask the students which of the following statements are true, and which are false.

1. Kansas City is at the top of the standings.
2. Texas is ahead of Minnesota in the standings.
3. Chicago is in last place.

	Won	Lost
Oakland	76	50
Kansas City. . .	68	55
Chicago	62	63
Texas.	61	66
Minnesota. . . .	58	68
California	58	69

Chalkboard Examples

1. Is $x + 5 = 9$ true if $x = 5$? if $x = 4$? no; yes

2. Is 4 a solution of $6 + y = 9$? no

3. Is 6 a solution of $12 = y + 6$? yes

Which of the numbers shown is a solution?

4. $y + 6 = 9$ 3 or 9? 3

5. $a - 3 = 6$ 9 or 3? 9

6. $48 = 6k$ 6 or 8? 8

7. $0 = 2(a - 6)$ 6 or 12? 6

Suggested Extension

Replace each * with +, −, ×, or ÷ to make a true sentence. You may need to use parentheses also.

1. $4 * 5 * 3 = 27$ $(4 + 5) \times 3 = 27$

2. $3 * 9 * 3 = 30$ $3 \times 9 + 3 = 30$

3. $16 * 4 * 2 = 2$ $16 \div 4 - 2 = 2$

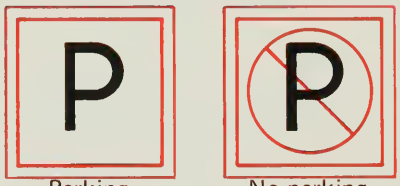
4. $12 * 3 * 2^2 = 0$ $12 \div 3 - 2^2 = 0$

11 • INTRODUCTION TO INEQUALITIES (pp. 24-25)

Teaching These Pages

Review the terms introduced in the last section and relate them to the development of inequalities in this section: number sentence, true number sentence, false number sentence, solution, root.

Point out that the slash is often used to mean *no*. Compare this example to the meanings of = and ≠.



Parking No parking

Chalkboard Examples

1. Is $x < 5$ true if x is replaced by 3? by 6? yes; no

2. Is $6 > y$ true if y is replaced by 4? by 9? yes; no

3. Is $8 \neq k$ true if k is replaced by 3? by 8? yes; no

Tell which of the numbers shown are solutions.

4. $6 > k$ 3, 4, 5, 6 3, 4, 5

5. $y < 21$ 0, 10, 20, 30 0, 10, 20

6. $m > 2$ 0, 1, 2, 3 3

7. $9 \neq k$ 7, 8, 9, 10 7, 8, 10

8. $x + 3 < 12$ 7, 8, 9, 10 7, 8

9. $12 > 2y - 3$ 6, 7, 8, 9 6, 7

Write <, >, or = to make a true sentence.

10. $4 \times 5 - 2$? $4 + 5 \times 2$ $>$

11. $4(6 \div 3) \times 0$? $0 \div 5 + 6 \times 0$ $=$

12. $(12 \div 3 + 2) \div 3$? 2^2 $<$

13. $(9 + 3) \div 3$? 2^1 $>$

Suggested Extension

Use all the symbols to make a true number sentence. (Answers may vary.)

1. 2, 3, 4, 14, ×, +, <

2. 2, 3, 4, +, ÷, >, 0

3. =, ÷, −, 24, 8, 3, 0

Chapter 2. Solving Equations

In the previous chapter students learned that equations are mathematical sentences. In this chapter the students will use equations as problem-solving tools. In order to overcome the difficulty that students have in solving word problems, practice is regularly provided in translating words into symbols.

Objectives for Pages 34-41

1. To solve equations by addition.

2. To solve equations by subtraction.

3. To solve equations by multiplication.

4. To solve equations by division.

1 • SOLVING EQUATIONS BY ADDITION (pp. 34-35)

Teaching These Pages

Ask the students to find a solution to each of these equations.

$x + 5 = 25$ $\quad 2 + m = 10$ $\quad 3b + 5 = 2b + 10$

The last equation illustrates that the "guess and evaluate" method of finding solutions will sometimes take quite a while. Point out the need to find organized ways of solving equations.

Use the balanced scale to show how an equation might be formed.

$$\begin{aligned} x &= 4 \\ x - 3 &= 4 - 3 \\ x - 3 &= 1 \end{aligned}$$

Point out that to solve the equation $x - 3 = 1$ we reverse or "undo" the steps shown.

Encourage students to check their solutions mentally in this section and in Sections 2, 3, and 4.

Chalkboard Examples

Solve. Encourage students to write a final statement of the solution. This will be a useful skill in later problem solving.

1. $x - 3 = 8$ The solution is 11.

2. $8 = y - 4$ The solution is 12.

3. $a - 4 = 11$ The solution is 15.

4. $12 = y - 7$ The solution is 19.

2 • SOLVING EQUATIONS BY SUBTRACTION (pp. 36–37)

Teaching These Pages

The balanced scale technique of building and unbuilding equations can be applied to this section also.

Chalkboard Examples

Solve.

1. $a + 9 = 15$ The solution is 6.

2. $19 = x + 10$ The solution is 9.

3. $y + 7 = 20$ The solution is 13.

4. $y - 7 = 20$ The solution is 27.

Suggested Extension

A path is hidden from *Start* to *Finish*. To find the path, solve each equation and study the solutions to find a pattern. Move only horizontally or vertically.

	$x - 4 = 0$ $x = 4$	$3 = k - 2$ $5 = k$	$12 = a - 12$ $24 = a$	$a - 6 = 15$ $a = 21$ → *Finish*
Start →	$x - 9 = 3$ $x = 12$	$3 = y - 1$ $4 = y$	$k - 2 = 4$ $k = 6$	$3 = p - 8$ $11 = p$
	$4 = y - 5$ $9 = y$	$x - 2 = 3$ $x = 5$	$21 = y - 6$ $27 = y$	$n - 9 = 5$ $n = 14$
	$15 = a - 3$ $18 = a$	$x - 12 = 3$ $x = 15$	$7 = m - 5$ $12 = m$	$16 = m - 12$ $28 = m$

The roots along the path are divisible by 3.

3 • SOLVING MORE EQUATIONS (pp. 38–41)

Teaching These Pages

The students are by now quite familiar with the balanced scale analogy. It can be used in this section to introduce solving equations by multiplication or division.

Chalkboard Examples

Solve.

1. $3x = 12$
$x = 4$

2. $20 = 4y$
$5 = y$

3. $\frac{m}{5} = 5$
$m = 25$

4. $9 = \frac{k}{3}$
$27 = k$

Think of an INPUT-OUTPUT machine. Tell what is missing in the table.

	5.	6.	7.	8.
INPUT	$y + 3 = 9$	$3y = 9$	$y - 3 = 9$	$\frac{y}{3} = 9$
Operation	Subtract 3 from both sides.	Divide both sides by 3.	?	?
OUTPUT	?	?	$y = 12$	$y = 27$

Add 3; Multiply by 3

$y = 6$; $y = 3$

Objective for Pages 42-45

1. To solve equations involving more than one step.

4 • EQUATIONS WITH MORE STEPS (pp. 42-45)

Teaching These Pages

The balanced scale technique can be used in this section to introduce solving equations with two steps.

Chalkboard Examples

Solve.

1.
$$3x - 2 = 4$$
$$3x - 2 + 2 = 4 + 2^*$$
$$3x = 6$$
$$\frac{3x}{3} = \frac{6^*}{3}$$
$$x = 2$$

Some students may do the starred steps mentally.

2. $10 = 2y + 2$
$4 = y$

3. $2x + 3 = 13$
$x = 5$

4. $4x - 3 = 9$
$x = 3$

5. $3m + 5 = 26$
$m = 7$

Think of two INPUT-OUTPUT machines. Tell what is missing in the table.

	6.	7.
INPUT	$4x - 2 = 10$	$4x + 2 = 10$
First Operation	Add 2 to both sides.	Subtract 2 from both sides.
OUTPUT	$4x = 12$	? $4x = 8$
Second Operation	? Divide both sides by 4.	? Divide both sides by 4.
OUTPUT	? $x = 3$	? $x = 2$

Write each sentence as an equation. Then solve.

8. 8 more than twice a number is 18. $2x + 8 = 18$; $x = 5$

9. 6 less than 3 times a number is 9. $3x - 6 = 9$; $x = 5$

10. When 9 is added to twice a number, the result is 21. $2x + 9 = 21$; $x = 6$

Suggested Extension

Choose a number between 10 and 25. Create four equations so that the sum of the solutions of the equations is equal to the number you chose. For example, choose the number 25. The equations may vary, but here are four.

	Solutions
$2x - 3 = 15$	9
$3x + 6 = 15$	3
$3x - 2 = 22$	8
$5x + 1 = 26$	5
	25

Objective for Pages 46–49

1. To solve equations by first combining like terms.

5 • A MIND-READING TRICK (pp. 46–47)

Teaching These Pages

Introduce this lesson by telling the class that you can read their minds. Have the class perform these steps.

	Algebra
1. Choose a whole number.	n
2. Add 2.	$n + 2$
3. Double the result.	$2n + 4$
4. Add 6.	$2n + 10$
5. Subtract 10.	$2n$

At this point you reveal the original number, n. Let's say the original number was 12. Outline the algebra shown above, pointing out that you need only solve the equation $2n = 24$ to find the original number.

Chalkboard Examples

1. Copy and complete.

Steps	Arithmetic	Algebra
Think of a number.	6	n
Multiply by 3.	?	?
Add 5.	?	?
Multiply by 4.	?	?
Subtract 20.	?	?

18; $3n$
23; $3n + 5$
92; $12n + 20$
72; $12n$

2. What equation would you solve to find the original number in Exercise 1? $12n = 72$

3. The result is 80 when these steps are completed. Find the original number.

Steps
Think of a number.
Multiply by 5.
Add 4.
Double your result.
Subtract 8.

n
$5n$
$5n + 4$
$10n + 8$
$10n$; $n = 8$

6 • COMBINING TERMS (pp. 48-49)

Teaching These Pages

Review some examples of combining terms.

$$9y - 3y = 6y \qquad 3a + 4a - 2a = 5a$$

Ask the class to solve the equation $3x + 2x = 20$. Then ask the class which step might be done first in solving $3x + 2x + 5 = 20$.

In Sections 1 through 4 the students were encouraged to mentally check their solutions of equations. In this section the check is set up formally.

Chalkboard Examples

Solve and check.

1. $2a + 4a - 3a = 12$

$$3a = 12$$

$$\frac{3a}{3} = \frac{12}{3}^*$$

$$a = 4$$

2. $3x - 2 + 2x = 13$

$$3x + 2x - 2 = 13^*$$

$$5x - 2 = 13$$

$$5x - 2 + 2 = 13 + 2^*$$

$$5x = 15$$

$$\frac{5x}{5} = \frac{15}{5}^*$$

$$x = 3$$

Some students may do the starred steps mentally.

Write an equation and solve.

3. Twice a number plus the number is 36.
What is the number?

$2n + 4n = 36$
$n = 6$

Point out that the result should be checked in the original problem, not in the equation.

Suggested Extension

Use the terms and symbols to write an equation. Solve. Check. (Answers may vary.)

1. $6n, 2, 30, 2n, -, +, =$ $6n - 2 + 2n = 30;\ n = 4$
2. $2x, 3, x, 12, +, -, =$ $2x + x - 3 = 12;\ x = 5$
3. $7y, 14, 6, 2y, -, -, =$ $7y - 6 - 2y = 14;\ y = 4$

Objectives for Pages 50–57

1. To translate sentences into algebraic equations.
2. To use equations to solve word problems.

7 • WRITING ALGEBRAIC EQUATIONS (pp. 50–53)

Teaching These Pages

Use a newspaper clipping to introduce a real-life problem. Omit some of the entries. Have the students write variable expressions based on the standings.

Philadelphia has won 10 more games than they have lost.
The number lost is n, the number won is $n + 10$.

NATIONAL LEAGUE
East

	Won	Lost
Pittsburg	82	?
Philadelphia . .	?	68
St. Louis	?	70
New York. . . .	74	72
Chicago	70	77
Montreal	68	81

Chalkboard Examples

1. Chicago lost 5 more games than they won.
Let x = number of games won.
Then ___?___ = number of games lost. $x + 5$

2. George is y years old today.
His age in 5 years will be ___?___. $y + 5$
His age last year was ___?___. $y - 1$

3. Georgina is 3 years older than Sam.
 Let g = Georgina's age.
 Then ___?___ = Sam's age. $g - 3$
 In 3 years, Georgina's age will be ___?___. $g + 3$
 In 3 years, Sam's age will be ___?___. g

Suggested Extension

Part of the standings shown have missing entries. Use the information that follows to complete the standings.

1. Chicago won 6 more games than they lost.
2. Vancouver lost 3 more games than St. Louis won.
3. Chicago won 3 more games than St. Louis won.
4. Minnesota lost 5 times as many games as Kansas City tied.
5. Kansas City lost as many games as Minnesota lost.
6. Minnesota won 6 more games than Vancouver tied.

NHL Standings

	W	L	T
Chicago	?	19	17
Vancouver . . .	25	?	11
St. Louis	?	29	9
Minnesota . . .	?	?	4
Kansas City . . .	12	?	?

8 • APPLIED PROBLEMS AND PUZZLES (pp. 54-57)

Teaching These Pages

Refer the students to the Guide for Problem Solving on student's page 54 as the following problem is solved.

Chalkboard Examples

The school play ran for two nights. A total of 535 people attended. There were 79 more people on the second night. How many people attended each night?

1. *Read the problem.*
2. Let n = number of people on the first night.
 Then $n + 79$ = number of people on the second night.
3. $n + n + 79 = 535$
 $2n + 79 = 535$
 $2n = 456$
 $n = 228$
4. *Answer:* 228 people attended the first night.
 (228 + 79) or 307 people attended the second night.
5. *Check:* $228 + 307 = 535$ ✓

Suggested Extension

On the first day you are given an amount of money. Each day after that your amount of money is doubled. On the sixth day you have 252 cents. How much money were you given on the first day? 4 cents

Objectives for Pages 58-63

1. To solve equations having the variable on both sides.
2. To use equations to solve word problems.

9 • VARIABLE ON BOTH SIDES OF THE EQUATION (pp. 58-59)

Teaching These Pages

Introduce this section by posing a puzzle problem that results in an equation with the variable on both sides.

> Six times a number decreased by 3 is equal to 4 times the number increased by 7.
>
> $6n - 3 = 4n + 7$

Point out that in this section the students will learn to solve equations like this.

Chalkboard Examples

Solve and check.

1. $4x = 8 + 2x$
$2x = 8$
$x = 4$

2. $25 - 3m = 2m$
$25 = 5m$
$5 = m$

3. $1 + x = 6x - 4$
$5 = 5x$
$1 = x$

4. $4a + 6 = 2a + 12$
$2a = 6$
$a = 3$

5. Six times a number is 15 more than 3 times the number. $6n - 15 = 3n$; $n = 5$

6. Solve the problem posed in the introduction to this lesson. Check the answer in the original problem. $6n - 3 = 4n + 7$; $n = 5$

10 • EQUATIONS WITH PARENTHESES (pp. 60-61)

Teaching These Pages

Review the various methods acquired thus far for solving equations by using examples. Encourage students to discuss how they would solve each one.

$x + 5 = 25$ $\qquad$ $x - 5 = 25$ $\qquad$ $5x = 25$ $\qquad$ $\frac{x}{5} = 25$

$3x + 2x = 25$ $\qquad$ $3x = 25 - 2x$ $\qquad$ $3x - 16 = 2x + 9$

Introduce an equation with parentheses and ask the students to discuss their first step in solving the equation.

Chalkboard Examples

Use the distributive property to simplify.

1. $4(x + 5) = 4x + 20$

2. $3(x - 6) = 3x - 18$

3. $4(2x - 1) = 8x - 4$

Solve and check.

4.
$$\begin{aligned} 2(x - 5) &= 8 \\ 2x - 10 &= 8 \\ 2x &= 18 \\ x &= 9 \end{aligned}$$

5.
$$\begin{aligned} 3(m + 1) &= 2m + 9 \\ 3m + 3 &= 2m + 9 \\ m &= 6 \end{aligned}$$

6.
$$\begin{aligned} 4(y + 2) + 9 &= 4(2y - 1) \\ y + 8 + 9 &= 8y - 4 \\ 21 &= 7y \\ 3 &= y \end{aligned}$$

11 • PUZZLES (pp. 62–63)

Teaching These Pages

Review the Guide for Problem Solving on student's page 54. Apply those steps to solve the following puzzles.

Chalkboard Examples

1. Jean has \$16 more than José.
Four times José's amount is the same as 2 times Jean's amount.
How much money does each have?

Let n = José's amount of money; $n + 16$ = Jean's amount

$$\begin{aligned} 2(n + 16) &= 4n \\ 2n + 32 &= 4n \\ 32 &= 2n \\ 16 &= n \end{aligned}$$

José has \$16; Jean has \$32.

2. One number is 6 more than another.
Three times the larger number is 5 more than 4 times the smaller.
What are the numbers?

Let n = the larger number; $n - 6$ = the smaller.

$$\begin{aligned} 3n - 5 &= 4(n - 6) \\ 3n - 5 &= 4n - 24 \\ 19 &= n \end{aligned}$$

The larger number is 19; the smaller number is 13.

Chapter 3. Positive and Negative Numbers

The main objective of this chapter is to teach the four operations with positive and negative numbers. The number line is used to develop the skills, as well as to help the students compare numbers. These skills are then applied to simplifying variable expressions and solving equations.

Objectives for Pages 72-75

1. To compare integers.
2. To graph the solutions of an inequality.

1 • POSITIVE AND NEGATIVE NUMBERS (pp. 72-73)

Teaching These Pages

To illustrate the occurrence of positive and negative numbers in everyday life, use the weather report in the newspaper, or a portion of the stock market listings in the newspaper. Discuss the need to use $+5$ to show a gain of 5 dollars and -4 to show a loss of four dollars.

CanCab		5	2⅞	2⅞	2⅞	+⅛
CoolCola		45	23⅝	23	23	−¾
ConCar		26	4¼	4⅛	4⅛	
DewPac	7	8	4	3⅞	3⅞	−⅛
DollCp	9	84	6¼	6⅛	6¼	
EverLst	16	133	13½	12⅝	13	−⅝
FlrCov	16	42	22¾	22	22¾	+½
FlVCorp	15	46	50	49⅝	50	

Have the students name other ways in which positive and negative numbers might be used.

Chalkboard Examples

Write a positive or negative number.

1. a debt of $40 -40
2. a deposit of $10 $+10$
3. 30 m below sea level -30
4. a profit of $10 $+10$

Compare the numbers. Write $>$ or $<$.

5. $-3 \,?\, 5$ $<$
6. $2 \,?\, -2$ $>$
7. $0 \,?\, -7$ $>$
8. $-6 \,?\, -9$ $>$

Suggested Extension

1. Use this newspaper clipping of the weather report. Which temperature is lowest? Which temperature is highest? Which temperature is closest to zero?

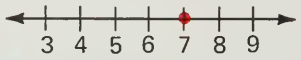
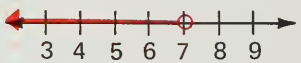
	High	Low
Albany	8°C	−5°C
Boise	18°C	6°C
Louisville	14°C	−1°C
Monterrey	22°C	16°C
Vancouver	17°C	7°C
Windsor	16°C	−2°C

2. Use a newspaper clipping for the weather in your area. Record the coldest temperature. Record the warmest temperature.

2 • INEQUALITIES AND GRAPHS (pp. 74–75)

Teaching These Pages

Remind the students that equations and inequalities are mathematical sentences. Compare the following.

Sentence	*Solution(s)*	*Graph of the Solution(s)*
$x - 2 = 5$	7	
$x < 7$	all numbers less than 7	

Point out that the open dot on the graph of the solutions of $x < 7$ means that 7 is not a solution.

Chalkboard Examples

Write an inequality to compare the numbers.

1. $-3, 2$ $-3 < 2$

2. $-6, -5$ $-6 < -5$

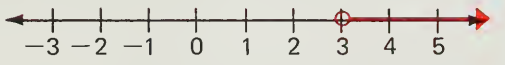
3. $3, -3, 0$ $3 > 0 > -3$

4. $-3, -1, 5$ $-3 < -1 < 5$

Graph the solutions of the sentence.

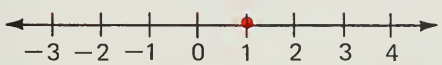
5. $x + 2 = 5$ 

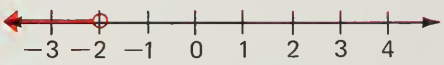
6. $x < 3$ 

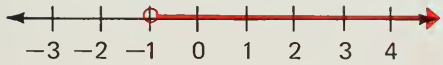
7. $x > 3$ 

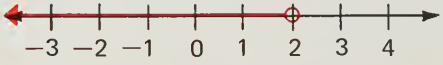
Suggested Extension

Write an equation or inequality for each solution shown on the number line.

1. 

2. 

3. 

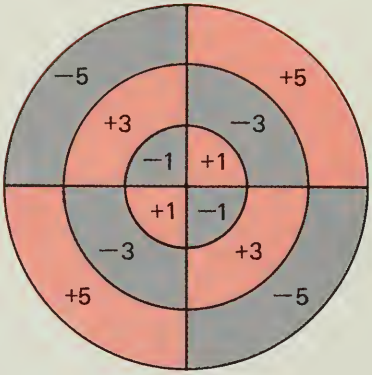
4. 

Objectives for Pages 76-83

1. To add positive and negative numbers.
2. To subtract positive and negative numbers.
3. To simplify expressions.

3 • ADDITION (pp. 76-77)

Teaching These Pages



Introduce the board for a dart game as shown. Explain that 5 means you gain 5 points, −5 means you lose 5 points. Have the students determine the net scores after two throws.

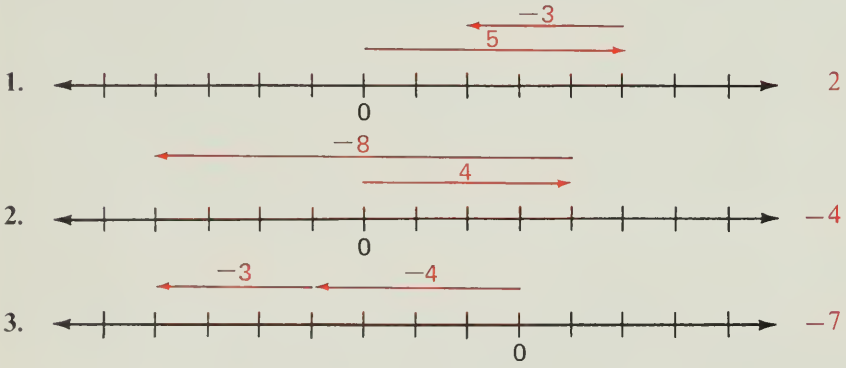
		Net Score
Sylvia	1, 3	4
Jean	5, −3	2
Mark	−5, 2	−3
Jonathan	−3, −2	−5

The scores can be related to a number line and the following results developed.

$$5 + (-3) = 2 \qquad (-3) + (-2) = -5$$

Chalkboard Examples

Read the sum from the drawing.



Add.

4. $5 + (-2) = 3$
5. $-3 + 4 = 1$
6. $-9 + 6 = -3$
7. $-3 + (-2) = -5$
8. $6 + (-3) + (-2) = 1$
9. $-4 + 8 + (-4) = 0$

4 • SUBTRACTION (pp. 78-81)

Teaching These Pages

Introduce opposite ideas.

below sea level, above sea level
3 floors up, 3 floors down
4 steps to the right, 4 steps to the left

Extend the list of opposites with examples provided by the class.

Chalkboard Examples

Complete.

1. $12 - 8 = 12 + \underline{?}\ -8$ **2.** $-3 - (-9) = -3 + \underline{?}\ 9$

Subtract.

3. $3 - 8 = -5$ **4.** $-8 - 9 = -17$ **5.** $8 - (-4) = 12$ **6.** $-3 - (-2) = -1$

7. From the sum of -3 and -2, subtract 6. -11

Suggested Extension

A path is hidden from *Start* to *Finish*. To find the path you must find a pattern for your answers by working each example. You may move along the path horizontally or vertically. (The values along the path are shown in red.)

Start ↓

$-6 + (-2) = -8$	$8 + (-2) = 6$	$-2 + (-1) = -3$	$8 - 6 = 2$
$-2 + (-5) = -7$	$-8 - (-8) = -6$	$6 - (-2) = 10$	$6 - (-1) = 7$
$-8 + 2 = -6$	$-4 + (-1) = -5$	$-2 + (-2) = -4$	$-6 + 3 = -3$
$-3 - (-4) = 1$	$-4 + 4 = 0$	$-3 - (-2) = -1$	$-11 - (-9) = -2$

↓ *Finish*

Each value along the path increases by 1.

5 • SIMPLIFYING EXPRESSIONS (pp. 82-83)

Teaching These Pages

Apply the distributive property to combine like terms.

$$4x + 5x = (4 + 5)x = 9x$$

Include examples with negative coefficients. It might be necessary to remind the students that $-3 - 2 = -3 + (-2)$ and that $-n$ means $-1n$.

Chalkboard Examples

Simplify.

1. $6n - 5n = n$
2. $-6n + 5n = -n$
3. $-6n - 5n = -11n$
4. $3x^2 - x^2 = 2x^2$
5. $-3x^2 + x^2 = -2x^2$
6. $-3x^2 - x^2 = -4x^2$
7. $2x - 3 - 3x = -x - 3$
8. $3a - (-2a) + 3 = 5a + 3$
9. $-x + 2y - (-3x) = 2x + 2y$

Solve.

10. Jeannette delivers $2x - 3y$ newspapers. George delivers $-2y - x$ newspapers. How many more newspapers does Jeannette deliver? How many newspapers do Jeannette and George deliver altogether? $3x - y$; $x - 5y$

Suggested Extension

Use the chart to answer the following questions.

	Points Scored	
	Week 1	Week 2
Robert	$2x - y$	$-2y$
Laurie	$3x$	$-x + y$

1. How many points did Robert and Laurie score altogether during Week 1? Week 2? $5x - y$; $-x - y$
2. How many points did Laurie score altogether? $2x + y$
3. Make up other questions about the chart.

Objectives for Pages 84-91

1. To multiply positive and negative numbers.
2. To simplify expressions.
3. To divide positive and negative numbers.
4. To solve equations.

6 • MULTIPLICATION (pp. 84-85)

Teaching These Pages

Finding the product of positive and negative numbers is a skill that students often find quite troublesome. An alternative strategy to finding the product of positive and negative numbers is shown here.

Have the students complete these patterns.

$4 \times 3 = 12$	$3 \times 2 = 6$
$4 \times 2 = 8$	$2 \times 2 = 4$
$4 \times 1 = 4$	$1 \times 2 =$?
$4 \times 0 =$?	$0 \times 2 =$?
$4 \times (-1) =$?	$(-1) \times 2 =$?
$4 \times (-2) =$?	$(-2) \times 2 =$?

Give additional examples that establish the following rules.

positive $\times$ negative = negative
negative $\times$ positive = negative

Similar patterns can be assigned to develop the following rule.

negative $\times$ negative = positive

Chalkboard Examples

Simplify.

1. $2(-3) = -6$ **2.** $-2(3) = -6$ **3.** $-2(-3) = 6$

4. $(-3)^2 = 9$ **5.** $(-2)^3 = 8$ **6.** $-(-4)^2 = -16$

Simplify.

7. $2(-8) - 3(-3) = 25$ **8.** $-2(-6) + (-3)(-1) = 15$ **9.** $-(-2)^3 = -8$

Suggested Extension

Complete.

$(-1)^2 =$? 1 $(-1)^3 =$? -1 $(-1)^4 =$? 1

$(-1)^5 =$? -1 $(-1)^6 =$? 1 $(-1)^7 =$? -1

Do you see a pattern? Use the pattern to predict the answer for $(-1)^{25}$ and $(-1)^{100}$. -1; 1

7 • SIMPLIFYING EXPRESSIONS (pp. 86-87)

Teaching These Pages

Review that $-12x$ means $(-12)x$. Apply this fact to simplifying expressions.

$$3(-4x) = 3(-4)x = (-12)x = -12x$$

Show how the distributive property can be applied to simplifying expressions.

$$-3(2a - 3b) = (-3)(2a) + (-3)(-3b) = -6a + 9b$$

Chalkboard Examples

Simplify.

1. $3(-6a) = -18a$

2. $-4(x - 3) = -4x + 12$

3. $2a(-3b) = -6ab$

4. $-2a(-3b) = 6ab$

5. $-(-3x + 5) = 3x - 5$

6. $2x - (x - 2) = x + 2$

Solve and check.

7. $3(y - 2) = -(y - 14)$

$y = 5$

Check: $3(5 - 2) \stackrel{?}{=} -(5 - 14)$

$9 = 9$ ✓

8 • DIVISION (pp. 88–89)

Teaching These Pages

Review the relationship between addition and subtraction.

$$3 + 2 = 5 \longleftrightarrow 5 - 2 = 3$$
$$x + y = 5 \longleftrightarrow 5 - y = x$$

Use whole numbers to review the relationship between multiplication and division.

$$3 \times 2 = 6 \longleftrightarrow 6 \div 2 = 3$$

Have the students state further examples to show they understand how multiplication and division are related. Then develop the rules for dividing positive and negative numbers.

Point out the similarities of these summaries.

Multiplication	*Division*
positive × positive = positive	positive ÷ positive = positive
negative × positive = negative	negative ÷ positive = negative
positive × negative = negative	positive ÷ negative = negative
negative × negative = positive	negative ÷ negative = positive

Chalkboard Examples

Divide.

1. $9 \div (-9) = -1$

2. $-9 \div 9 = -1$

3. $-9 \div (-9) = 1$

4. $\frac{-36}{-4} = 9$

5. $\frac{-18}{2} = -9$

6. $\frac{25}{-5} = -5$

Suggested Extension

Find the value of each expression if $a = -2$, $b = -3$, $c = 3$ and $d = -1$.

1. $a^2 = 4$ **2.** $ab = 6$ **3.** $a^2b = -12$ **4.** $-a^2 = -4$

5. $a + b = -5$ **6.** $abc = 18$ **7.** $ab + cd = 3$ **8.** $d^5 = -1$

9 • SOLVING EQUATIONS (pp. 90-91)

Teaching These Pages

These four examples review that to solve an equation, the same number is added (subtracted, multiplied, divided) to both sides of the equation.

$$k - 3 = -4 \qquad x + 6 = -3 \qquad -3y = -24 \qquad \frac{m}{-2} = -8$$

Review the vocabulary related to solving equations as the Chalkboard Examples are completed.

Chalkboard Examples

Solve and check.

1. $8x - 3x = -5$
$x = -1$

2. $3y + 15 = 2y$
$y = -15$

3. $3x - (5x - 2) = -2$
$-4 = x$

10 • ABSOLUTE VALUE (OPTIONAL) (pp. 92-93)

Teaching These Pages

Introduce the symbol for absolute value. Point out that $|x| = x$ if $x \geq 0$ and $|x| = -x$ if $x < 0$.

Chalkboard Examples

Find the value of the expression.

1. $|-4| = 4$ **2.** $-|7| = -7$ **3.** $|-9| - |3| = 6$

Solve.

4. $|a| + 2 = 4$
$a = 6$ *or* -6

5. $|x| - 4 = 10$
$x = 14$ *or* -14

6. $|y| - 3 = -2$
$y = 1$ *or* -1

Chapter 4. Formulas

Skills needed to apply formulas to problem solving are developed in this chapter. The students will work with the following formulas.

$P = 2l + 2w$ (p. 102) $A = lw$ (p. 104) $A = \frac{1}{2}bh$ (p. 104)

$V = Bh$ (p. 108) $D = rt$ (p. 112) $C = pn$ (p. 112)

The students' earlier work with solving equations is related to work with formulas. Also, throughout the chapter the students obtain practice in writing formulas.

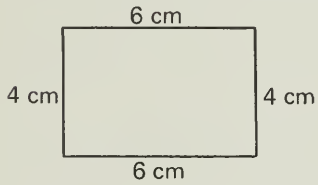
Objectives for Pages 102-111

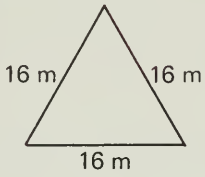
1. To use the formula $P = 2l + 2w$ to find perimeters.
2. To use the formulas $A = lw$ and $A = \frac{1}{2}bh$ to find areas.
3. To use the formula $V = Bh$ to find volumes.

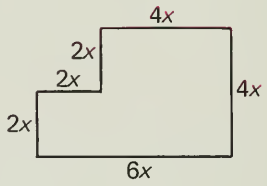
1 • PERIMETER FORMULAS (pp. 102-103)

Teaching These Pages

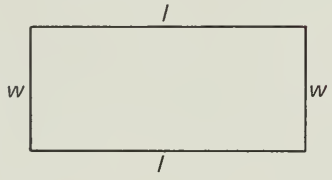
Reinforce the meaning of perimeter as "distance around" by having the students find the perimeters of these figures.







Develop the formula for the perimeter of a rectangle by finding the distance around this figure.



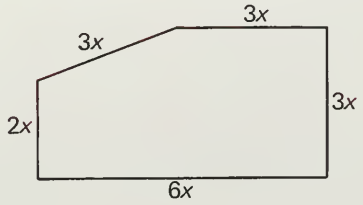
$P = l + w + l + w$ (Why?)
$= 2l + 2w$ (Why?)

Chalkboard Examples

1. The width of a rectangle is 6 cm shorter than the length. The perimeter is 60 cm. Find the length and width. (Remind students to check their answers in the original problem.)
 $60 = 2l + 2(l - 6)$; $l = 18, l + 6 = 2$

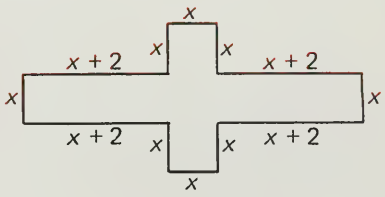
Write a formula for the perimeter of each figure.

1.



$P = 17x$

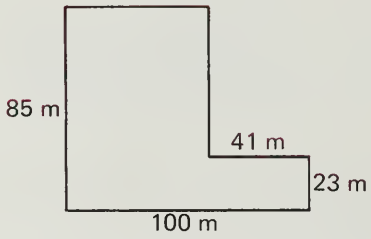
2.



$P = 12x + 8$

Suggested Extension

Chain-link fencing sells for \$2.10 per meter. Find the cost of fencing the field. \$777



2 • AREA FORMULAS (pp. 104–107)

Teaching These Pages

Compare the metric units for measuring length and area.

Length	*Area*
centimeter	square centimeter
meter	square meter
kilometer	square kilometer

Have the class suggest the unit used to measure various objects.

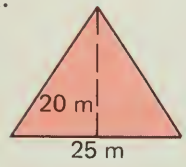
Length of an airplane, meters
Area of the plane's wing, square meters
Length of the coastline, kilometers
Area of a postage stamp, square centimeters

Chalkboard Examples

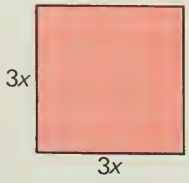
Find the value of A.

1. $A = s^2$, $s = 12$ $A = 144$
2. $A = lw$, $l = 26$, $w = 13$ $A = 338$
3. $A = \frac{1}{2}bh$, $b = 12$, $h = 16$ $A = 128$

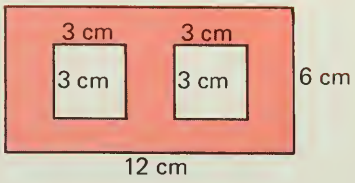
Find the area of each shaded region.

1. 

$A = 250 \text{ m}^2$

2. 

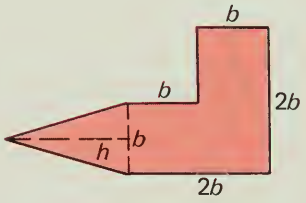
$A = 9x^2$

3. 

$A = 54 \text{ cm}^2$

Suggested Extension

1. Write a formula to find the area of the shaded region. $A = 3b^2 + \frac{1}{2}bh$



2. Use the formula you found above to find the area of the region if $h = 8$ and $b = 6$. 132

3 • VOLUME FORMULAS (pp. 108–111)

Teaching These Pages

To review the meaning of volume, use a small box and ask the class to estimate how many centimeter cubes will fill the box. (Sugar cubes might be used to represent centimeter cubes.) If the box holds 10 cubes, the volume is 10 cubic centimeters. Introduce the symbol 10 cm^3.

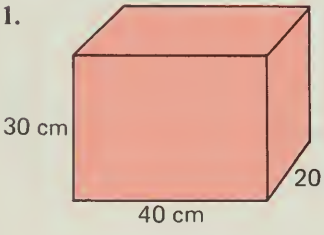
Use the term *prism* to name the shape of the box. Point out other shapes in the classroom to provide meaning to the term: closet, eraser, etc.

Compare the different units. Point out the significance of the words *square* and *cubic*.

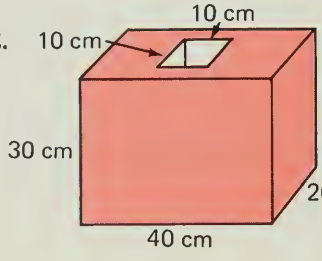
Point out that 1 liter = 1000 cubic centimeters.

Chalkboard Examples

Find the volume.

1. 

$V = 24{,}000 \text{ cm}^3$

2. 

$V = 21{,}000 \text{ cm}^3$

3. Express the volumes in Exercises 1 and 2 as liters. 24 L; 21 L

4. A box measures 12 cm by 15 cm by 20 cm. Find the volume. 3600 cm^3

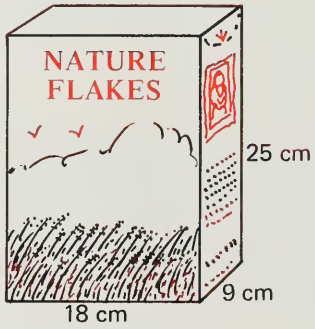
5. A cube has sides that measure 3 cm. Find the volume. 27 cm^3

Suggested Extension

1. How much paper is needed to cover the cereal box shown? 1724 cm^2

2. Find the volume of the box in cubic centimeters. 4050 cm^3

3. When the box is opened, the top of the cereal is 8 cm from the top. How many cubic centimeters of cereal are in the box? 2754 cm^3



Objectives for Pages 112–121

1. To write formulas.

2. To solve motion problems.

3. To solve cost problems.

4 • WRITING FORMULAS (pp. 112–113)

Teaching These Pages

Sports standings from the newspaper can be used to introduce this topic.

When a soccer team wins a game, they are given 2 points. After winning n games, the number of winning points w is given by the formula $w = 2n$.

Develop the formulas for distance and cost. These formulas will be dealt with in detail in Sections 5 and 6.

SOCCER STANDINGS

	Won	Lost	Tied
Northampton	15	4	2
Lincoln	15	3	2
Reading.	13	4	4
Tranmere.	12	4	5
Bournemouth	10	5	5

Chalkboard Examples

Complete.

1. You walk 5 km/h for t hours.
Your distance $D =$ ___?___. $5t$

2. You rode your bicycle at y km/h for n hours.
Your distance $D =$ ___?___. yn

3. You buy 5 tennis balls at s cents each.
The cost $C =$ ___?___. $5s$

4. In hockey, 2 points are given for a win.
Winning 2 games scores __?__ points. 4

5. For a tie game, a hockey team receives 1 point.
Tying t games scores __?__ points. t

6. A hockey team won w games and tied t games.
The total points $T =$ __?__. $2w + t$

Suggested Extension

Clip the hockey standings from the newspaper. Use the formula developed in Exercises 4–6 above to check the total points for the various teams.

For example, St. Louis has won 10 games and tied 5 games.

$$T = 2w + t = 2(10) + 5 = 25$$

NATIONAL LEAGUE	W	L	T
Chicago	11	10	11
Vancouver	11	14	6
St. Louis	10	16	5
Kansas City	10	19	4
Minnesota	10	19	1

5 • MOTION FORMULAS (pp. 114–117)

Teaching These Pages

Review the problem-solving methods of Chapter 2. They are applied throughout this section.

Chalkboard Examples

Jean and John live 18 km apart.
They leave at the same time and walk toward each other.
Jean walks at 5 km/h and John walks at 4 km/h.
How long will it take them to meet?

Let t = number of hours until they meet.

	rate × time = Distance		
Jean	5	t	$5t$
John	4	t	$4t$
			18

$$5t + 4t = 18$$
$$9t = 18$$
$$t = 2$$

Answer: Jean and John will meet 2 hours later.

Check: Jean walks $5 \cdot 2$, or 10 km
John walks $4 \cdot 2$, or 8 km
18 km ✓

6 • COST AND MONEY PROBLEMS (pp. 118–121)

Teaching These Pages

Use several examples to review the formula for cost. Point out the meaning of *profit*.

1. A pencil costs 15¢. Find the cost of 10 pencils.
2. A tennis ball costs $1.25. Find the cost of 3 tennis balls.
3. You bought 10 pens at 35¢ each, and sold them for 50¢ each. Find the profit.

Solve the Chalkboard Example and ask the class to indicate each problem-solving step.

Chalkboard Examples

George and Robin bought greeting cards at 70¢ a box.
They lost 2 boxes and sold the rest for $1.20 each.
Their profit was $17.10.
How many boxes did they sell?

Let n = number of boxes they bought.
Then $n - 2$ = number of boxes they sold. (Why?)

	p ×	n =	Cost
Sell	120	$n - 2$	$120(n - 2)$
Buy	70	n	$70n$
			1710

They sold 37 boxes of greeting cards.

$$\begin{aligned} 120(n - 2) - 70n &= 1710 \\ 120n - 240 - 70n &= 1710 \\ 50n - 240 &= 1710 \\ 50n &= 1950 \\ n &= 39 \end{aligned}$$

Suggested Extension

Clip newspaper ads for various items and have the students create their own problems about profit.

Objective for Pages 122–125

1. To solve a formula for a given variable.

7 • NEW FORMULAS FROM OLD (pp. 122–125)

Teaching These Pages

Use the cost formula to introduce this section. If the total cost of n record albums is $36 and each album costs $3, we can substitute these values into

$C = pn$ and solve for n to find the number of record albums. *Or* we could solve for n first and substitute the values into $\frac{C}{p} = n$ to obtain $12 = n$. Have the class compare these two methods.

Chalkboard Examples

1. Solve $D = rt$ for r. $r = \frac{D}{t}$

Solve for y.

2. $2x + y = 8$
$y = 8 - 2x$

3. $2x - y = 6$
$2x - 6 = y$

4. $3x + y = 9$
$y = 9 - 3x$

Complete.

5. $V = lwh$; $V = 72$, $l = 6$, $h = 4$
$w =$? 3

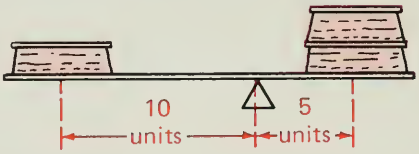
6. The Samuels family traveled 3120 km on their vacation. They drove a total of 65 hours. Find their average rate. $3120 = 65r$; $r = 48$

8 • FORMULAS FROM MECHANICS (OPTIONAL) (pp. 126-127)

Teaching These Pages

A bicycle can be used to introduce the gear formula. Ask the students to count the number of teeth on the gears of a bicycle, then to observe the relationship between the number of turns of the wheel for each turn of the pedal.

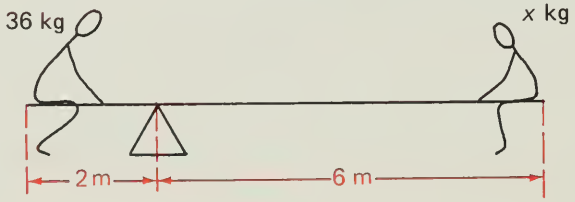
A meter stick and chalk erasers can be used to give meaning to the lever formula.



Chalkboard Examples

1. On Terry's bicycle, the pedal gear has 48 teeth and the wheel gear has 16 teeth. If the pedal gear turns once per second, how fast does the wheel gear turn? 3 turns per second

2. Find the value of x in the lever diagram shown. 12



Chapter 5. Working with Polynomials

This chapter introduces skills for adding, subtracting, multiplying and dividing polynomials. In appropriate sections, the students are provided with methods that will help them develop mental skills for working with polynomials. Throughout the chapter the students' arithmetic skills are used to develop similar algebraic skills.

Objectives for Pages 136-139

1. To add polynomials.
2. To subtract polynomials.

1 • ADDITION (pp. 136-137)

Teaching These Pages

The meaning of *polynomial* can be introduced as follows. A small box contains slips of paper, each with a different algebraic term written on it. Another box contains the symbols + and −. Terms are drawn from one box and symbols from the other to create polynomials.



Compare the meaning of *poly* in *poly*nomial and *poly*gon.

Point out that the distributive property allows us to simplify polynomials.

$$2a^2 + 6a^2 = (2 + 6)a^2 = 8a^2$$

Chalkboard Examples

Use a special name to name each polynomial.

1. $3x^2$ monomial

2. $a + b + c$ trinomial

3. $2x^2 + 2xy + y^2$ trinomial

4. $x^2 + y^2$ binomial

5. x^2y^2 monomial

6. $m - 3$ binomial

Add.

7.
$$\begin{array}{r} 2x + 6 \\ 4x + 3 \\ \hline 6x + 9 \end{array}$$

8.
$$\begin{array}{r} 2y - 6 \\ 2y + 4 \\ \hline 4y - 2 \end{array}$$

9.
$$\begin{array}{r} x^2 + y^2 \\ x^2 - y^2 \\ \hline 2x^2 \end{array}$$

10.
$$\begin{array}{r} 5m - 3n \\ -2m + 2n \\ \hline 3m - n \end{array}$$

11. $(k^2 + 2k - 1) + (k^2 - 3k + 1) = 2k^2 - k$

12. $(y^2 + 2y - 3) + (y^2 - 6) = 2y^2 + 2y - 9$

Suggested Extension

1. Let $x = 2$ and $y = -1$. Find the value of $(5x^2 + y) + (-2x^2 + 3y) + (3x^2 - y)$. 21

Some students may evaluate the expression by direct substitution. Point out the advantage of simplifying the polynomials first.

2 • SUBTRACTION (pp. 138–139)

Teaching These Pages

Use oral examples to review some of the skills needed for the exercises in this section.

$$6 - 7 = 6 + (-7) = -1$$
$$2 - (-3) = 2 + 3 = 5$$

Students may find the vertical form for subtracting polynomials easier to use at first. Eventually, however, they should be encouraged to complete the exercises in the form given.

Chalkboard Examples

Name the opposite.

1. $(2y^2 - 3)$
$-2y^2 + 3$

2. $(3x^2 + 5x + 4)$
$-3x^2 - 5x - 4$

3. $(x^2 + y^2)$
$-x^2 - y^2$

4. $(-m + 3n)$
$m - 3n$

Subtract.

5.
$$\begin{array}{r} 3x - 2 \\ -(2x - 5) \\ \hline x + 3 \end{array}$$

6.
$$\begin{array}{r} y^2 + xy \\ -(2y^2 - xy) \\ \hline 3y^2 + 2xy \end{array}$$

7.
$$\begin{array}{r} a + 2b \\ -(-a - 2b) \\ \hline 2a + 4b \end{array}$$

8.
$$\begin{array}{r} x^2 + 2xy + y^2 \\ -(3x^2 + 2xy - y^2) \\ \hline -2x^2 \qquad + 2y^2 \end{array}$$

9. $(x - 5) - (2x + 5) = -x - 10$

10. $(3x^2 - 2xy - y^2) - (2x^2 - y^2) = x^2 - 2xy$

11. Solve $(5x - 3) - (2x - 9) = 33$. $x = 9$

Objectives for Pages 140–145

1. To multiply monomials.
2. To simplify powers of monomials.
3. To multiply polynomials by monomials.

3 • MULTIPLYING MONOMIALS (pp. 140–141)

Teaching These Pages

Remind students that exponents are a shorthand for writing like products.

Work these examples of multiplication in detail, with the students looking for a short cut.

$$\begin{aligned} y^3 \cdot y^5 &= (y \cdot y \cdot y) \cdot (y \cdot y \cdot y \cdot y \cdot y) = y^8 \\ (2x^3)(3x^4) &= (2 \cdot x \cdot x \cdot x) \cdot (3 \cdot x \cdot x \cdot x \cdot x) \\ &= 2 \cdot 3 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \\ &= 6x^7 \end{aligned}$$

Point out that $x^a \cdot x^b = x^{a+b}$

Chalkboard Examples

Multiply.

1. $(3a^2)(4a^3) = 12a^5$ **2.** $(-2y^2)(4y^3) = -8y^5$ **3.** $(3x^2y)(-2x^3y^2) = -6x^5y^3$

4 • POWERS OF MONOMIALS (pp. 142–143)

Teaching These Pages

The rule $x^a \cdot x^b = x^{a+b}$ is used to develop the rule $(x^a)^b = x^{ab}$.

$$\begin{aligned} (n^2)^2 &= n^2 \cdot n^2 = n^{2+2} = n^{2 \cdot 2} \\ (y^2)^3 &= y^2 \cdot y^2 \cdot y^2 = y^{2+2+2} = y^{3 \cdot 2} = y^{2 \cdot 3} \\ (a^3)^4 &= a^3 \cdot a^3 \cdot a^3 \cdot a^3 = a^{3+3+3+3} = a^{4 \cdot 3} = a^{3 \cdot 4} \end{aligned}$$

The following examples can be used to establish the rule $(xy)^n = x^n y^n$.

$$\begin{aligned} (ab)^3 &= a \cdot b \cdot a \cdot b \cdot a \cdot b = a^3b^3 \\ (a^2b)^3 &= a^2 \cdot b \cdot a^2 \cdot b \cdot a^2 \cdot b = (a^2)^3b^3 = a^6b^3 \\ (a^2b^3)^3 &= a^2 \cdot b^3 \cdot a^2 \cdot b^3 \cdot a^2 \cdot b^3 = (a^2)^3(b^3)^3 = a^6b^9 \end{aligned}$$

Chalkboard Examples

Simplify.

1. $(2x)^4 = 16x^4$ **2.** $(-2x)^4 = 16x^4$ **3.** $(-3xy)^3 = -27x^3y^3$

4. $4(-2xy^3)^3 = -32x^3y^9$ **5.** $(2x^3y^2)^3 = 8x^9y^6$ **6.** $(-2ab^2)^2(-3a^2b)^3 = -108a^8b^7$

5 • POLYNOMIALS TIMES MONOMIALS (pp. 144–145)

Teaching These Pages

Review the distributive property. Point out that the variables in a polynomial represent numbers and so the distributive property can be used to multiply polynomials times monomials.

Chalkboard Examples

Multiply.

1. $-3x(4 - 4y) = -12x + 12xy$ **2.** $2(x + 4y) = 2x + 8y$ **3.** $-x(3x^2 + 2x) = -3x^3 - 2x^2$

4. Solve $3(2x - 5) - 2(x - 6) = -11$. $x = -2$

Objectives for Pages 146-153

1. To multiply polynomials.

2. To multiply binomials at sight.

3. To square binomials.

6 • MULTIPLYING POLYNOMIALS (pp. 146-147)

Teaching These Pages

Use the distributive property to show how to multiply polynomials.

$$(m + 2)(m - 3) = (m + 2)m - (m + 2)3 = m^2 - m - 6$$

Students may find the vertical form easier to use. Although they may rewrite the exercises in this section in the vertical form, in subsequent sections they will be encouraged to work mentally with the horizontal form.

Chalkboard Examples

Multiply.

1. $(n + 3)(n + 5) = n^2 + 8n + 15$ **2.** $(x - 3)(x - 4) = x^2 - 7x + 12$

3. $(2x + 3)(3x - 9) = 6x^2 - 9x - 27$ **4.** $(n - 2)(n^2 - 3n + 4) = n^3 - 5n^2 + 10n - 8$

5. $(x^2 - 2x + 1)(2x^2 - 4x - 5) = 2x^4 - 8x^3 + 5x^2 + 6x - 5$

Suggested Extension

Here is a method students may use to check their work, at the same time obtaining practice with computation. Exercise 18 on page 147 is used to illustrate the method.

1. *Choose a value for the variable.* Let $x = 2$.

2. *Evaluate the expression in the exercise.*
$(4x + 1)(4x - 1) = (8 + 1)(8 - 1) = 9 \cdot 7 = 63$

3. *Evaluate your answer.* $16x^2 - 1 = (16 \cdot 4) - 1 = 64 - 1 = 63$

4. *Are the two results equal?* $63 = 63$; the answer is correct.

7 • MULTIPLYING AT SIGHT (pp. 148–151)

Teaching These Pages

Point out that just as it is useful to multiply numbers at sight, it is useful to multiply binomials at sight. Have the students find these products. Ask them if they notice any patterns.

$(x + 2)(x + 3) = x^2 + 5x + 6$ $\quad$ $(x - 2)(x + 3) = x^2 + x - 6$
$(x + 2)(x - 3) = x^2 - x - 6$ $\quad$ $(x - 2)(x - 3) = x^2 - 5x + 6$

Use the model on page 148 to develop the FOIL method.

Chalkboard Examples

Complete.

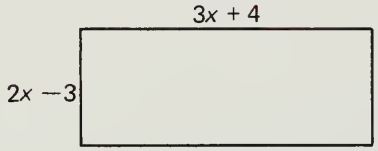
1. $(y + 3)(y + 2) = __?__ + 5y + 6$ y^2

2. $(x + 2)(x + 3) = x^2 + 5x + __?__$ 6

3. $(x - 6)(x + 8) = x^2 + __?__ - 48$ $2x$

4. $(x + 6)(x - 8) = x^2 - __?__ - 48$ $2x$

5. Find the area.



$A = (3x + 4)(2x - 3)$
$= 6x^2 - x - 12$

8 • SQUARE OF A BINOMIAL (pp. 152–153)

Teaching These Pages

Review what it means to square a number. Have the students square these expressions: 2, y, y^2, $(x + 2)^2$.

Have the students look for patterns as you work these examples.

$(x + 2)^2 = (x + 2)(x + 2) = x^2 + 4x + 4$
$(x - 2)^2 = (x - 2)(x - 2) = x^2 - 4x + 4$
$(2x + 3)^2 = (2x + 3)(2x + 3) = 4x^2 + 12x + 9$
$(2x - 3)^2 = (2x - 3)(2x - 3) = 4x^2 - 12x + 9$

Outline the steps for squaring a binomial. Point out that since squaring a binomial is a special case of multiplying two binomials, the method used in the previous section will work also.

Chalkboard Examples

Complete.

1. $(k + 3)^2 = __?__ + 6k + 9$ k^2

2. $(k - 3)^2 = k^2 - __?__ + 9$ $6k$

3. $(m - 9)^2 = m^2 - 18m + __?__$ 81

4. $(3x + 2y)^2 = 9x^2 + __?__ + 4y^2$ $12xy$

Express as a trinomial.

5. $(3x - 2y)^2 = 9x^2 - 12xy + 4y^2$

6. $3(x - 2y)^2 = 3(x^2 - 4xy + 4y^2)$
$= 3x^2 - 12xy + 12y^2$

Objectives for Pages 154-159

1. To divide monomials.
2. To divide polynomials by monomials.

9 • DIVISION WITH MONOMIALS (pp. 154-155)

Teaching These Pages

Remind students that a division in arithmetic is often shown as a fraction. In arithmetic we can simplify $\frac{12}{4}$ as shown.

$$\frac{12}{4} = \frac{3 \cdot 4}{4} = 3 \cdot \frac{4}{4} = 3 \cdot 1 = 3$$

We simplify $\frac{12x}{4}$ and $\frac{12x}{4x}$ in a similar way.

$$\frac{12x}{4} = \frac{3 \cdot 4 \cdot x}{4} = \frac{3 \cdot x \cdot 4}{4} = 3x \cdot \frac{4}{4} = 3x \cdot 1 = 3x$$

$$\frac{12x}{4x} = \frac{3 \cdot 4 \cdot x}{4 \cdot x} = 3 \cdot \frac{4x}{4x} = 3 \cdot 1 = 3$$

Point out that we often use the method shown on page 154 to shorten our work.

In the next section the students will develop the rule $\frac{x^a}{x^b} = x^{a-b}$, $a > b$. Some students may discover this pattern and should be permitted to use it in the exercises, if they understand why it works.

Chalkboard Examples

Divide.

1. $\frac{9x^2}{x} = 9x$

2. $\frac{16a^4}{8a^2} = 2a^2$

3. $\frac{48x^2y^4}{-6xy^2} = -8xy^2$

4. $\frac{-12m^3n}{-6m^2} = 2mn$

Simplify.

5. $\frac{25a^2b^3}{-5ab^3} + \frac{-6a^4b}{-3a^3b} = -5a + 2a = -3a$

6. $\frac{5m^3n^2}{m^2} - \frac{3m^2n^2}{m} = 2mn^2$

Suggested Extension

A path is hidden from *Start* to *Finish*. To find the path, simplify each expression and find a pattern. You may move horizontally or vertically. (The answers along the path are shown in red.)

Start

$\frac{21a^2b^5}{7b^5a} = 3a$	$(3a - b) - (a - b) = 2a$	$\frac{6a^2x^3}{6ax^3} = a$	$8a - 10a = -2a$
$\frac{25a^2y^2}{5ay^2} = 5a$	$\frac{-20a^2}{-5a} = 5a$	$\frac{5ab}{b} = 5a$	$\frac{9a^2y^2}{3y^2a} = 3a$
$(3a + 2b) - (2b - 4a) = 7a$	$\frac{18a^5b^3}{-3a^4b^3} = -6a$	$\frac{(25a^2)(3b^2)}{15ab^2} = 5a$	$\frac{-16a^2b^3}{4ab^3} = -4a$
$\frac{-64a^3b^3x^2}{8a^2x^2b^3} = -8a$	$\frac{-63a^2b}{9ab} = -7a$	$\frac{a^3b^2}{-a^2b^2} = -a$	$\frac{36a^6}{6a^5} = 6a$

Finish

10 • EXPONENT RULES (OPTIONAL) (pp. 156–157)

Teaching These Pages

Discuss the exponent rules on page 156. Encourage students to use these rules, but point out that they may use the methods of the previous section when necessary.

Chalkboard Examples

Divide.

1. $\frac{x^6}{x^2} = x^4$ **2.** $\frac{n^3}{n^8} = \frac{1}{n^5}$ **3.** $\frac{-9a^3b^2}{3a^2b} = \frac{-3b}{a}$ **4.** $\frac{-9a^3b}{3a^3b^2} = \frac{-3}{ab}$

11 • POLYNOMIALS DIVIDED BY MONOMIALS (pp. 158–159)

Teaching These Pages

Use examples from arithmetic to illustrate $\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}$. The second example below illustrates a common error.

$\frac{6 + 12}{3} \stackrel{?}{=} \frac{6}{3} + \frac{12}{3}$

$\frac{18}{3}$	$2 + 4$
6	6 ✓

$\frac{6 + 12}{3} \stackrel{?}{=} \frac{6}{3} + 12$

$\frac{18}{3}$	$2 + 12$
6	14 No!

Show students that this idea is valid for polynomials divided by monomials also.

Chalkboard Examples

Divide.

1. $\frac{20a - 12}{4} = 5a - 3$ **2.** $\frac{3a^2b + 6ab^2}{ab} = 3a + 6b$ **3.** $\frac{6a^2b - 3ab^2}{3ab} = 2a - b$

4. $\frac{6m^2n + 4mn - 12mn^2}{-2mn} = -3m - 2 + 6n$ **5.** $\frac{12ab^2 - 3ab + 3ab^2}{3ab} = 5b - 1$

Suggested Extension

Find the errors in the examples below.

1. $\frac{4a + 6b}{2} = 2a + 6b$ Did not divide both terms by 2.

2. $\frac{-12m^3n^4}{-4mn^2} = 3m^3n^2$ Overlooked m term in denominator.

12 • DIVIDING POLYNOMIALS (OPTIONAL) (pp. 160-161)

Teaching These Pages

To introduce this topic, have the class find the product.

$$(x + 4)(x + 5) = x^2 + 9x + 20$$

Then have the class explain these equations.

$$\frac{x^2 + 9x + 20}{x + 4} = x + 5 \qquad \frac{x^2 + 9x + 20}{x + 5} = x + 4$$

Use the example on page 160 to develop a method for dividing polynomials. Have the students suggest a method to check that the division is correct.

Chalkboard Examples

Divide. Explain each step.

1.

$$\begin{array}{r} x + 5 \\ x + 6\overline{)x^2 + 11x + 30} \\ \underline{x^2 + 6x} \\ 5x + 30 \\ \underline{5x + 30} \\ 0 \end{array}$$

$(x + 6)(x + 5) = x^2 + 11x + 30$ ✓

2.

$$\begin{array}{r} x + 3 \\ x - 4\overline{)x^2 - x - 12} \\ \underline{x^2 - 4x} \\ 3x - 12 \\ \underline{3x - 12} \\ 0 \end{array}$$

$(x - 4)(x + 3) = x^2 - x - 12$ ✓

Chapter 6. Factoring

This chapter extends work with polynomials to factoring. The students will learn three basic factoring techniques: finding a common monomial factor, factoring trinomials with first term having coefficient of 1, and factoring the difference of squares.

In later chapters the skills developed in this chapter will be applied to equation solving as well as other algebraic problems.

Objectives for Pages 170–175

1. To factor positive integers.

2. To find a common monomial factor in a polynomial.

1 • FACTORING IN ARITHMETIC (pp. 170–171)

Teaching These Pages

Discuss the importance of factoring in adding fractions. Factoring can help us find a common denominator.

$$\frac{5}{12} + \frac{9}{20} = \frac{5 \cdot 5}{2 \cdot 2 \cdot 3 \cdot 5} + \frac{3 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 3 \cdot 5} = \frac{25}{60} + \frac{27}{60}$$

Introduce the meanings of *prime number, prime factor, common factor* and *greatest common factor* as they relate to the above example.

Point out that factors can be found by division. If the remainder is zero, the divisor is a factor.

Chalkboard Examples

Find the prime factors.

1. $24 = 2 \cdot 2 \cdot 2 \cdot 3$ **2.** $42 = 2 \cdot 3 \cdot 7$ **3.** $99 = 3 \cdot 3 \cdot 11$ **4.** $53 = 53$

5. Find a common factor of 12 and 20. 2 *or* 4

Find the greatest common factor.

6. 12, 30 6 **7.** 100, 125 25 **8.** 17, 27 1 **9.** 18, 54 18

Suggested Extension

A number has 9 as a factor if the sum of its digits is divisible by 9. For example, 135 is divisible by 9 since $1 + 3 + 5$ is divisible by 9.

Which numbers are divisible by 9?

1. 45 yes **2.** 147 no **3.** 909 yes **4.** 242 no

A number has 4 as a factor if its last two digits are divisible by 4. For example, 2348 is divisible by 4 since 48 is divisible by 4.

Which numbers are divisible by 4?

5. 1323 no **6.** 4156 yes **7.** 324 yes **8.** 3929 no

2 • COMMON MONOMIAL FACTORS (pp. 162–173)

Teaching These Pages

Review the distributive property.

$$2x(3x + 4y) = 6x^2 + 8xy$$

Use the terms $6x^2$ and $8xy$ to introduce the meaning of greatest common factor.

$6x^2 = 2 \cdot 3 \cdot x \cdot x$
$8xy = 2 \cdot 2 \cdot 2 \cdot x \cdot y$ The greatest common factor is $2x$.

Encourage students to factor completely, that is, to find the greatest common factor, and to check their factorizations by applying the distributive property.

Chalkboard Examples

Find the greatest common factor.

1. $50mn, 10m^2n^2$ $10mn$ **2.** $12x^2y, 16xy^2$ $4xy$ **3.** $81ab^3, 18a^2b^2$ $9ab^2$

Factor.

4. $3y^3 - 6y^2 = 3y^2(y - 2)$ **5.** $10x^3 + 30x^2 + 50x = 10x(x^2 + 3x + 5)$

6. $6a^2 - 24ab + 18b^2 = 6(a^2 - 4ab + 3b^2)$ **7.** $y(x + y) - x(x + y) = (x + y)(y - x)$

3 • USING FACTORING (pp. 174–175)

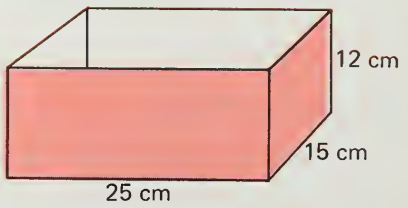
Teaching These Pages

Ask the class to suggest different ways to evaluate the following expression. Some students will suggest finding each product and then adding. Encourage the students to look for a common factor, using the methods of the previous section.

$$25 \times 32 + 25 \times 18 + 25 \times 50 = 25(32 + 18 + 50) = 25(100) = 2500$$

Chalkboard Examples

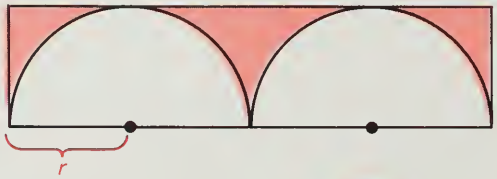
1. Find the total area of the sides of the box.



$$\begin{aligned} &12 \times 15 + 12 \times 25 + 12 \times 15 + 12 \times 25 \\ &= 12(15 + 25 + 15 + 25) \\ &= 12(80) \\ &= 960 \end{aligned}$$

The area is 960 cm^2.

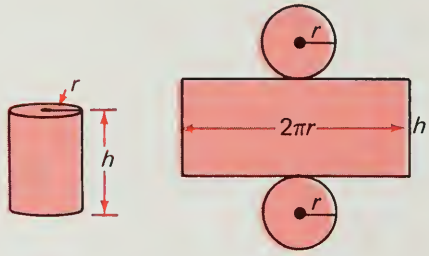
2. Find the area of the shaded part. Leave your answer in terms of π and r.



Area of rectangle: $4r^2$
Area of 2 semicircles: πr^2
Area of shaded part: $4r^2 - \pi r^2 = r^2(4 - \pi)$

Suggested Extension

The surface area of a cylinder is shown by the model. Find an expression for the surface area.



Area of 2 ends: $2\pi r^2$
Area of curved surface: $2\pi rh$
Total surface area: $2\pi r^2 + 2\pi rh = 2\pi r(r + h)$

Objectives for Pages 176-187

1. To factor trinomials with factors of the form $(x + a)(x + b)$.

2. To factor trinomials with factors of the form $(x - a)(x - b)$.

3. To factor trinomial squares.

4. To factor trinomials with factors of the form $(x - a)(x + b)$.

4 • FACTORING TRINOMIALS—TWO SUMS (pp. 176-179)

Teaching These Pages

Have the students find these products at sight.

$$\begin{aligned} (x + 2)(x + 4) &= x^2 + 6x + 8 \\ (x + 3)(x + 5) &= x^2 + 8x + 15 \\ (x + 6)(x + 3) &= x^2 + 9x + 18 \end{aligned}$$

Discuss the method the students used to find the products. Point out that in this section we reverse that method to find the factors that yield each trinomial.

Discuss the examples on pages 176 and 177. Ask the class to suggest a way to check that the factorizations are correct.

Chalkboard Examples

Name the two numbers.

1. Their sum is 9, their product is 18. 3, 6

2. Their sum is 12, their product is 36. 6, 6

3. Their sum is 13, their product is 40. 5, 8

4. Factor $a^2 + 13a + 30$. $(a + 3)(a + 10)$

Suggested Extension

The work of the previous section is used in this alternative approach to factoring.

1. Tina factored $x^2 + 12x + 27$ as shown. Explain the steps she used.

$$\begin{aligned} x^2 + 12x + 27 &= x^2 + 3x + 9x + 27 \\ &= x(x + 3) + 9(x + 3) \\ &= (x + 3)(x + 9) \end{aligned}$$

2. Use Tina's method to factor Exercises 31–36 on page 179.

5 • FACTORING TRINOMIALS—TWO DIFFERENCES (pp. 180-181)

Teaching These Pages

Have the students find these products at sight.

$$(x - 2)(x - 4) = x^2 - 6x + 8$$
$$(x - 3)(x - 5) = x^2 - 8x + 15$$
$$(x - 6)(x - 3) = x^2 - 9x + 18$$

Discuss the examples on page 180. Be sure students understand why we look for negative numbers in Step 3.

Chalkboard Examples

Name the two numbers.

1. Their sum is -8, their product is 12. $-2, -6$

2. Their sum is -12, their product is 32. $-4, -8$

3. Their sum is -10, their product is 25. $-5, -5$

4. Factor $m^2 - 10m + 24$. $(m - 6)(m - 4)$

Suggested Extension

The factoring method used in the Suggested Extension for the previous section can be used for this section also.

6 • FACTORING TRINOMIAL SQUARES (pp. 182–183)

Teaching These Pages

Have the students find the squares of these binomials and note how each term of the trinomial square is related to the original factors.

$(x + 3)^2 = x^2 + 6x + 9$ $\quad$ $(x - 3)^2 = x^2 - 6x + 9$

$(x + 5)^2 = x^2 + 10x + 25$ $\quad$ $(x - 5)^2 = x^2 - 10x + 25$

Develop the test outlined on page 182 for recognizing trinomial squares. Point out that the students can always use the methods of the previous sections for factoring if they forget how to recognize trinomial squares.

Chalkboard Examples

Complete.

1. $x^2 + 16x + 64 = (\underline{?} + 8)^2$ x

2. $a^2 + 12a + 36 = (a \underline{?})^2$ $+6$

3. $m^2 - 10m + 25 = (m \underline{?})^2$ -5

4. $y^2 - 8y + 16 = (\underline{?} - 4)^2$ y

Factor.

5. $m^2 - 18m + 81 = (m - 9)^2$

6. $x^2 + 16x + 64 = (x + 8)^2$

7. $a^2 - 8a + 15 = (a - 3)(a - 5)$

8. $x^2 - 14x + 49 = (x - 7)^2$

Suggested Extension

We can use what we know about trinomial squares to find squares of numbers. For example, we can write 25 as $20 + 5$ and find 25^2 as shown below.

$$(20 + 5)^2 = 20^2 + 2(5 \cdot 20) + 5^2 = 400 + 200 + 25 = 625$$

Write the square of each number. How many can you do mentally?

1. 21 441 $\quad$ **2.** 16 256 $\quad$ **3.** 35 1225 $\quad$ **4.** 28 784

7 • FACTORING TRINOMIALS (pp. 184–187)

Teaching These Pages

Have the students find these products.

$(x + 8)(x - 6) = x^2 + 2x - 48$ $\quad$ $(x - 8)(x + 6) = x^2 - 2x - 48$

Ask students to compare the two examples. How are they the same? How are they different? Relate the students' observations to the examples on page 184.

Chalkboard Examples

Name the two numbers.

1. Their sum is -5, their product is -14. $-7, 2$
2. Their sum is -4, their product is -5. $-5, 1$
3. Their sum is 6, their product is -16. $-2, 8$

Factor.

4. $x^2 + 2x - 35 = (x + 7)(x - 5)$
5. $x^2 - 3x - 40 = (x - 8)(x + 5)$

Objectives for Pages 188-193

1. To multiply the sum of two monomials by their difference.
2. To factor the difference of two squares.

8 • A SPECIAL PRODUCT (pp. 188-189)

Teaching These Pages

Have the students find these products.

$(x + 1)(x - 1)$ $\quad$ $(x - 2)(x + 2)$
$(x - 3)(x + 3)$ $\quad$ $(x + 4)(x - 4)$

Ask the students if they notice anything special about the products.

Chalkboard Examples

Complete.

1. $(y - 3)(y + 3) = \underline{?} - \underline{?}$ y^2; 9
2. $(2m - 4)(2m + 4) = \underline{?} - \underline{?}$ $4m^2$; 16

Multiply.

3. $(y + 9)(y - 9) = y^2 - 81$
4. $(4x - y)(4x + y) = 16x^2 - y^2$
5. $(m^2 + 5)(m^2 - 5) = m^4 - 25$
6. $(x - 2y)(x + 2y) = x^2 - 4y^2$
7. Find the product (45)(55). Use the rule $(a - b)(a + b) = a^2 - b^2$.

$$\begin{aligned}(45)(55) &= (50 - 5)(50 + 5)\\ &= 50^2 - 5^2\\ &= 2500 - 25\\ &= 2475\end{aligned}$$

Suggested Extension

Extensions for Chapter 1, Sections 6 and 7; Chapter 6, Section 6; and Example 6 above suggest methods for computing products mentally. Combine exercises for each method to have a Mental Multiplication Quiz. The students might work in teams to arrive at a class championship.

9 • FACTORING THE DIFFERENCE OF SQUARES (pp. 190–191)

Teaching These Pages

Review a few examples from the previous section on finding products of the form $(a + b)(a - b)$.

Discuss the examples on page 190. Emphasize that this method is for factoring the *difference* of two squares. Point out that $x^2 + 4y^2$ and $9x^2 + 1$ cannot be factored.

Introduce the need to recognize the difference of two squares. For example, ask the class to factor $-16 + y^2$. Point out that $-16 + y^2$ can be written as $y^2 - 16$.

Chalkboard Examples

Complete.

1. $a^2 - b^2 = (a __?__)(a __?__)$ $+b;\ -b$

2. $4x^2 - y^2 = (__?__ + y)(__?__ - y)$ $2x;\ 2x$

Factor.

3. $m^2 - 16 = (m - 4)(m + 4)$

4. $x^2 - 9y^2 = (x - 3y)(x + 3y)$

5. $m^4 - 16 = (m^2 - 4)(m^2 + 4) = (m - 2)(m + 2)(m^2 + 4)$

Suggested Extension

1. Show why $x^2 + y^2 \neq (x - y)(x + y)$.

Students may show that $x^2 - y^2 = (x - y)(x + y)$.
Then since $x^2 - y^2 \neq x^2 + y^2$ (unless $y = 0$), $x^2 + y^2 \neq (x - y)(x + y)$.

The students might also assign values to x and y then evaluate $x^2 + y^2$ and $(x - y)(x + y)$ to show that the two expressions are not equal.

10 • MANY TYPES OF FACTORING (pp. 192–193)

Teaching These Pages

Review the various factoring techniques developed in this chapter. Have the class work a few examples using each method.

Point out that in this section more than one technique may be needed to factor each example.

Chalkboard Examples

Factor.

1. $2x^2 + 12x + 18 = 2(x^2 + 6x + 9)$
$= 2(x + 3)^2$

2. $x^3 - 6x^2 + 8x = x(x^2 - 6x + 8)$
$= x(x - 4)(x - 2)$

3. $3x^2 - 48 = 3(x^2 - 16)$
$= 3(x + 4)(x - 4)$

4. $-x^2 + 6x + 16 = -1(x^2 - 6x - 16)$
$= -1(x - 8)(x + 2)$

Suggested Extension

Factor.

1. $x^4 + 5x^2 - 36 = (x^2 + 9)(x^2 - 4)$
$= (x^2 + 9)(x - 2)(x + 2)$

2. $x^4 - 5x^2 - 36 = (x^2 - 9)(x^2 + 4)$
$= (x - 3)(x + 3)(x^2 + 4)$

3. $x^4 - 13x^2 + 36 = (x^2 - 4)(x^2 - 9)$
$= (x - 2)(x + 2)(x - 3)(x + 3)$

11 • MORE DIFFICULT FACTORING (OPTIONAL) (pp. 194-195)

Teaching These Pages

Introduce this section by having the students find products.

$$(3x + 1)(x + 2) = 3x^2 + 7x + 2$$
$$(2x - 1)(x + 3) = 2x^2 + 5x - 3$$

Discuss the examples on page 194. Point out that when the coefficient of the first term of the trinomial is not 1, it may be necessary to try many factors before finding the correct ones.

Chalkboard Examples

Factor.

1. $3x^2 - 13x + 4 = (3x - 1)(x - 4)$

2. $3y^2 + 5y - 2 = (3y - 1)(y + 2)$

Chapter 7. Graphs

Every day we are confronted with graphs occurring in newspapers, magazines, reports, and so on. Thus it is important to be able to interpret information displayed by means of a graph. This chapter develops skills for working with pictographs and bar graphs and then extends the development to include broken line graphs as well as graphs of linear equations. In preparation for the next chapter, the graphs of equations in two unknowns are studied and related vocabulary is introduced.

Objectives for Pages 204–215

1. To read a pictograph.

2. To read and draw a bar graph.

3. To read and draw a broken line graph.

4. To identify and graph ordered pairs of numbers on a coordinate plane.

1 • GRAPHS YOU OFTEN SEE (pp. 204–207)

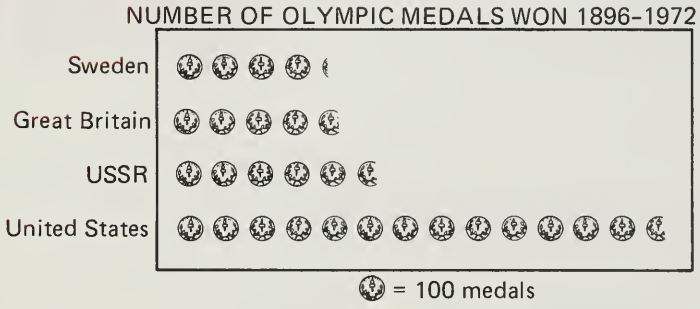
Teaching These Pages

Throughout this chapter, emphasize the usefulness of a graph to display information in a concise manner.

Have the class collect examples of graphs from newspapers and magazines. Textbooks for other subjects might also provide examples for discussion. Use the example on page 205 to introduce the nature of misleading graphs.

Point out that the scale on a graph should always start at zero. When this would cause the size of the graph to be too large, we delete a portion of the scale and indicate the deletion by a jagged rule. (See page 206.)

Chalkboard Examples



1. What information does the graph show? The number of medals won in the Olympics from 1896 to 1972.

2. How many medals does each symbol represent? 100

3. What does a portion of a symbol represent? Fewer than 100 medals.

4. Draw a bar graph that shows the same information.

2 • GRAPHS WITH LINES AND CURVES (pp. 208–211)

Teaching These Pages

Record the temperature for a certain city for 7 consecutive days. Make a table to illustrate the data.
Then show the class the use of a bar graph to show the data, the construction of a broken line graph from the bar graph, and the drawing of a "smooth" line graph. Discuss how the three graphs are related and why we can draw a smooth curve in the third graph.

Have the class create questions based on the information shown by the graphs.

Chalkboard Example

Each year a group of students hold a car wash-a-thon to raise money for charity. The amount of money collected for the last 6 years is shown on the bar graph.



Refer to the graph to answer the questions.

1. Estimate the amount of money collected in 1972 and in 1975. $482; $525

2. How much more money was collected in 1976 than in 1971? $60

3. Estimate the average amount of money collected each year since 1971. about $500

4. Predict the amount of money the group might collect in 1977. probably more than $500

Suggested Extension

Each day the newspaper publishes high and low temperatures for various cities. Have the class, or groups of students, select a city and collect data about the daily temperatures.

1. Draw a graph to show the data.

2. Use the graph to predict the next day's temperature.

3 • POINTS ON GRAPH (pp. 212-215)

Teaching These Pages

The temperature data used in the previous section can be used here to introduce graphing points on a coordinate plane.

Introduce the vocabulary related to the graph on page 212: ordered pair, origin, coordinate, horizontal axis, vertical axis.

Chalkboard Examples

Give the coordinates of each point.

1. A $(-4, 2)$ **2.** C $(2, -2)$ **3.** D $(-2, 0)$

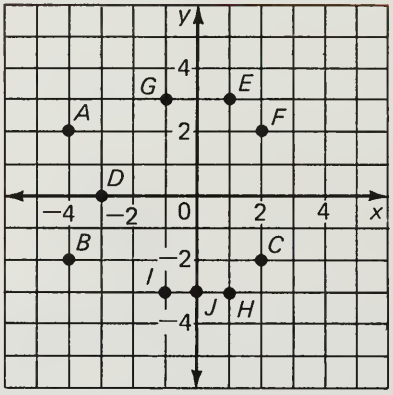
Name the point by its letter.

4. $(1, 3)$ E **5.** $(-1, 3)$ G **6.** $(-1, -3)$ I

Give the coordinates of the point closest to each point.

7. F E **8.** G E **9.** H J

10. Name 4 points that are at the vertices of a rectangle.



Objectives for Pages 216-227

1. To identify solution pairs for a linear equation in two variables.

2. To draw the graph of a linear equation in two variables.

3. To find the slope of a line.

4 • SOLUTION PAIRS (pp. 218-219)

Teaching These Pages

Show the class an equation in one variable and an equation in two variables.

$$3x + 6 = 12 \qquad x + y = 6$$

List some solutions for the equation in two variables.

$$x = 1, y = 5 \qquad x = 2, y = 4 \qquad x = -3, y = 9$$

Point out that the equation in one variable has only one solution, 2, but the equation in two variables has many solution pairs.

Introduce the use of an ordered pair (x, y) to record a solution for an equation in two variables.

Chalkboard Examples

1. Find x if $y = 3$ in $2x + y = 11$. 4

2. Find y if $x = 2$ in $x + y = 8$. 6

3. Which ordered pair is *not* a solution pair for $x - 2y = 8$?
$(1, -4)$, $(-2, -5)$, $(6, 1)$ (6, 1)

4. Find a solution pair for $x + 3y = 9$.

5. Write an equation to go with the table. $y = 3x$

x	y
2	6
3	9
4	12
5	15

5 • GRAPHS OF EQUATIONS (pp. 220-221)

Teaching These Pages

Have the class find some solution pairs that satisfy $x + y = 5$. Plot each of the solution pairs.

Point out that the points seem to lie along a straight line. Have the class name the coordinates of another point that *seems* to lie along the same line. Check that this point is the graph of a solution by substituting the values for x and y in $x + y = 5$.

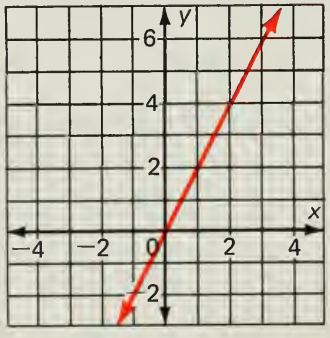
Introduce the meaning of the graph of the equation $x + y = 5$. Emphasize that the graph of any ordered pair that is a solution pair will lie on the graph of $x + y = 5$ and vice versa.

Chalkboard Examples

1. Draw the graph of $2x + y = 6$.

Point out that at least two points are needed to draw the line and a third point is useful to assure accuracy.

2. The graph of an equation is shown. Write the equation for the graph $y = 2x$



Suggested Extension

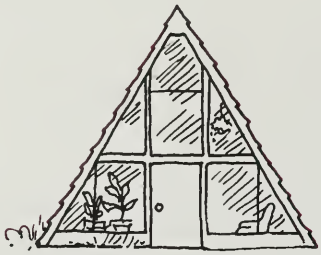
1. Draw the graph of $x + y = 5$.
2. Draw the graph of $x - y = 1$ on the same set of axes.
3. Name the point that lies on both lines. (3, 2)

6 • SLOPE OF A LINE (pp. 222-223)

Teaching These Pages

Introduce the meaning of slope by comparing the slopes of the roofs of various houses.





The roof of an A-frame cottage has a steeper slope than many roofs, allowing rain and snow to run off more quickly.

Introduce the meanings of *rise* and *run*.

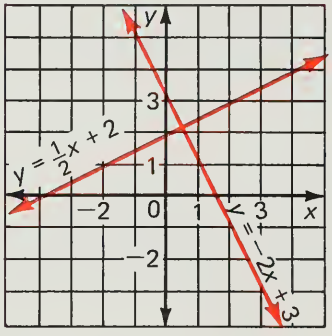
Chalkboard Examples

Find the slope.

1. $y = \frac{1}{2}x + 2$ $\frac{1}{2}$

2. $y = -2x + 3$ -2

3. How is the slope of each graph of the equations in Exercises 1 and 2 related to the coefficient of the x term in each equation? the same



Use the answer to Exercise 3 to name the slope of the graph of each equation.

4. $y = 6x - 3$ 6

5. $y = -\frac{1}{4}x + 4$ $-\frac{1}{4}$

6. $y = -2x + 5$ -2

7. Draw a line with slope $\frac{2}{3}$. Can you write an equation for the line?

Suggested Extension

Draw the graph of each equation on the same set of axes.

1. $y = 3x + 1$ **2.** $y = 3x - 1$

3. Do the graphs in Exercises 1 and 2 cross each other? No.

4. What do you notice about the slopes of the graphs in Exercises 1 and 2? They are the same.

Draw the graph of each equation on the same set of axes.

5. $y = 2x + 3$ **6.** $y = x - 4$

7. Do the graphs in Exercises 5 and 6 cross each other? Yes.

8. What do you notice about the slopes of the graphs in Exercises 5 and 6? They are different.

7 • FUNCTIONS (pp. 224–227)

Teaching These Pages

Discuss the examples of functions on pages 224 and 225.

Although this section takes a very intuitive approach to functions, some students will learn in later courses that a function can be thought of as a set of ordered pairs in which no two different pairs have the same first element.

Chalkboard Examples

1. A machine adds 2 to the input. Complete the table.

x = INPUT	8	6	−12
y = OUTPUT	?	?	?

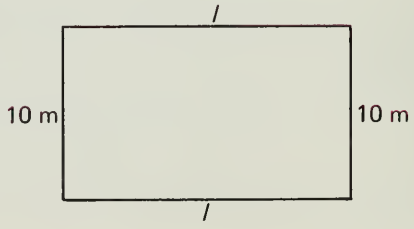
10 8 −10

2. An equation relating x and y in Exercise 1 is: $y =$ ___?___. $x + 2$

3. Study the function in Example 2 on page 224. Is there an equation relating the age of the car to the value of the car? No.

Suggested Extension

1. Use the formula $P = 2l + 2w$. For the rectangle shown, write an equation to show how P depends on l. $P = 2l + 20$



Chapter 8. Equations with Two Variables

In this chapter students will learn to solve a pair of linear equations in two variables. The graphic method is first used, but for greater accuracy the methods of substitution and add-or-subtract are developed. These skills are applied to problem solving in Sections 5, 7, and 8.

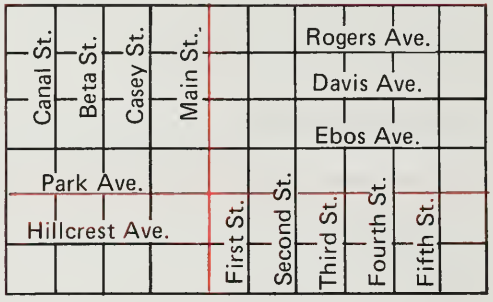
Objectives for Pages 236–243

1. To find the solutions of pairs of linear equations by graphing.
2. To recognize whether a pair of linear equations shares one solution pair, no solution pairs, or all solution pairs.

1 • THE GRAPHING METHOD (pp. 236–239)

Teaching These Pages

Use a street map to review locating points on a coordinate system and to introduce the meaning of *intersection*.



Chalkboard Examples

Complete the table.

1.

$x + y = 12$	
x	y
2	?
?	14
6	?
7	?

10
−2
6
5

2.

$x - y = 2$	
x	y
0	?
?	2
?	5
?	4

−2
4
7
6

3. Name an ordered pair that satisfies $x + y = 12$ *and* $x - y = 2$. (7, 5)

4. Draw the graphs of $x + y = 12$ and $x - y = 2$ on the same set of axes.

5. Name the coordinates of the point where $x + y = 12$ and $x - y = 2$ intersect. Compare this answer to the answer for Exercise 3. (7, 5)
Both answers are the same.

6. Name the ordered pair that is the solution of $x + y = 12$ and $x - y = 2$. (7, 5)

2 • NO SOLUTION, MANY SOLUTIONS (pp. 240-243)

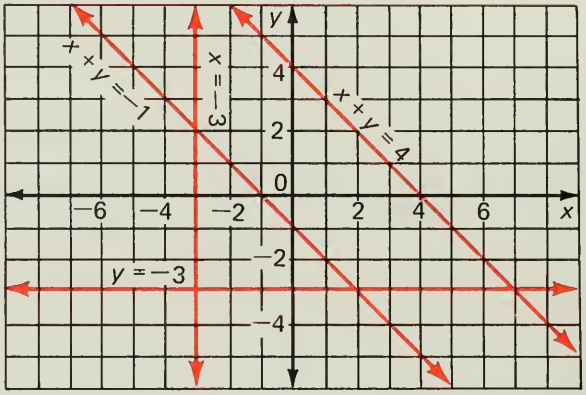
Teaching These Pages

Use the following graph to introduce this lesson. Ask the students to name a pair of equations that have intersecting graphs.

Introduce the meaning of *parallel* lines. You may want to point out that parallel lines lie in the same plane.

Ask the students to name a pair of equations that have graphs that are parallel lines. Point out that the slopes are the same.

Point out that the graphs of $x + y = 4$ and $2x + 2y = 8$ are the same, and so $x + y = 4$ and $2x + 2y = 8$ share all solutions.



Chalkboard Examples

Find the slope of the line defined by each equation.

1. $y = 6x + 3$ 6 **2.** $x + y = 4$ -1 **3.** $x + 2y = 4$ $-\frac{1}{2}$

4. Do $x - y = 3$ and $x - y = 6$ share any solution pairs? Why? No, their graphs are parallel lines.

5. Do $x + 2y = 6$ and $2x + 4y = 12$ share any solution pairs? Why? Yes, their graphs are the same line.

Objectives for Pages 244-249

1. To use the substitution method to find the solutions of pairs of linear equations.
2. To use the add-or-subtract method to find the solutions of pairs of linear equations.

3 • THE SUBSTITUTION METHOD (pp. 244-245)

To introduce the need to find other methods of finding solution pairs for a pair of linear equations, have the students use the graphing method to find the ordered pair which is a solution of the equations below.

$$4x - 3y = 1$$
$$2x + 3y = 2$$

The solution pair is $\left(\frac{1}{2}, \frac{1}{3}\right)$.

Chalkboard Examples

Solve.

1. $x + y = 8$
$x - y = 4$ $(6, 2)$

2. $y = 4x$
$x - y = -9$ $(3, 12)$

3. $a + 2b = 4$
$b - 3a = -26$ $(8, -2)$

4. $P = 2l + 2w$; $l = 4k$, $w = 2k$,
Find P in terms of k. $P = 12k$

4 • THE ADD-OR-SUBTRACT METHOD (pp. 246-249)

Teaching These Pages

Use the substitution method to find a solution pair for these equations.

$$\begin{aligned} 3x - y &= 7 \quad (1) \\ 2x + y &= 8 \quad (2) \end{aligned}$$

$$\begin{aligned} y &= 3x - 7 \\ 2x + (3x - 7) &= 8^* \\ 5x - 7 &= 8 \\ x &= 3, y = 2 \end{aligned}$$

Introduce the add-or-subtract method. Point out that if we add equations (1) and (2) we will obtain the starred equation. Suggest that for certain pairs of equations the add-or-subtract method is more convenient than the substitution method.

Chalkboard Examples

Solve. Encourage students to check the solution in the original equations.

1. $3y - 2x = 11$
$2y + 2x = 4$ $(-1, 3)$

2. $3y + 2x = 4$
$2y + 2x = 2$ $(-1, 2)$

Objectives for Pages 250-257

1. To use a pair of linear equations to solve problems.

2. To use multiplication and the add-or-subtract method to find the solutions of pairs of linear equations.

3. To use a pair of linear equations to solve cost problems.

5 • WORD PROBLEMS, TWO VARIABLES (pp. 250-253)

Teaching These Pages

Review the Guide for Problem Solving on page 54. Point out that we follow similar steps when solving problems with two variables. Solve a problem with one variable and compare its solution to the solutions of the Chalkboard Examples.

Chalkboard Examples

Solve. Use two variables and two equations.

1. The sum of two numbers is 26. One number is 6 more than the other. Find the numbers.

Let x = the larger number;
y = the smaller.

$x + y = 26$
$x - 6 = y$ $x = 16; y = 10$

2. Mike and Lesley scored 145 points altogether. Lesley scored 7 points more than Mike. How many points did each score?

Let x = Lesley's score; y = Mike's score

$x + y = 145$
$x - y = 7$ $x = 76, y = 69$

Suggested Extension

1. Write a problem to "fit" these equations. Solve your problem.

$$x + y = 28$$
$$x - y = 2$$

Sample: The sum of Alec's and Georgina's ages is 28 years.
The difference in their ages is 2.
Find their ages.

6 • USING MULTIPLICATION (pp. 254-255)

Teaching These Pages

Show an example of a pair of equations that cannot be solved by the substitution method or the add-or-subtract method.

Review that we can multiply members of an equation by the same number to obtain an equivalent equation. Thus we can transform the second equation and solve the pair as shown.

$$\begin{array}{l} 4x + 5y = 7 \\ 2x + 3y = 3 \end{array} \longrightarrow \begin{array}{r} 4x + 5y = 7 \\ 4x + 6y = 6 \\ \hline -y = 1;\ y = -1 \\ 4x - 5 = 7;\ x = 3 \end{array}$$

Point out that we can multiply members of an equation by the same fractional number if it is more convenient.

Chalkboard Examples

Solve.

1. $3x - 2y = 10$
$2x + y = 9$ $(4, 1)$

2. $3x - 2y = -7$
$2x + 3y = 4$ $(-1, 2)$

7 • COST PROBLEMS (pp. 256–257)

Teaching These Pages

Review the cost formula from Chapter 4. $C = p \times n$. Also review the Guide for Problem Solving on page 54.

Chalkboard Examples

1. A hot dog costs 50¢. A hamburger costs 60¢.
 a. Write the total cost of x hot dogs and y hamburgers. $50x + 60y$ cents
 b. Write the total cost of $3x$ hot dogs and $2y$ hamburgers. $150x + 120y$ cents

2. Movie tickets cost \$3 for adults and \$2 for children. On Saturday, \$66 was paid for 28 tickets. How many adult tickets were sold?

 Let a = number of tickets for adults; c = number of tickets for children.

 $$\begin{aligned} a + \ c &= 28 \\ 3a + 2c &= 66 \end{aligned}$$

 There were 10 adult tickets sold.

Suggested Extension

Clip advertisements from the newspaper to obtain prices for various items. Have the students create and exchange cost problems based on the ads.

8 • BOAT AND AIRCRAFT PROBLEMS (OPTIONAL) (pp. 258–259)

Teaching These Pages

Review the skills developed in Chapter 4 for solving motion problems with one variable.

Show the students the difficulty in attempting to solve the example on page 258 with only one variable.

Chalkboard Example

1. Janie and Jacques traveled 12 km upstream in a boat in 6 hours. The return trip took only 3 hours. Find the rate of the flowing water and the rate of the boat (in still water).

 Let x = rate in km/h of the boat in still water;
 y = rate in km/h of the current.

 $$\begin{aligned} 6(x - y) &= 12 \\ 3(x + y) &= 12 \end{aligned}$$ $x = 3;\ y = 1$

Chapter 9. Working with Fractions

In earlier chapters the students have developed skills for working with polynomials, and applied these skills to solving problems. In this chapter students develop skills for working with fractions in algebra, essential to solving problems. Throughout the chapter skills with fractions in algebra are related to similar skills the students have learned in working with arithmetic. Once the students have learned to add, subtract, multiply and divide algebraic expressions involving fractions, these skills become useful for solving equations that involve fractions. This work with equations is then applied to solving problems that deal with rates of work.

Objective for Pages 268-273

1. To use factoring to simplify fractions.

1 • FRACTIONS IN ALGEBRA (pp. 268-269)

Teaching These Pages

Review that a number has different names.

$$9 \qquad 12 - 3 \qquad 3^2 \qquad 5 + 4$$

Remind students that a fractional number also has different names.

$$\frac{1}{4} \qquad \frac{2}{8} \qquad 4 \div 16 \qquad \frac{8}{32}$$

Point out that to simplify a fraction means to find a simpler name for the fraction.

Use the examples on page 268 to compare simplifying fractions in arithmetic to simplifying fractions in algebra.

Before assigning the exercises it would be helpful to review the rules for dividing positive and negative numbers (page 88), and the exponent rules (page 156).

Chalkboard Examples

Simplify.

1. $\frac{-8a}{16a} = -\frac{1}{2}$ **2.** $\frac{30m}{-25m} = -\frac{6}{5}$ **3.** $\frac{-12a^2b}{-24ab^2} = \frac{a}{2b}$ **4.** $\frac{-36ab^3c^2}{-12a^2bc} = \frac{3b^2c}{a}$

State the value of x for which the fraction has no meaning.

5. $\frac{3}{x-2}$ 2 **6.** $\frac{x}{1-x}$ 1 **7.** $\frac{3}{2x-4}$ 2 **8.** $\frac{7}{6-3x}$ 2

Suggested Extension

Simplify each fraction then add or subtract.

1. $\frac{-36a^2b}{12ab} + \frac{12a^2b^2}{-3a^2b^2} = -7a$

2. $\frac{-24m^3n}{-4m^3} - \frac{-4m^2n^3}{2m^2n^2} = 8n$

3. $\frac{16r^4s^2}{-8r^3s} - \frac{-16r^3s^2}{-3r^2s} = -4rs$

4. $\frac{6k^3m^2}{3km} + \frac{-16k^5m^4}{4k^3m^3} = -2k^2m$

2 • SIMPLIFYING FRACTIONS (pp. 270-271)

Teaching These Pages

Review the various factoring techniques students learned in Chapter 6.

$y^3 - 2y^2 = y^2(y - 2)$ $\qquad$ $a^2 + 2a - 8 = (a + 4)(a - 2)$

$m^2 + 6m + 9 = (m + 3)^2$ $\qquad$ $4x^2 - y^2 = (2x - y)(2x + y)$

Chalkboard Examples

Simplify.

1. $\frac{x + 2}{2x + 4} = \frac{x + 2}{2(x + 2)} = \frac{1}{2}$

2. $\frac{3y - 6}{3} = \frac{3(y - 2)}{3} = y - 2$

3. $\frac{2ab - 6a}{3b^2 - 9b} = \frac{2a(b - 3)}{3b(b - 3)} = \frac{2a}{3b}$

4. $\frac{3k + 6}{k^2 + 4k + 4} = \frac{3(k + 2)}{(k + 2)(k + 2)} = \frac{3}{k + 2}$

3 • THE -1 FACTOR (pp. 272-273)

Teaching These Pages

Ask the students to compare these pairs of binomials.

$x - 1$	$a - b$	$x^2 - y^2$
$\updownarrow$	$\updownarrow$	$\updownarrow$
$1 - x$	$b - a$	$y^2 - x^2$

Use the distributive property to show that $1 - x = -1(x - 1)$.

$$\begin{aligned} -1(x - 1) &= -x + 1 \\ &= 1 - x \end{aligned}$$

Point out the usefulness of the -1 factor in simplifying fractions.

$$\frac{x - 1}{1 - x} = \frac{x - 1}{-1(x - 1)} = \frac{1}{-1} = -1$$

Chalkboard Examples

Simplify.

1. $\frac{x-y}{y-x} = -1$

2. $\frac{y^2-x^2}{x+y} = -x+y$

3. $\frac{y-x}{-x^2+2xy-y^2} = \frac{1}{x-y}$

Objectives for Pages 274-283

1. To solve problems involving ratios.
2. To solve problems involving proportions.

4 • RATIO (pp. 274-275)

Teaching These Pages

Introduce the meaning of *ratio* with familiar examples.

1. Ratio of the number of students over 165 cm tall to the number of students under 165 cm tall.
2. Ratio of the number of pages in the math book to the number of pages in the dictionary.

Point out that the following two statements are equivalent.

The ratio of the number of students to the number of teachers is 20 to 1.

There are 20 students for every 1 teacher.

Chalkboard Examples

Give the ratio in simplest form.

1. 20 meters to 100 meters. $\frac{20}{100} = \frac{1}{5}$

2. 2 years to 2 months. $\frac{24}{2} = \frac{12}{1}$

3. A team won 24 games and lost 18. What is the ratio of wins to losses? What is the ratio of losses to wins? $\frac{4}{3}; \frac{3}{4}$

5 • PROBLEMS INVOLVING RATIOS (pp. 276-277)

Teaching These Pages

Introduce this topic by posing a problem.

The ratio of the number of adults to the number of children who attended the school play is 2 to 3. Three hundred seventy-five persons attended the play. How many adults were there?

Point out that if the ratio of two numbers is 2 to 3, then we can write the ratio as $2x$ to $3x$.

$$\frac{2x}{3x} = \frac{2}{3}$$

Review the Guide to Problem Solving on page 54.

Chalkboard Examples

1. The ratio of the number of adults to the number of children who attended the school play was 2 to 3. Three hundred seventy-five persons attended the play. How many adults were there?

$$2x + 3x = 375$$
$$x = 75$$

There were 150 adults there.

2. There are 480 cars in the parking lot. The ratio of compact cars to full-size cars is 3 to 5. How many compact cars were there?

$$3x + 5x = 480$$
$$x = 60$$

There were 180 compact cars.

Suggested Extension

A box is filled with 500 beans, brown beans and white beans. Have the students take a sample from the box and record the ratio of brown beans to white beans. After the students have taken a number of samples, have them predict the number of brown beans in the box.

6 • PROPORTION (pp. 278–279)

Teaching These Pages

Review that fractional numbers have different names.

$$\frac{2}{3} = \frac{4}{6}$$

In a similar way, ratios may be equal.

$$\begin{array}{ccc} 2 \text{ to } 3 & \text{equals} & 4 \text{ to } 6 \\ \updownarrow & & \updownarrow \\ \frac{2}{3} & = & \frac{4}{6} \end{array}$$

Introduce the meaning of *proportion* and the property of proportions.

Chalkboard Examples

Solve.

1. $\frac{3}{x} = \frac{36}{12}$

$36 = 36x$

$1 = x$

2. $\frac{5}{4} = \frac{30}{a}$

$5a = 120$

$a = 24$

3. $\frac{y+1}{18} = \frac{y-1}{6}$

$6(y+1) = 18(y-1)$

$2 = y$

4. $\frac{x+3}{x+5} = \frac{x-4}{x-3}$

$(x+3)(x-3) = (x+5)(x-4)$

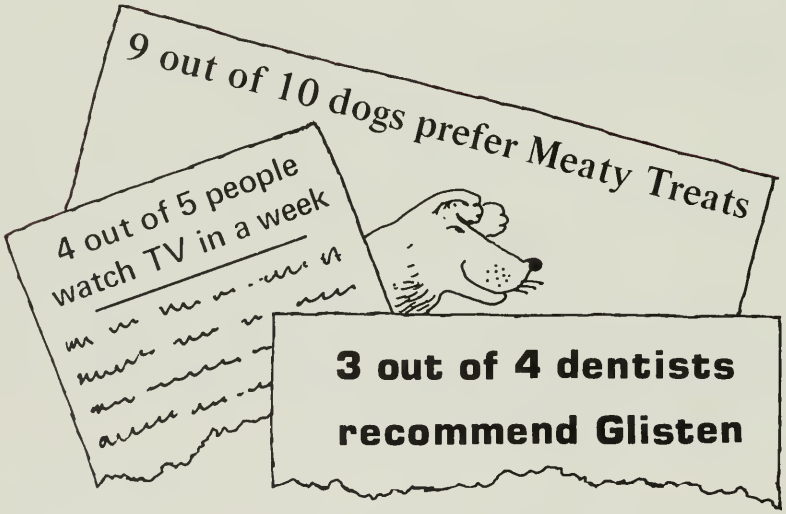
$x^2 - 9 = x^2 + x - 9$

$x = 11$

7 • APPLYING PROPORTIONS (pp. 280–283)

Teaching These Pages

Use newspaper clippings about polls or surveys to introduce the topic.



Show that proportions can be used to solve real-life problems based on surveys.

Review the steps for solving problems.

Chalkboard Examples

1. During a survey it was found that 88 out of 135 drivers had their seat belts fastened. If 540 drivers were interviewed, how many would you expect to have their seat belts fastened?

$$\frac{88}{135} = \frac{x}{540}$$

$$x = 352$$

352 drivers would have their seat belts fastened.

Objectives for Pages 284-289

1. To multiply fractions.
2. To divide fractions.
3. To add and subtract fractions with the same denominators.

8 • MULTIPLYING FRACTIONS (pp. 284-285)

Teaching These Pages

Compare how we multiply in arithmetic to how we multiply in algebra.

$$\frac{3}{5}\cdot\frac{2}{7}=\frac{3\cdot 2}{5\cdot 7}=\frac{6}{35} \qquad \frac{x}{y}\cdot\frac{k}{m}=\frac{x\cdot k}{y\cdot m}=\frac{xk}{ym}$$

Compare an arithmetic example to an algebra example to show how we often simplify the product of two fractions.

$$\frac{4}{5}\cdot\frac{3}{8}=\frac{4}{5}\cdot\frac{3}{2\cdot 4}=\frac{3}{10} \qquad \frac{a}{5}\cdot\frac{3}{2a}=\frac{a}{5}\cdot\frac{3}{2\cdot a}=\frac{3}{10}$$

Chalkboard Examples

Simplify.

1. $\frac{3x}{4}\cdot\frac{8}{x^2}=\frac{6}{x}$ **2.** $\frac{x-y}{4}\cdot\frac{x+y}{x-y}=\frac{x+y}{4}$ **3.** $\frac{x^2-y^2}{y}\cdot\frac{y^2}{x+y}=y(x-y)$

9 • DIVIDING FRACTIONS (pp. 286-287)

Teaching These Pages

Introduce the meaning of *reciprocal.*

Compare dividing in arithmetic to dividing in algebra.

$$\frac{3}{5}\div\frac{2}{7}=\frac{3}{5}\cdot\frac{7}{2}=\frac{21}{10} \qquad \frac{3}{5}\div\frac{k}{m}=\frac{3}{5}\cdot\frac{m}{k}=\frac{3m}{5k}$$

Chalkboard Examples

Simplify.

1. $\frac{4x}{3y}\div\frac{8xy}{3}=\frac{4x}{3y}\cdot\frac{3}{8xy}=\frac{1}{2y^2}$ **2.** $\frac{15}{4}\div\frac{3}{2x-2}=\frac{15}{4}\cdot\frac{2(x-1)}{3}=\frac{5(x-1)}{2}$

3. $\frac{2x-4}{3x+6}\div\frac{x-2}{2x+4}=\frac{2(x-2)}{3(x+2)}\cdot\frac{2(x+2)}{x-2}=\frac{4}{3}$

Suggested Extensions

1. Have the students discuss the restrictions on the denominators of the fractions in the exercises on page 287.

2. To review earlier work as well as emphasize the need to watch out for restrictions, have students discuss the "proof" that $2 = -2$.

$$\begin{aligned} a &= b \\ 2a &= 2b && \text{Multiply both sides by 2.} \\ 2a - 2b &= 2b - 2a && \text{Subtract the same values from both sides. } (2a = 2b) \\ 2(a - b) &= 2(b - a) && \text{Factor.} \\ -2(b - a) &= 2(b - a) && a - b = -(b - a) \\ -2 &= 2 && \text{Divide both sides by } b - a. \end{aligned}$$

10 • ADD, SUBTRACT—SAME DENOMINATORS (pp. 288-289)

Teaching These Pages

Point out that in algebra as well as in arithmetic we add fractions having the same denominators by adding the numerators.

$$\frac{1}{5} + \frac{2}{5} = \frac{1+2}{5} = \frac{3}{5} \qquad \frac{x}{5} + \frac{2x}{5} = \frac{x+2x}{5} = \frac{3x}{5}$$

Chalkboard Examples

Simplify.

1. $\frac{3x}{a} + \frac{2x}{a} = \frac{3x+2x}{a} = \frac{5x}{a}$ **2.** $\frac{2x-4}{3} + \frac{x-1}{3} = \frac{2x-4+x-1}{3} = \frac{3x-5}{3}$

3. $\frac{5a}{6} - \frac{6-a}{6} = \frac{5a-6+a}{6} = a - 1$ **4.** $\frac{m-n}{n} - \frac{2m-n}{n} = \frac{m-n-2m+n}{n} = -\frac{m}{n}$

Objectives for Pages 290-293

1. To name equivalent fractions.

2. To add and subtract fractions with different denominators.

11 • RENAMING FRACTIONS (pp. 290-291)

Teaching These Pages

Remind students that in arithmetic, to add or subtract fractions that have different denominators, we need to rename the fractions.

$$\frac{1}{2} + \frac{3}{4} = \frac{2}{4} + \frac{3}{4} = \frac{5}{4}$$

And in algebra, to add or subtract fractions that have different denominators we need to rename the fractions. This section provides practice in renaming fractions and the next section applies this skill to adding and subtracting fractions.

Chalkboard Examples

1. $\frac{4}{x} = \frac{?}{3x}$ 12 **2.** $\frac{2a}{3} = \frac{?}{6}$ $4a$ **3.** $\frac{5}{8x} = \frac{?}{16x^2}$ $10x$ **4.** $\frac{6}{8a} = \frac{?}{16ab}$ $12b$

12 • ADD, SUBTRACT—DIFFERENT DENOMINATORS (pp. 292-293)

Teaching These Pages

Compare the steps of each example.

$$\frac{1}{2} + \frac{1}{3} = \frac{3}{3 \cdot 2} + \frac{2}{2 \cdot 3} = \frac{3}{6} + \frac{2}{6} = \frac{3 + 2}{6} = \frac{5}{6}$$

$$\frac{1}{2x} + \frac{1}{3x} = \frac{3}{3 \cdot 2x} + \frac{2}{2 \cdot 3x} = \frac{3}{6x} + \frac{2}{6x} = \frac{3 + 2}{6x} = \frac{5}{6x}$$

Chalkboard Examples

Simplify.

1. $\frac{2}{3y} + \frac{4}{y} = \frac{2}{3y} + \frac{12}{3y} = \frac{14}{3y}$

2. $\frac{3}{x} - \frac{1}{2} = \frac{6}{2x} - \frac{x}{2x} = \frac{6 - x}{2x}$

3. $\frac{3}{2x} - \frac{5}{4xy} = \frac{6y}{4xy} - \frac{5}{4xy} = \frac{6y - 5}{4xy}$

4. $\frac{2}{x} + \frac{x}{x^2} - \frac{3}{2x} = \frac{4x + 2x - 3x}{2x^2} = \frac{3x}{2x^2} = \frac{3}{2x}$

13 • MORE DIFFICULT FRACTIONS (OPTIONAL) (pp. 294-295)

Teaching These Pages

This section extends the work of the previous sections to fractions with binomial numerators. Compare these examples.

$$\frac{x}{3} + \frac{x}{5} = \frac{5x}{5 \cdot 3} + \frac{3x}{3 \cdot 5} = \frac{5x + 3x}{15} = \frac{8x}{15}$$

$$\frac{x + 2}{3} + \frac{x + 3}{5} = \frac{5(x + 2)}{5 \cdot 3} + \frac{3(x + 3)}{3 \cdot 5} = \frac{5x + 10 + 3x + 9}{15} = \frac{8x + 19}{15}$$

If students have difficulty with any exercises, they may find it helpful to relate the troublesome exercises to an arithmetic example.

Chalkboard Examples

Simplify.

1. $\frac{x-2}{2} - \frac{x-1}{4} = \frac{2(x-2)-(x-1)}{4} = \frac{x-3}{4}$

2. $\frac{x-2}{2} - \frac{x-1}{5} = \frac{5(x-2)-2(x-1)}{10} = \frac{3x-8}{10}$

3. $\frac{a-1}{3a} + \frac{a+1}{2a} - \frac{a-2}{6a} = \frac{2(a-1)+3(a+1)-(a-2)}{6a} = \frac{4a+3}{6a}$

14 • EQUATIONS WITH FRACTIONS (pp. 296-297)

Teaching These Pages

Review the basic skills developed earlier for solving equations. Each of the following examples reviews a skill.

$$x + 6 = 12 \rightarrow x + 6 - 6 = 12 - 6$$
$$x = 6$$

$$x - 6 = 12 \rightarrow x - 6 + 6 = 12 + 6$$
$$x = 18$$

$$3x = 6 \rightarrow \frac{3x}{3} = \frac{6}{3}$$
$$x = 2$$

$$\frac{x}{3} = 6 \rightarrow \frac{3 \cdot x}{3} = 3 \cdot 6$$
$$x = 18$$

$$3x + 2x = 10 \rightarrow 5x = 10$$
$$x = 2$$

Now ask the class how they would solve $\frac{x}{2} + \frac{x}{4} = 3$. Suggest that we need to add the fractions before solving the equation.

Chalkboard Examples

Solve.

1. $\frac{x}{2} + \frac{x}{4} = 3$

$$\frac{3x}{4} = 3$$
$$x = 4$$

2. $\frac{x+1}{6} - \frac{x+2}{3} = -1$

$$\frac{-x-3}{6} = -1$$
$$3 = x$$

Suggested Extension

Which is greater?

1. $\frac{1}{2}$ or $\frac{1+1}{2+1}$ $\quad\frac{1+1}{2+1}$ **2.** $\frac{4}{5}$ or $\frac{4+1}{5+1}$ $\quad\frac{4+1}{5+1}$ **3.** $\frac{3}{2}$ or $\frac{3+1}{2+1}$ $\quad\frac{3+1}{2+1}$

Let $x = a + b$ and $y = a - b$. Evaluate.

4. $\frac{x^2 + y^2}{xy} = \frac{2a^2 + 2b^2}{(a+b)(a-b)}$ **5.** $\frac{(x+y)}{xy} = \frac{2a}{(a-b)(a+b)}$

15 • WORK PROBLEMS (pp. 298-301)

Teaching These Pages

Discuss different types of jobs where many persons contribute to complete the job. (Building houses, making cars, delivering newspapers.) Some students in class may deliver newspapers. Ask whether it takes $\frac{1}{2}$ as much time to deliver the newspapers when another person helps. Chances are that the time is not reduced by $\frac{1}{2}$ when 2 students do the paper route together. Discuss why or why not.

Chalkboard Examples

1. Steve delivers newspapers and it takes him 70 minutes. His sister Sandra can do the job in half the time. How long would it take Steve and Sandra to deliver the newspapers together?

Let n = number of minutes needed to do the job together.

	Steve	Sandra	Together
Minutes needed	70	35	n
Part done in one minute	$\frac{1}{70}$	$\frac{1}{35}$	$\frac{1}{n}$

$$\frac{1}{70} + \frac{1}{35} = \frac{1}{n}$$

$$n = \frac{70}{3}$$

It will take Sandra and Steve $23\frac{1}{3}$ minutes to deliver the newspapers.

Suggested Extension

1. A rectangular tank has dimensions 20 m × 10 m and is 6 m deep. Which of the following will empty the tank faster?

A: 2 holes in the bottom of the tank each with diameter 4 cm? *or* B: 1 hole in the bottom of the tank with radius 3 cm?

The rate at which the tank empties depends on the area of the hole.

A	B
Area of Holes	Area of Hole
$A = 2 \cdot \pi \cdot 2^2 \text{ cm}^2$	$A = \pi \cdot 3^2 \text{ cm}^2$
$= 8\pi \text{ cm}^2$	$= 9\pi \text{ cm}^2$

Thus B is the fastest.

16 • BINOMIAL DENOMINATORS (OPTIONAL) (pp. 302-303)

Teaching These Pages

This section extends the work of Sections 10, 12 and 13. Compare these examples.

$$\frac{4}{x} + \frac{3}{y} \quad \leftrightarrow \quad \frac{4}{a+1} + \frac{3}{a-1}$$

$$= \frac{4y}{xy} + \frac{3x}{xy} \quad \leftrightarrow \quad = \frac{4(a-1)}{(a+1)(a-1)} + \frac{3(a-1)}{(a+1)(a-1)}$$

$$= \frac{4(a-1) + 3(a+1)}{(a+1)(a-1)}$$

$$= \frac{4a - 4 + 3a + 3}{(a+1)(a-1)}$$

$$= \frac{4y + 3x}{xy} \quad \leftrightarrow \quad = \frac{7a - 1}{(a+1)(a-1)}$$

Chalkboard Examples

1. $\dfrac{3}{y} + \dfrac{2}{y-1} = \dfrac{3(y-1) + 2y}{y(y-1)} = \dfrac{3y - 3 + 2y}{y(y-1)} = \dfrac{5y - 3}{y(y-1)}$

2. $\dfrac{3}{(a+1)} - \dfrac{2}{(a-1)} = \dfrac{3(a-1) - 2(a+1)}{(a+1)(a-1)} = \dfrac{3a - 3 - 2a - 2}{(a+1)(a-1)} = \dfrac{a - 5}{(a+1)(a-1)}$

3. $\dfrac{2}{x-3} + \dfrac{3}{x^2-9} = \dfrac{2(x+3)}{(x-3)(x+3)} + \dfrac{3}{(x-3)(x+3)} = \dfrac{2x + 6 + 3}{(x-3)(x+3)} = \dfrac{2x + 9}{(x-3)(x+3)}$

Chapter 10. Decimals and Percents

As consumers, students need to use their skills with decimals, fractions, and percents. The first three sections of this chapter review the skills needed to work with decimals and fractions, and Section 7 deals with the topic of percents. Once the skills have been established they are applied to equations, which are then applied to real-life problems that involve interest and investment. In the last section, mixture problems are solved using the skills developed in the chapter.

Objectives for Pages 312–321

1. To add, subtract, multiply, and divide decimals.
2. To express fractions as decimals.
3. To solve equations with decimal coefficients.
4. To solve problems using decimal equations.

1 • DECIMALS (pp. 312–313)

Teaching These Pages

To motivate the class, show examples of different ways that we use decimals every day.



Review the meaning of place value. This will help the students not only to read decimals but also to understand why we need to line up the decimal points when we add or subtract decimals.

$$26.396 \text{ means } 2 \cdot 10 + 6 \cdot 1 + 3 \cdot \frac{1}{10} + 9 \cdot \frac{1}{100} + 6 \cdot \frac{1}{1000}$$

Compare *place value* and *face value* as they relate to the example.

Chalkboard Examples

Add or subtract.

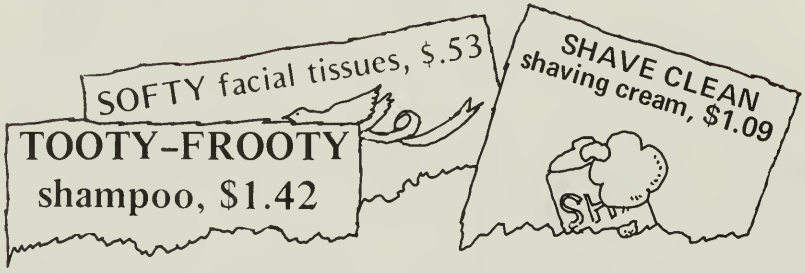
1. $4.63 + 389.2 = 393.83$ **2.** $0.57 - 0.3 = 0.27$ **3.** $492.19 - 273.284 = 218.906$

Multiply.

4. $69.8 \times 0.04 = 2.792$ **5.** $0.69 \times 46.21 = 31.8849$ **6.** $7.32 \times 0.12 = 0.8784$

Suggested Extension

Newspaper advertisements can be used to provide additional practice.



1. A case of shampoo holds 24 bottles. How much does a case of shampoo cost? $34.08
2. How much would 6 cans of shaving cream and 3 boxes of facial tissues cost? $8.13

2 • DIVISION WITH DECIMALS (pp. 314-315)

Teaching These Pages

Review that to rename $19.55 \div 2.3$ we can think of the fraction form. We rename by multiplying numerator and denominator by the same number, namely 10.

$$\frac{19.55}{2.3} = \frac{19.55 \times 10}{2.3 \times 10} = \frac{195.5}{23}$$

Before beginning the exercises, review the meaning of rounding to one decimal place, to two decimal places.

Introduce the symbol $\doteq$ for "is approximately equal to."

Chalkboard Examples

Divide. Round to one decimal place.

1. $2.3\overline{)19.55} \rightarrow 23\overline{)195.5}$ (8.5)

2. $0.17\overline{)4.39} \rightarrow 17\overline{)439}$ ($25.82 \doteq 25.8$)

Suggested Extension

1. The diameter of a nickel is about 2.15 cm. How many nickels placed edge to edge are needed to stretch 1 km? about 46,512

2. Measure the diameter of a quarter. How many quarters are needed to stretch 1 km? about 40,816

3. Which is worth more: a kilometer of nickels or a kilometer of quarters? quarters

3 • DECIMALS FOR FRACTIONS (pp. 316-317)

Teaching These Pages

Use the sports pages to show that it is useful to find a decimal form for a rational number. The ratios do not help us quickly to decide which team is the best in the standings. To compare the standings we need to find the decimal forms for the rational numbers.

Chalkboard Examples

Express the fraction as a decimal.

1. $\frac{3}{4} = 0.75$ **2.** $\frac{2}{3} = 0.666\ldots$ **3.** $\frac{5}{33} = 0.151515\ldots$ **4.** $\frac{3}{8} = 0.375$

4 • EQUATIONS WITH DECIMALS (pp. 318-319)

Teaching These Pages

Have the class compare the following two equations. How are they the same? How are they different?

$$8y = 24 \qquad 0.8y = 2.4$$

Point out that to solve equations, it is easier to work with whole numbers than with decimals.

Review multiplying by 10, 100, and 1000.

Chalkboard Examples

Would you multiply by 10, 100, or 1000 to obtain an expression with the smallest whole number coefficient?

1. $0.4x$ 10

2. $40.23y$ 100

3. $4.036k$ 1000

4. $0.26x - 4.1x$ 100

5. $0.25y + 0.5y$ 100

6. $0.025m - 0.25n$ 1000

Solve.

7. $0.9x = 3.6$
$9x = 36$
$x = 4$

8. $0.12y + 0.03y = 0.3$
$12y + 3y = 30$
$y = 2$

9. $0.15(y - 2) = 0.3$
$15(y - 2) = 30$
$y - 2 = 2$
$y = 4$

Suggested Extension

1. Express $\frac{1}{33}$ as a decimal. 0.030303 . . .

2. Express $\frac{2}{33}$ as a decimal. 0.060606 . . .

3. Use your answers for 1 and 2 to predict the decimal form of $\frac{3}{33}$.
0.090909 . . .

Express each fraction as a decimal.

4. $\frac{6}{33} = 0.1818\ldots$

5. $\frac{8}{33} = 0.2424\ldots$

6. $\frac{15}{33} = 0.4545$

7. $\frac{32}{33} = 0.9696\ldots$

5 • USING DECIMAL EQUATIONS (pp. 320-321)

Teaching These Pages

Review the Guide for Problem Solving on page 54 as you discuss the examples on page 320.

Chalkboard Examples

1. On Monday Jenny jogged 2.6 times as far as she did on Tuesday. The total distance she jogged on the two days was 14.4 km. How far did she jog each day?

Let x = distance jogged on Tuesday; $2.6x$ = distance jogged on Monday.

$$\begin{aligned} 2.6x + x &= 14.4 \\ 26x + 10x &= 144 \\ 36x &= 144 \\ x &= 4 \end{aligned}$$

She jogged 4 km on Tuesday and 10.4 km on Monday.

Suggested Extension

The data below was taken from a cereal box. Have students create problems based on the data they find on nutrition labels.

VITAMINS AND MINERALS	Per 100 g
Niacinamide	21.2 mg
Riboflavin	3.6 mg
Thiamine	2.1 mg
Iron	14.1 mg

NUTRIENTS	Per 100 g
Protein	20.4 g
Fat	0.53 g
Carbohydrate	72.4 g

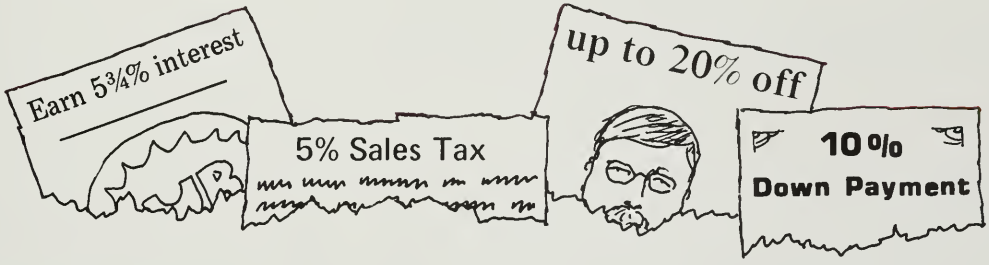
Objectives for Pages 322–331

1. To express decimals as percents.
2. To express percents as decimals or fractions.
3. To solve problems involving percents.

6 • PERCENTS (pp. 322–325)

Teaching These Pages

Use examples from the newspaper to illustrate the many ways percents are used in everyday life.



Chalkboard Examples

Express as a percent.

1. $0.48 = 48\%$ **2.** $0.375 = 37.5\%$ **3.** $\frac{1}{4} = 0.25 = 25\%$ **4.** $\frac{1}{5} = 0.2 = 20\%$

Express as a decimal.

5. $38\% = 0.38$ **6.** $10\% = 0.1$ **7.** $46.5\% = 0.465$ **8.** $4.5\% = 0.045$

The circle graph shows how Mike spends his time.



9. What percent of the day does he sleep and eat? 50%

10. What percent of the day does he spend in school? 25%

Suggested Extension

1. What percent of its games has Rochester won? about 61%

2. What percent of its games has Springfield lost? about 68%

3. What percent of its games has Nova Scotia won? about 67%

AMERICAN LEAGUE
Northern Division

	Games	W	L	T
Nova Scotia . . .	33	20	11	2
Rochester	31	19	8	4
Providence	35	16	15	4
Springfield	37	11	25	1

7 • USING PERCENTS (pp. 326-327)

Teaching These Pages

Introduce this topic with an advertisement of a sale from the newspaper.



Introduce the meanings of *original price, discount,* and *sale price* as they relate to the advertisement.

Chalkboard Examples

Compute.

1. 40% of 1200 400

2. 35% of 1800 450

3. 75% of $23 $17.25

4. A pen set is regularly $8.
A 15% discount is offered at a sale.
Find the discount.
$0.15 \times 8 = 1.20$

5. A badminton set regularly sells for $7.50.
It is on sale at a 20% discount.
Find the sale price.
$0.2 \times 7.5 = 1.5$; $\$7.50 - 1.50 = \$6.00$

Suggested Extension

Have the students collect sale advertisements and create problems based on the ads.

8 • PERCENTS IN EQUATIONS (pp. 328-331)

Teaching These Pages

In this section, as well as the sections on interest, investment, and mixtures, the students will be solving equations involving percents. Point out some of the equations that occur in the real-life problems in Sections 9, 10, and 11 to show students the need to learn skills for solving equations involving percents.

Chalkboard Examples

1. The stadium holds 4500 seats.
80% of the seats are filled.
How many seats are occupied?
$0.80 \times 4500 = 3600$ 3600 seats are occupied.

2. A total of 360 bottles were returned and 18 of them were broken.
What percent were broken?

$\frac{18}{360} = 0.05 = 5\%$ 5% of the bottles were broken.

3. After the game, the spectators were asked if they enjoyed the game.
63 persons said they enjoyed the game.
If 6% of the persons interviewed enjoyed the game, how many persons were interviewed?

$0.06x = 63$
$6x = 6300$
$x = 1050$ 1050 persons were interviewed.

Objectives for Pages 332-339

1. To solve simple interest problems.
2. To solve investment problems.
3. To solve mixture problems.

9 • INTEREST (pp. 332-333)

Teaching These Pages

Introduce this topic with a bank brochure showing the current rate of interest paid on deposits. Collect different brochures to show that different rates of interest may be paid by different banks. Discuss the interest formula $I = prt$.

Chalkboard Examples

1. How much money does \$200 earn in one year at 6% interest?
$I = 200 \times 0.06 \times 1 = 12$ The interest is \$12 for one year.

2. George deposited \$250 in the bank.
After one year, the amount of interest added to his account was \$13.75.
Find the interest rate.

$13.75 = 250r$
$0.055 = r$ The interest rate is 5.5%

3. In January Jan put \$150 into an account that earns 6% interest per year.
Four months later she added \$100 to the account.
How much interest did the money earn at the end of the year?

Interest on \$150: $I = 150 \times 0.06 = 9$ Interest on \$100: $I = 100 \times 0.06 \times \frac{2}{3} = 4$

The total interest earned is \$9 + \$4 = \$13.

10 • INVESTMENT (pp. 334–335)

Teaching These Pages

Point out some of the different ways money can be invested: stocks, bonds, investment certificates. Obtain advertisements that give the interest rates for various types of investments. Relate these rates of interest to the rates that occur in the exercises on page 335.

Chalkboard Examples

1. The school band saved \$2400 for their trip.
One part of the money was invested at 6%, and the rest at 8%.
The interest earned in one year is \$180.
How much is invested at each rate?

Let x = amount invested at 8%; $2400 - x$ = amount invested at 6%.

	p	$\times$ r	t =	I
Amount at 6%	$2400 - x$	0.06	1	$0.06(2400 - x)$
Amount at 8%	x	0.08	1	$0.08x$
				180

$$6(2400 - x) + 8x = 18{,}000$$
$$x = 1800$$

\$1800 is invested at 8%; \$600 is invested at 6%.

11 • MIXTURE PROBLEMS (pp. 336–339)

Teaching These Pages

Obtain a package of mixed candy to introduce this lesson. Discuss why a manufacturer makes packages of mixed candy. Have the class list various types of merchandise that are sold in mixtures.

Chalkboard Examples

1. A manufacturer decides to mix two types of candy and sell the mixture for \$1.72 per kilogram. One kind sells for \$1.50 per kilogram, and the other for \$1.90 per kilogram. In a 100 kilogram mixture, how many kilograms of each candy should be used?

Let x = number of kilograms of \$1.50 candy; $100 - x$ = number of kilograms of other candy.

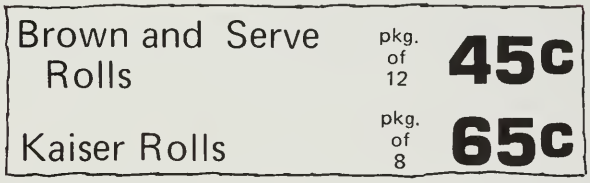
	p	$\times$ n	$=$ C
\$1.50 candy	150	x	$150x$
\$1.90 candy	190	$100x$	$190(100 - x)$
Mixture	172	100	17,200

$$\begin{aligned} 150x + 190(100 - x) &= 17{,}200 \\ 15x + 19(100 - x) &= 1720 \\ 15x + 1900 - 19x &= 1720 \\ -4x &= -180 \\ x &= 45 \end{aligned}$$

45 kilograms of the \$1.50 candy is used and 55 kilograms of the \$1.90 candy is used.

Suggested Extension

Use ads from the newspaper to motivate different types of problems. For example, this ad can be used for the following challenging problem.



1. For a party, two types of rolls were purchased. If the total cost for 216 rolls was \$12.30, find how many packages of each type of roll was bought.

Let x = number of brown and serve rolls; $216 - x$ = number of kaiser rolls

	p	$\times$ n	$=$ C
brown and serve	$\frac{45}{12}$	x	$\frac{45}{12}x$
kaiser	$\frac{65}{8}$	$216 - x$	$\frac{65}{8}(216 - x)$
			1230

$$\begin{aligned} \frac{45}{12}x + \frac{65}{8}(216 - x) &= 1230 \\ x &= 120 \end{aligned}$$

There are 120 brown and serve rolls, or 12 packages, and 96 kaiser rolls, or 8 packages.

Chapter 11. Squares and Square Roots

In problems that apply the Pythagorean Theorem, the need arises to find square roots. This chapter provides skills for working with radicals as well as finding their decimal values from a square root table. Work on square roots leads to the introduction to irrational numbers.

The skills of finding square roots are also used in solving equations. The Pythagorean Theorem provides an opportunity to solve a new type of problem, as well as apply the skill of solving equations of the type $x^2 = a^2$. The chapter concludes with developing skills to multiply and divide, as well as add and subtract radicals. An important skill for simplifying computation is to rationalize the denominator of a radical expression.

Objectives for Pages 348–355

1. To recognize perfect squares and find their square roots.
2. To use a square root table to find square roots.
3. To recognize rational and irrational numbers.
4. To simplify radical expressions.

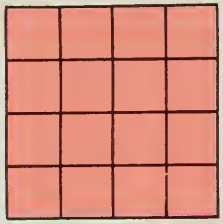
1 • SQUARE ROOTS (pp. 348–349)

Teaching These Pages

Review the meaning of the *square* of a number and relate it to the diagrams below.



$A = 3^2$ or 9 units



$A = 4^2$ or 16 units

Point out the following.

1. The square of 3 is 9.
2. A *square root* of 9 is 3.
3. $(-3)^2$ is also equal to 9, so -3 is also a square root of 9.
4. The *positive* or *principal* square root of 9, represented by $\sqrt{9}$, is 3.

Point out the need to find square roots. For example, we are given a square region. What is the measure of each side? We need to know how to find square roots to answer the question. Relate the symbol $\sqrt{\ }$ to the example, pointing out that the side of the square would measure $\sqrt{12}$ cm.



$A = 12$ cm^2

Summarize that the next few sections will show how to find a value for $\sqrt{12}$.

Introduce the symbol $\pm$.

Chalkboard Examples

Which numbers are perfect squares?

1. 36 yes **2.** 37 no **3.** 65 no **4.** 64 yes

Find the value.

5. $\sqrt{36} = \sqrt{6^2} = 6$ **6.** $-\sqrt{100} = -\sqrt{10^2} = -10$ **7.** $\sqrt{5^2 - 4^2} = \sqrt{26 - 16} = \sqrt{9} = 3$

8. Find the perimeter of a square if the area is 81 cm^2.

$4 \times \sqrt{81} = 4 \times 9 = 36$ The perimeter is 36 cm.

Suggested Extension

The numbers 3, 4, and 5 have a special property, namely $3^2 + 4^2 = 5^2$. Such numbers are called *Pythagorean triples*. Find other Pythagorean triples. Some are 6, 8, 10; 9, 12, 15; 5, 12, 13.

2 • USING A SQUARE ROOT TABLE

Teaching These Pages

Introduce the use of tables to show values concisely. For example, in supermarkets the cash registers often have a table summarizing the sales tax posted on them. Thus the operator does not need to calculate the tax each time but refers to the handy chart.

To provide some background for the square root table, point out that $\sqrt{6}$ lies between 3 and 2.

$$\sqrt{9} = 3 > \sqrt{6} > \sqrt{4} = 2$$

An approximate value for $\sqrt{6}$ is $\frac{3 + 2}{2}$, or 2.5. Note that the value in the table is 2.449.

Point out that a computer has calculated the square roots for all the numbers in the table.

Chalkboard Examples

Use the table to find the value.

1. $\sqrt{39} \doteq 6.245$ **2.** $\sqrt{74} \doteq 8.602$ **3.** $\sqrt{98} \doteq 9.899$ **4.** $-\sqrt{11} \doteq -3.317$

5. $-\sqrt{21} \doteq -4.583$ **6.** $-\sqrt{53} \doteq -7.280$ **7.** $\sqrt{80} \doteq 8.944$ **8.** $-\sqrt{28} \doteq 5.292$

Round to one decimal place.

9. $\sqrt{35} \doteq 5.9$ **10.** $\sqrt{86} \doteq 9.3$ **11.** $\sqrt{97} \doteq 9.8$ **12.** $-\sqrt{67} \doteq 8.2$

13. Find the length of a side of the square correct to one decimal place.
$\sqrt{13} \doteq 3.6$



14. Find the perimeter of the square correct to one decimal place.
$4\sqrt{13} \doteq 14.4$

Suggested Extension

Have the students find the length of a side of the square to one decimal place and then find the perimeter using that value. $\sqrt{59} \doteq 7.7$; $4 \times 7.7 = 30.8$



Now have the students find the length of a side correct to two decimal places and then find the perimeter using that value. $\sqrt{59} \doteq 7.68$; $4 \times 7.68 = 30.72$

Have the student round 30.72 to one decimal place and compare the answer to the first perimeter they found, 30.8. The difference will provide a useful source for discussion about adding numbers that have been rounded.

3 • IRRATIONAL NUMBERS (pp. 352–353)

Teaching These Pages

Review that rational numbers are numbers that can be expressed as the ratio of two integers. Point out that the prefix "ir" in irrational means "not" and thus irrational means not rational. Have the students give other examples of words with the prefix "ir." (For example, irresponsible, irrelevant.)

Review that rational numbers have decimal forms that either terminate or do not terminate, but repeat. Discuss the examples of irrational numbers on page 352.

Chalkboard Examples

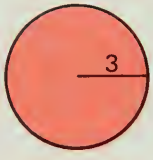
Is the number rational or is it irrational?

1. $\frac{-3}{2}$ rational **2.** $\frac{3}{\sqrt{5}}$ irrational **3.** $\frac{-4}{-\sqrt{5}}$ irrational **4.** $\sqrt{12}$ irrational

5. $1\frac{2}{11}$ irrational **6.** $\sqrt{9}$ rational **7.** $\sqrt{2.5}$ irrational **8.** 3.1515 . . . rational

9. Find the area to one decimal place.

$A = \pi r^2 \doteq 3.14 \cdot 3^2 = 28.26$; $A \doteq 28.3$



4 • SIMPLIFYING SQUARE ROOTS (pp. 354-355)

Teaching These Pages

This section uses square roots of small perfect squares, such as 4, 9, 16, 25, to find the square roots of larger numbers.

Provide examples to illustrate the rule $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

Chalkboard Examples

Find the value.

1. $\sqrt{3600} = \sqrt{36} \cdot \sqrt{100} = 60$ **2.** $\sqrt{32} = \sqrt{16 \cdot 2} = 4\sqrt{2}$ **3.** $\sqrt{27y^2} = \sqrt{9 \cdot 3 \cdot y^2} = 3y\sqrt{3}$

Suggested Extension

Have the students investigate the prime factors of perfect square as well as numbers that are not perfect squares.

$$100 = 2 \cdot 2 \cdot 5 \cdot 5 \qquad 125 = 5 \cdot 5 \cdot 5$$

Point out that the prime factors of squares occur in pairs and that we can write the square root by taking one member of each pair.

$$100 = 2 \cdot 2 \cdot 5 \cdot 5 \qquad \sqrt{100} = 2 \cdot 5 = 10$$

The students can use this method to do Exercises 1–20 on page 355.

Objectives for Pages 356-361

1. To solve quadratic equations by taking the square root of each side.

2. To use the Pythagorean Theorem.

5 • SOLVING EQUATIONS (pp. 356-357)

Teaching These Pages

Review that 25 has two square roots, -5 and 5. Introduce the equation $x^2 = 25$ and have the students name the solutions. Point out that the solutions can be found by taking the square root of both sides of the equation.

Chalkboard Examples

Solve and check.

1. $x^2 = 49$
$x = \pm 7$

2. $\frac{x}{32} = \frac{2}{x}$
$x^2 = 64$
$x = \pm 8$

3. $4^2 - 36 = 45$
$x^2 = 81$
$x = \pm 9$

4. $x^2 + 49 = 89$
$x^2 = 40$
$x = \pm\sqrt{40}$
$x = \pm 2\sqrt{10}$

6 • LAW OF PYTHAGORAS

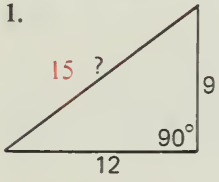
Teaching These Pages

Students may have dealt with this topic in earlier courses. To review, draw the diagram on page 358 on the chalkboard and have the students count the number of squares in each square region. Have the students study the relationship among the numbers 9, 16, and 25. They will discover $9 + 16 = 25$. Write this equation as $3^2 + 4^2 = 5^2$ and relate it to the sides of the triangle and the squares drawn on each side of the triangle.

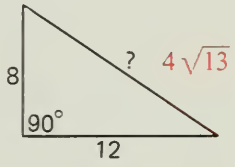
Chalkboard Examples

Find the length of the third side of the triangle.

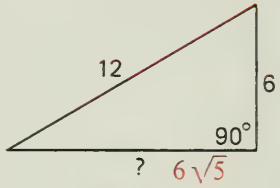
1.



2.

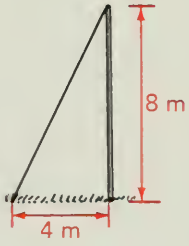


3.



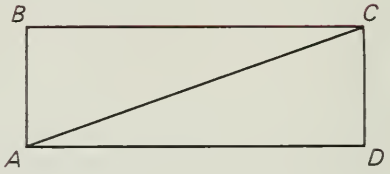
4. A wire is attached to a pole as shown. Find the length of the wire correct to one decimal place.

$c^2 = 16 + 64 = 80$
$c = \sqrt{80} \doteq 8.9$



Suggested Extension

Discuss taking short cuts across corner lots. Use the dimensions of the corner lot shown to see how much further it is to walk along AB and BC than to walk along AC.



Objectives for pp. 362-369

1. To find the square root of a fraction.
2. To multiply and divide radical expressions.
3. To rationalize the denominator of a radical expression.
4. To simplify radical expressions by addition or subtraction.

7 • QUOTIENTS OF SQUARE ROOTS (pp. 362–363)

Teaching These Pages

Illustrate the rule $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, $b \neq 0$, with simple examples.

$$\sqrt{\frac{36}{9}} = \sqrt{4} = 2 \text{ but also } \sqrt{\frac{36}{9}} = \frac{\sqrt{36}}{\sqrt{9}} = \frac{6}{3} = 2$$

$$\sqrt{\frac{100}{25}} = \sqrt{4} = 2 \text{ but also } \sqrt{\frac{100}{25}} = \frac{\sqrt{100}}{\sqrt{25}} = \frac{10}{5} = 2$$

Chalkboard Examples

Find the square root.

1. $\sqrt{\frac{81}{9}} = \frac{\sqrt{81}}{\sqrt{9}} = \frac{9}{3} = 3$

2. $\sqrt{\frac{10}{36}} = \frac{\sqrt{10}}{\sqrt{36}} = \frac{\sqrt{10}}{6}$

3. $\sqrt{\frac{25}{49}} = \frac{\sqrt{25}}{\sqrt{49}} = \frac{5}{7}$

4. $\sqrt{\frac{1}{225}} = \frac{1}{\sqrt{225}} = \frac{1}{15}$

5. $\sqrt{\frac{25y^2}{100}} = \frac{\sqrt{25y^2}}{\sqrt{100}} = \frac{5y}{10}$

6. $\sqrt{\frac{3a^2}{64}} = \frac{\sqrt{3a^2}}{\sqrt{64}} = \frac{a\sqrt{3}}{8}$

7. Find the square root of $\frac{35}{25}$. Express your result correct to one decimal place.

$$\sqrt{\frac{35}{25}} = \frac{\sqrt{35}}{\sqrt{25}} = \frac{\sqrt{35}}{5} \doteq \frac{5.916}{5} \doteq 1.183 \doteq 1.2$$

8 • MULTIPLICATION AND DIVISION (pp. 364–365)

Teaching These Pages

Show how rules can be written in a different way to be useful in different ways. For example, $a(b + c) = ab + ac$ can be written $ab + ac = a(b + c)$ to show how to factor out a common monomial factor.

In a similar way we can rewrite the rules of the previous sections to show us how to multiply and divide with radicals.

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b} \quad \text{can be written} \quad \sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \quad \text{can be written} \quad \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

Chalkboard Examples

Simplify.

1. $\sqrt{3} \cdot \sqrt{27} = \sqrt{3 \cdot 27} = \sqrt{81} = 9$

2. $2\sqrt{5} \cdot 4\sqrt{2} = 8\sqrt{5 \cdot 2} = 8\sqrt{10}$

3. $3\sqrt{x} \cdot 27\sqrt{x} = 81\sqrt{x^2} = 81x$

4. $\dfrac{\sqrt{96}}{\sqrt{8}} = \sqrt{\dfrac{96}{8}} = \sqrt{12} = 2\sqrt{3}$

5. $\dfrac{\sqrt{125}}{\sqrt{50}} = \sqrt{\dfrac{125}{50}} = \sqrt{\dfrac{5}{2}}$

6. $\dfrac{\sqrt{72x^2}}{\sqrt{36}} = \sqrt{\dfrac{72x^2}{36}} = \sqrt{2x^2} = x\sqrt{2}$

9 • RATIONALIZING THE DENOMINATOR (pp. 366-367)

Teaching These Pages

Have the students find the value of $\dfrac{\sqrt{15}}{\sqrt{2}}$ to one decimal place, using $\sqrt{15} \doteq 3.87$ and $\sqrt{2} \doteq 1.41$.

$$\frac{\sqrt{15}}{\sqrt{2}} \doteq \frac{3.87}{1.41} \doteq 2.74 \doteq 2.7$$

The calculation will take about 5 minutes of tedious work. (Record the time that is actually taken.) With this experience, the students will appreciate the method of rationalizing the denominator. Introduce the method and have students find the value of $\dfrac{\sqrt{15}}{\sqrt{2}}$ again, recording the amount of time they take.

$$\frac{\sqrt{15}}{\sqrt{2}} = \frac{\sqrt{15} \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{30}}{2} \doteq \frac{5.48}{2} = 2.74 \doteq 2.7$$

This calculation will take students only about 1 minute.

Chalkboard Examples

Express in simplest form.

1. $\sqrt{\dfrac{6}{7}} = \dfrac{\sqrt{6}}{\sqrt{7}} = \dfrac{\sqrt{42}}{7}$

2. $\sqrt{\dfrac{1}{3}} \cdot \sqrt{5} = \sqrt{\dfrac{5}{3}} = \dfrac{\sqrt{15}}{3}$

3. $\dfrac{2\sqrt{32}}{\sqrt{3}} = \dfrac{2\sqrt{96}}{3} = \dfrac{8\sqrt{6}}{3}$

Find the decimal value correct to one decimal place.

4. $\dfrac{\sqrt{6}}{\sqrt{5}} = \dfrac{\sqrt{30}}{5} \doteq 1.09 \doteq 1.1$

5. $3\sqrt{\dfrac{2}{3}} = \dfrac{3\sqrt{6}}{3} = \sqrt{6} \doteq 2.44 \doteq 2.4$

10 • ADDITION AND SUBTRACTION (pp. 368-369)

Teaching These Pages

Review the meaning of like terms and point out that radical expressions also have like terms.

$$3x + 5x = 8x \qquad 3\sqrt{11} + 5\sqrt{11} = 8\sqrt{11}$$

A common error that should be pointed out if it occurs is to write $\sqrt{4} + \sqrt{6} = \sqrt{10}$ and $3\sqrt{6} + 6\sqrt{3} = 9\sqrt{9}$.

Chalkboard Examples

Express in simplest form.

1. $3\sqrt{6} + 2\sqrt{6} = 5\sqrt{6}$ **2.** $8\sqrt{7} - 10\sqrt{7} = -2\sqrt{7}$ **3.** $3\sqrt{5} + \sqrt{5} - 3\sqrt{3} = 4\sqrt{5} - 3\sqrt{3}$

4. $\sqrt{8} + \sqrt{18} = \sqrt{4 \cdot 2} + \sqrt{9 \cdot 2} = 2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}$

5. $\sqrt{12y} - \sqrt{3y} - \sqrt{75y} = \sqrt{4 \cdot 3y} - \sqrt{3y} - \sqrt{25 \cdot 3y} = 2\sqrt{3y} - \sqrt{3y} - 5\sqrt{3y} = -4\sqrt{3y}$

11 • RADICALS AND BINOMIALS (OPTIONAL) (pp. 370-371)

Teaching These Pages

Introduce this topic by having students consider the value of $\frac{1}{2 + \sqrt{2}}$. To find the value we could evaluate as shown.

$$\frac{1}{2 + \sqrt{2}} \doteq \frac{1}{2 + 1.414} = \frac{1}{3.414}$$

Students may indicate there is a simpler way to find the value. Point out that we would like to find an expression that does not have a radical in the denominator, that is, we would like to rationalize the denominator.

$$\frac{1}{2 + \sqrt{2}} = \frac{1 \cdot (2 - \sqrt{2})}{(2 + \sqrt{2})(2 - \sqrt{2})} = \frac{2 - \sqrt{2}}{4 - 2} = \frac{2 - \sqrt{2}}{2}$$

Chalkboard Examples

Express in simplest form.

1. $\sqrt{5}(2 - \sqrt{6}) = 2\sqrt{5} - \sqrt{30}$ **2.** $4\sqrt{3}(6 + \sqrt{6}) = 24\sqrt{3} + 12\sqrt{2}$

3. $(2 - 3\sqrt{3})(3\sqrt{2} - 2) = 6\sqrt{2} - 4 - 9\sqrt{6}$ **4.** $(1 - \sqrt{3})(\sqrt{3} - 1) = -4$

Rationalize the denominator.

5. $\frac{1}{3 - \sqrt{3}} = \frac{3 + \sqrt{3}}{6}$ **6.** $\frac{3}{2 + \sqrt{5}} = \frac{3(2 - \sqrt{5})}{4 - 5} = -6 - 3\sqrt{5}$

7. $\frac{\sqrt{3}}{1 - \sqrt{4}} = \frac{\sqrt{3}(1 + \sqrt{4})}{-3} = \frac{\sqrt{3} + 2\sqrt{3}}{-3} = \frac{3\sqrt{3}}{-3} = -\sqrt{3}$

Simplify.

8. $(1 + \sqrt{5})^2 = 1 + 2\sqrt{5} + 5 = 6 + \sqrt{5}$ **9.** $(2\sqrt{3} - 3\sqrt{2})^2 = 30 - 12\sqrt{6}$

Chapter 12. Quadratic Equations

In earlier chapters the students have dealt with the graphs of linear equations. The intersection of lines was related to the solutions of equations having two variables, and the meaning of function was introduced. In this chapter, the students first solve quadratic equations using the zero property: if $ab = 0$ then $a = 0$ or $b = 0$. Quadratic equations are then solved using factoring, taking square roots, and finally using the quadratic formula. These skills are applied to solving problems that require quadratic equations. The chapter concludes with a study of the graph of this nonlinear function, the quadratic function.

Objective for Pages 380–385

1. To use the zero property and factoring to solve quadratic equations.

1 • ZERO PRODUCTS (pp. 380–381)

Teaching These Pages

Introduce this section by having the students complete each of the following examples.

$0 \cdot 5 =$ ___?___ $\quad 15 \cdot 0 =$ ___?___ $\quad \frac{1}{3} \cdot 0 =$ ___?___ $\quad 0 \cdot \frac{1}{4} =$ ___?___

Summarize that the product of two numbers, one of which is zero, must be zero. Then ask the students what they know about the factors in the following examples.

$$5x = 0 \qquad 14y = 0 \qquad \frac{1}{2}k = 0 \qquad ky = 0$$

Students should conclude that if two numbers have a zero product, one of the numbers must be zero. Extend this to equations with a binomial factor.

Chalkboard Examples

Complete.

1. If $x(x + 4) = 0$ then $x =$ ___?___ or $x + 4 =$ ___?___. 0; 0

2. If $(x - 1)(x + 5) = 0$ then ___?___ $= 0$ or ___?___ $= 0$. $x - 1; x + 5$

Solve.

3. $y(y + 6) = 0$
$y = 0$ *or* $y + 6 = 0$
$y = -6$

4. $3k(k - 5) = 0$
$k = 0$ *or* $k - 5 = 0$
$k = 5$

5. $(2x + 8)(4x - 16) = 0$
$2x + 8 = 0$ *or* $4x - 16 = 0$
$x = -4 \qquad x = 4$

Suggested Extensions

1. Find $_?_$ in $(x - 3)(x + _?_) = 0$ if the solution is $x = 3$ or $x = -7$. 7

2. Find $_?_$ in $(2x + _?_)(x - 4)$ if the solution is $x = 4$ or $x = -\frac{3}{2}$. 3

3. The solution to an equation is $x = \frac{1}{4}$ or $x = \frac{5}{3}$. Write the equation in the form $(ax - b)(cx - d) = 0$. $(4x - 1)(3x - 5) = 0$

2 • SOLVING QUADRATIC EQUATIONS (pp. 382–383)

Teaching These Pages

The skills developed in this section and in subsequent sections will be applied to solving problems that involve the quadratic equation (in Section 7).

Have the students find the product in each example.

$$(x - 4)(x - 5) = 0 \quad \rightarrow \quad x^2 - 9x + 20 = 0$$

$$(x + 2)(x - 5) = 0 \quad \rightarrow \quad x^2 - 3x - 10 = 0$$

Point out that in this section we will solve quadratic equations like $x^2 - 9x + 20 = 0$, but the first step will be to factor them.

Review the types of factoring before assigning the exercises, and remind students to always check for a common monomial factor first.

Chalkboard Examples

Solve.

1. $x^2 + 8x + 12 = 0$
$(x + 6)(x + 2) = 0$
$x + 6 = 0$ *or* $x + 2 = 0$
$x = -6$ $\quad$ $x = -2$

2. $a^2 - 5a - 24 = 0$
$(a - 8)(a + 3) = 0$
$a - 8 = 0$ *or* $a + 3 = 0$
$a = 8$ $\quad$ $a = -3$

3. $y^2 + 12y + 36 = 0$
$(y + 6)(y + 6) = 0$
$y + 6 = 0$ *or* $y + 6 = 0$
$y = -6$ $\quad$ $y = -6$

4. $m^2 - 16 = 0$
$(m - 4)(m + 4) = 0$
$m - 4 = 0$ *or* $m + 4 = 0$
$m = 4$ $\quad$ $m = -4$

Suggested Extension

Solve.

1. $2y^2 + 3y + 1 = 0$
$y = -\frac{1}{2}$ *or* $y = -1$

2. $2k^2 + k - 3 = 0$
$k = -\frac{3}{2}$ *or* $k = 1$

3. $6x^2 - 13x + 6 = 0$
$x = \frac{2}{3}$ *or* $x = \frac{3}{2}$

3 • SOLVING QUADRATIC EQUATIONS (pp. 382-385)

Teaching These Pages

Use the examples on page 384 to point out the need to write zero on one side of the equation. A common error students make is factoring to change $x^2 - x = 6$ into $x(x - 1) = 6$ and then concluding that $x = 6$ or $x - 1 = 6$.

Chalkboard Examples

Solve.

1. $x^2 - 5x = 24$
$(x - 8)(x + 3) = 0$
$x = 8$ *or* $x = -3$

2. $(y + 2)(y - 13) = -50$
$y = 3$ *or* $y = 8$

3. $\frac{x - 4}{x} = \frac{-3}{x + 1}$
$x = 2$ *or* $x = -2$

Objectives for Pages 386-389

1. To solve a quadratic equation by taking square roots.

4 • USING SQUARE ROOTS (pp. 386-387)

Teaching These Pages

This section is primarily a review of Section 4, Chapter 11, in preparation for the next section in which students will solve equations of the form $(x + a)^2 = c$. Throughout the exercises, students should be encouraged to solve the equations by taking square roots.

Chalkboard Examples

Solve.

1. $x^2 = 36$
$x = \pm 6$

2. $x^2 = 12$
$x = \pm\sqrt{12}$
$x = \pm 2\sqrt{3}$

3. $a^2 = 64$
$a = \pm 8$

4. $2m^2 = 50$
$m^2 = 25$
$m = \pm 5$

5 • USING SQUARE ROOTS (pp. 388-389)

Teaching These Pages

Have the students compare the following equations.

$$x^2 = 36 \quad \leftrightarrow \quad (x + 1)^2 = 36$$
$$y^2 = 49 \quad \leftrightarrow \quad (y + 3)^2 = 49$$

Use the above examples and find the roots by inspection.

Some students will solve $(x + 3)^2 = 16$ as $x + 3 = 16$ and $x + 3 = -16$. If this problem occurs, encourage those students to write 16 as 4^2 before taking the square roots of both sides.

Chalkboard Examples

Complete.

1. $(x + 2)^2 = 9$
$x + 2 =$ ___?___ ± 3

2. $(x - 4)^2 = 16$
$x - 4 =$ ___?___ ± 4

3. $(2x + 1)^2 = 25$
$2x + 1 =$ ___?___ ± 5

Solve.

4. $(x + 3)^2 = 16$
$x + 3 = \pm 4$
$x = -3 \pm 4$
$x = -7$ *or* 1

5. $(x - 3)^2 = 12$
$x - 3 = \pm 2\sqrt{3}$
$x = 3 \pm 2\sqrt{3}$

6. $3(x - 2)^2 = 12$
$x - 2 = \pm 2$
$x = 2 \pm 2$
$x = 0$ *or* 4

Suggested Extension

Students can solve any quadratic equation by the process of *completing the square,* illustrated below.

Solve $x^2 - 2x - 2 = 0$.

1. Write the equation with the constant term in the right member. $x^2 - 2x = 2$

2. Add to each member the square of one-half the coefficient of x. $x^2 - 2x + 1 = 3$

3. Write the left member as a perfect square. $(x - 1)^2 = 3$

4. Take the square root of both members. $x - 1 = \pm\sqrt{3}$
$x = 1 \pm \sqrt{3}$

If the coefficient of x^2 in a quadratic equation is not 1, first find an equivalent equation in which the coefficient of x^2 is 1.

Students can use this technique to solve the quadratic equations in Section 6.

Objectives for Pages 390-399

1. To use the quadratic formula to solve quadratic equations.

2. To use quadratic equations to solve problems.

3. To draw the graphs of quadratic functions.

6 • THE QUADRATIC FORMULA (pp. 390–393)

Teaching These Pages

So that students will appreciate the "power" of the quadratic formula, have them attempt to solve the following equation by factoring.

$$6x^2 + 13x + 6 = 0$$

The formula is applied to this equation as the first Chalkboard Example.

Some students may find it easier to work with the formula if they record the values of a, b, and c for each exercise.

Chalkboard Examples

Solve.

1. $6x^2 + 13x + 6 = 0$

$$x = \frac{-13 \pm \sqrt{169 - 144}}{12}$$

$$x = \frac{-13 \pm \sqrt{25}}{12}$$

$$x = -\frac{2}{3} \text{ or } -\frac{3}{2}$$

2. $2x^2 + 4x + 1 = 0$

$$x = \frac{-4 \pm \sqrt{16 - 8}}{4}$$

$$x = \frac{-4 \pm 2\sqrt{2}}{4}$$

$$x = -2 + \sqrt{2} \text{ or } -2 - \sqrt{2}$$

3. $x^2 - 3x - 4 = 0$

$$x = \frac{3 \pm \sqrt{9 + 16}}{2}$$

$$x = \frac{3 \pm \sqrt{25}}{2}$$

$$x = 4 \text{ or } -1$$

Suggested Extension

Students may be interested in the derivation of the quadratic formula. The formula is found by completing the square using the standard form $ax^2 + bx + c = 0$.

7 • PROBLEM SOLVING (pp. 394–395)

Teaching These Pages

Review the various methods for solving quadratic equations. Throughout the exercises, suggest that when a quadratic equation is obtained, students first check whether it can be factored or can be solved by taking square roots. If the other methods don't work, use the quadratic formula.

Review the Guide for Problem Solving on page 54.

Chalkboard Examples

1. The length of a rectangle is 4 cm longer than the width. The area of the rectangle is 165 cm^2. Find the length and width.

Let y = the width; $y - 4$ = the length.

$$y(y - 4) = 165$$
$$y^2 - 4y - 165 = 0$$
$$(y + 11)(y - 15) = 0$$

(Is it possible to have a negative length?)

The length is 15 cm and the width is 11 cm.

2. The perimeter of a rectangle is 32 cm.
The area of the rectangle is 55 cm^2.
Find the length and width.

$$2l + 2w = 32$$
$$l + w = 16$$
$$l = 16 - w$$

$$(16 - w)w = 55$$
$$w = 11 \text{ or } w = 5$$

The length is 11 cm and the width is 5 cm. (Point out why we do not use $w = 11$.)

Suggested Extension

1. Two numbers differ by 4.
The sum of their squares is 208.
Find the numbers.

$$(x + 4)^2 + x^2 = 208$$

The numbers are 8 and 12 *or* -8 and -12.

2. A garden is 4 m longer than it is wide.
The diagonal of the garden is 20 m.
Find the length and width.

$$(x + 4)^2 + x^2 = 400$$

The length is 16 m and the width is 12 m.

8 • QUADRATIC GRAPHS (pp. 396-399)

Teaching These Pages

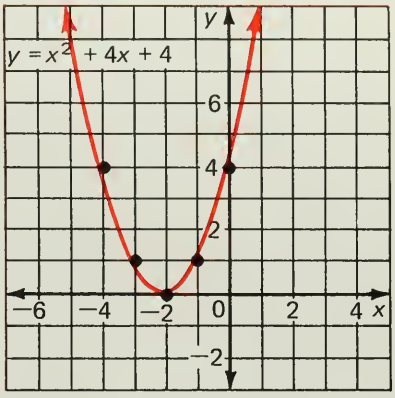
Have the students draw the graphs of the following equations.

$$y = x \qquad y = x^2 \qquad y = -x^2$$

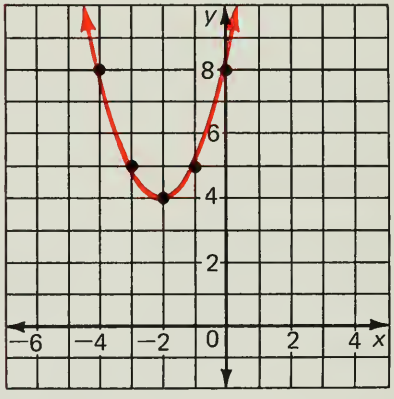
Discuss the graphs. How are they the same? How are they different? Introduce the terms *parabola* and *vertex*. Introduce the meaning of a function as it relates to the graphs of $y = x^2$ and $y = -x^2$.

Chalkboard Examples

1. Draw the graph of $y = x^2 + 4x + 4$.
2. What are the coordinates of the vertex of this parabola? $(-2, 0)$
3. What is the smallest value of y? 0 What is the largest value of y? There is no largest value.
4. How many values of x make $y = 1$? two
5. How many values of y make $x = 0$? one



6. Draw the graph of $y = x^2 + 4x + 8$.
7. What are the coordinates of the vertex of this parabola? $(-2, 4)$
8. What is the smallest value of y? 4 What is the largest value of y? There is no largest value.
9. How many values of x make $y = 8$? two
10. How many values of y make $x = 0$? one



Basic Algebra

Basic Algebra

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Symbols

		Page
$=$	equals, is equal to	3
$\cdot$	$\times$ (times)	3
()	parentheses—a grouping symbol	6
$<$	is less than	24
$>$	is greater than	24
$\neq$	is not equal to	24
$+$	positive	72
$-$	negative	72
$^\circ$	degree	72
$-a$	opposite of a	78
$\lvert a \rvert$	absolute value of a	92
π	pi, a number approximately equal to $\frac{22}{7}$	174
(a, b)	ordered pair whose first component is a and whose second component is b	212
$\doteq$	is approximately equal to	314
$\sqrt{}$	positive square root	348
$\pm$	read: positive or negative	348

Metric system symbols

mm	millimeter	g	gram
cm	centimeter	mg	milligram
m	meter	kg	kilogram
km	kilometer	L	liter
km/h	kilometers per hour	mL	milliliter
°C	degrees Celsius		

Diagnostic Tests in Arithmetic*

1. Whole Numbers—Addition

1. $\begin{array}{r} 6 \\ +9 \\ \hline 15 \end{array}$ **2.** $\begin{array}{r} 2 \\ 2 \\ +3 \\ \hline 7 \end{array}$ **3.** $\begin{array}{r} 4 \\ 6 \\ +7 \\ \hline 17 \end{array}$ **4.** $\begin{array}{r} 55 \\ +25 \\ \hline 80 \end{array}$ **5.** $\begin{array}{r} 68 \\ +15 \\ \hline 83 \end{array}$

6. $\begin{array}{r} 575 \\ +371 \\ \hline 946 \end{array}$ **7.** $\begin{array}{r} 778 \\ +177 \\ \hline 955 \end{array}$ **8.** $\begin{array}{r} 814 \\ 857 \\ +311 \\ \hline 1982 \end{array}$ **9.** $\begin{array}{r} 7456 \\ 3701 \\ 6101 \\ +5592 \\ \hline 22{,}850 \end{array}$

2. Whole Numbers—Subtraction

1. $\begin{array}{r} 7 \\ -2 \\ \hline 5 \end{array}$ **2.** $\begin{array}{r} 11 \\ -9 \\ \hline 2 \end{array}$ **3.** $\begin{array}{r} 97 \\ -54 \\ \hline 43 \end{array}$ **4.** $\begin{array}{r} 769 \\ -608 \\ \hline 161 \end{array}$ **5.** $\begin{array}{r} 684 \\ -308 \\ \hline 376 \end{array}$

6. $\begin{array}{r} 736 \\ -365 \\ \hline 371 \end{array}$ **7.** $\begin{array}{r} 6508 \\ -4395 \\ \hline 2113 \end{array}$ **8.** $\begin{array}{r} 643 \\ -177 \\ \hline 466 \end{array}$ **9.** $\begin{array}{r} 4220 \\ -3686 \\ \hline 534 \end{array}$ **10.** $\begin{array}{r} 40{,}704 \\ -27{,}978 \\ \hline 12{,}726 \end{array}$

3. Whole Numbers—Multiplication

1. $\begin{array}{r} 6 \\ \times 6 \\ \hline 36 \end{array}$ **2.** $\begin{array}{r} 21 \\ \times 4 \\ \hline 84 \end{array}$ **3.** $\begin{array}{r} 28 \\ \times 3 \\ \hline 84 \end{array}$ **4.** $\begin{array}{r} 66 \\ \times 8 \\ \hline 528 \end{array}$ **5.** $\begin{array}{r} 375 \\ \times 3 \\ \hline 1125 \end{array}$ **6.** $\begin{array}{r} 69 \\ \times 20 \\ \hline 1380 \end{array}$

7. $\begin{array}{r} 125 \\ \times 200 \\ \hline 25{,}000 \end{array}$ **8.** $\begin{array}{r} 1291 \\ \times 2000 \\ \hline 2{,}582{,}000 \end{array}$ **9.** $\begin{array}{r} 96 \\ \times 65 \\ \hline 6240 \end{array}$ **10.** $\begin{array}{r} 734 \\ \times 29 \\ \hline 21{,}286 \end{array}$ **11.** $\begin{array}{r} 825 \\ \times 955 \\ \hline 787{,}875 \end{array}$ **12.** $\begin{array}{r} 801 \\ \times 203 \\ \hline 162{,}603 \end{array}$

4. Whole Numbers—Division

1. $9\overline{)54}$ 6 **2.** $2\overline{)42}$ 21 **3.** $8\overline{)448}$ 56 **4.** $5\overline{)2285}$ 457 **5.** $13\overline{)455}$ 35 **6.** $16\overline{)216}$ 13R8

7. $568\overline{)416090}$ 732R314

8. Express the quotient as a mixed numeral: $482\overline{)285385}$ 592R41

592 41/482

9. Express the quotient to the nearest hundredth: $6\overline{)38}$ 6.33

5. Fractions—Basic Skills

1. Identify by letter the figure that is divided into thirds. A



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2. What is the denominator in $\frac{1}{3}$? 3

3. The area shaded in figure D is represented by which fraction in row E? $\frac{1}{5}$

D 

E $\frac{2}{5}$, $\frac{3}{5}$, $\frac{1}{5}$, $\frac{5}{1}$

4. Which of the figures below represents the fraction $\frac{1}{3}$? G



5. Write the fraction represented by the set diagram shown below. $\frac{2}{3}$



6. Identify by letter the set diagram at the right that represents the fraction $\frac{2}{6}$. M



7. Which fraction represents the number 1? $\frac{4}{4}$ $\frac{4}{1}$, $\frac{4}{4}$, $\frac{4}{8}$, $\frac{1}{4}$

8. Which fraction represents C on the number line? $\frac{3}{2}$ $\frac{5}{3}$, $\frac{3}{2}$, $\frac{3}{3}$, $\frac{5}{2}$



9. Write the consecutive multiples of 6: 6, ___?___, ___?___, ___?___ 12, 18, 24

10. Find the least common multiple of 3, 4, and 6. 12

11. Write 36 as the product of prime factors. $2 \times 2 \times 3 \times 3$

12. Find the greatest common factor of 12 and 28. 4

13. Find the fraction in row Y that is equal to each fraction or mixed numeral in row X.
$\frac{4}{6} = \frac{6}{9}$, $\frac{3}{5} = \frac{9}{15}$, $2\frac{3}{4} = \frac{11}{4}$

X: (a) $\frac{4}{6}$ (b) $\frac{3}{5}$ (c) $2\frac{3}{4}$

Y: $\frac{2}{2}$, $\frac{6}{4}$, $\frac{11}{4}$, $\frac{3}{4}$, $\frac{9}{15}$, $\frac{6}{9}$

14. Find the least common denominator for the fractions $\frac{2}{4}$, $\frac{1}{6}$, and $\frac{7}{8}$. 24

15. Find the fraction or mixed numeral in row Y that is equal to each fraction or mixed numeral in row X. $\frac{5}{4} = 1\frac{1}{4}$, $2\frac{3}{5} = \frac{13}{5}$

X: (a) $\frac{5}{4}$ (b) $2\frac{3}{5}$

Y: $\frac{1}{4}$, $1\frac{1}{4}$, $\frac{13}{5}$, $\frac{5}{13}$

6. Fractions—Addition

1. $\frac{1}{5} + \frac{2}{5} =$? $\frac{3}{5}$ 2. $3\frac{1}{8} + 4\frac{1}{8} =$? $7\frac{1}{4}$ 3. $3\frac{7}{12} + 4\frac{11}{12}$ $8\frac{1}{2}$ 4. $\frac{1}{5} + \frac{1}{4} =$? $\frac{9}{20}$ 5. $4\frac{2}{3} + 3\frac{4}{7}$ $8\frac{5}{21}$

7. Fractions—Subtraction

1. $\frac{2}{4} - \frac{1}{4}$ $\frac{1}{4}$ 2. $3\frac{3}{8} - 1\frac{1}{8}$ $2\frac{1}{4}$ 3. $7 - 5\frac{1}{6}$ $1\frac{5}{6}$ 4. $7\frac{1}{9} - 1\frac{7}{9}$ $5\frac{1}{3}$ 5. $\frac{2}{3} - \frac{1}{7}$ $\frac{11}{21}$ 6. $14\frac{4}{6} - 3\frac{4}{5}$ $10\frac{13}{15}$

8. Fractions—Multiplication

1. $\frac{1}{3} \times \frac{1}{9} =$? $\frac{1}{27}$ 2. $\frac{7}{8} \times \frac{6}{7} =$? $\frac{3}{4}$ 3. $\frac{1}{2} \times 4 =$? 2

4. $4\frac{5}{6} \times \frac{3}{5} =$? $\frac{29}{10}$ or $2\frac{9}{10}$ 5. $1\frac{3}{5} \times 2\frac{1}{2} =$? 4

9. Fractions—Division

1. $5 \div \frac{1}{9} =$? 45 2. $6 \div \frac{3}{4} =$? 8 3. $\frac{1}{6} \div \frac{1}{2} =$? $\frac{1}{3}$ 4. $5\frac{5}{9} \div \frac{2}{7} =$? $\frac{175}{9} = 19\frac{4}{9}$

5. $5\frac{1}{8} \div 4 =$? $\frac{41}{32}$ or $1\frac{9}{32}$ 6. $5\frac{5}{8} \div 5\frac{5}{6} =$? $\frac{27}{28}$

10. Decimals

1. Write the decimal which represents "two and six thousandths." 2.006

2. $\frac{5}{1000} =$? 0.005 (Decimal) 3. 0.206 = ? $\frac{206}{1000}$ or $\frac{103}{500}$ (Fraction) 4. $3\frac{4}{5} =$? 3.80 (Decimal)

5. Round 40.5656 to the nearest tenth. 40.6 6. 1.033 + 0.1 + 10.066 = ? 11.199

7. 856.175 − 20.05 = ? 836.125 8. 71.60 + 500.42 + 53.96 = 625.98 9. 735.02 − 7.709 = 727.311 10. 6639 × .0031 = 20.5809 11. $.07\overline{)21.707}$ 310.1

11. Percents

1. $\frac{7}{4} = \frac{?}{100}$ 175 2. 0.69 = ? % 69 3. $\frac{1}{3} =$? % $33\frac{1}{3}$ 4. $6\frac{1}{2} =$? % 650

5. 2.405 = ? % 240.5 6. 289% = ? $2\frac{89}{100}$ (Mixed numeral in simplest form)

7. $38\frac{1}{2}\% =$? 0.385 (Decimal) 8. 25% of 23 = ? 5.75

9. ? 40 % of 20 = 8 10. 20% of ? 1700 = 340

Why Study Algebra?

You've probably asked this question at least once. Here are a few reasons to think about.



Through algebra you'll learn to organize and express your thoughts concisely. That's important in today's world. You know this is true if you've ever worked with a calculator, for you have to have a *plan* for solving the problem before the machine can begin to help you at all.



You'll get a chance to view from a different angle all those concepts you've already learned in math. This will help you firm in your mind what math is all about and how it works.



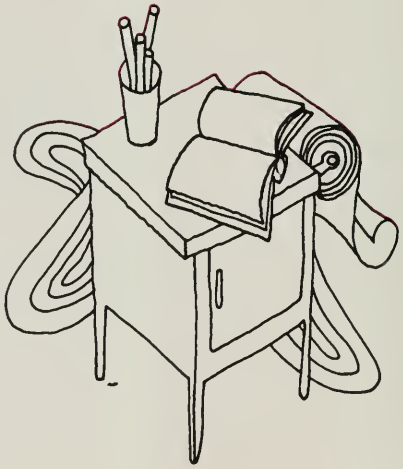
Algebra will prepare you for further work in math and the sciences, if this is for you. Also, hundreds of careers rely upon a good, solid math background. Algebra can help give this to you.

Once you've learned basic algebra skills, you'll probably find yourself using some of them to solve many problems in your everyday life.

UNIT A

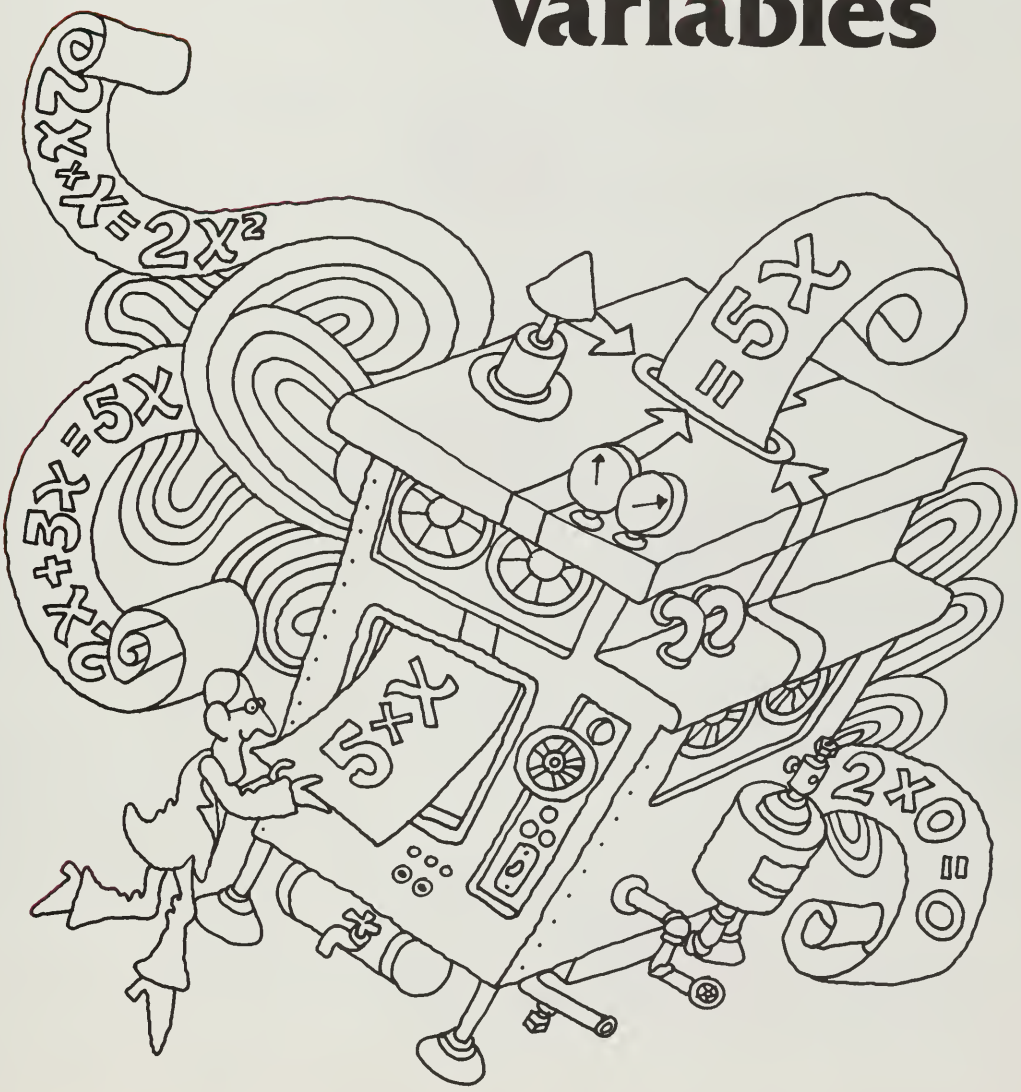
Here's what you'll learn in this chapter:

1. To find the values of expressions with variables.
2. To simplify expressions.
3. To find the value of expressions with exponents.
4. To check whether a given number is a solution of an equation or inequality.



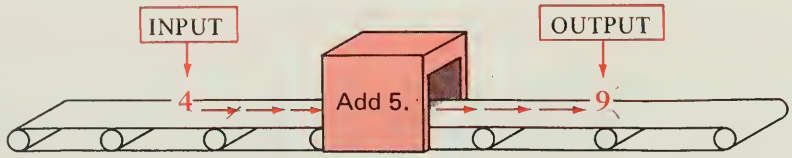
Chapter 1

Working With Variables

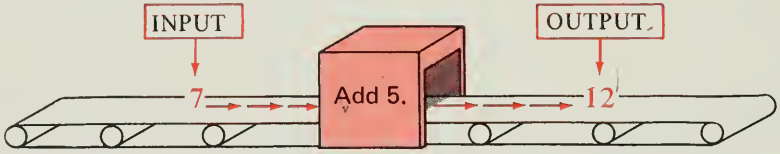


1 • Using Letters for Numbers

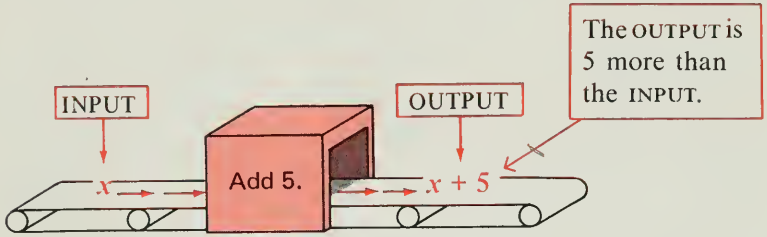
Imagine a calculating machine that adds 5 to any number you put into it. It might look like this.



Now suppose you put 7 into the machine.

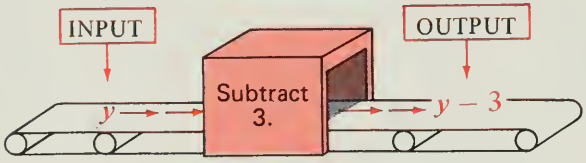


You could go on drawing pictures like this all day. However, you could draw just one picture to represent all possibilities. You can use a letter to represent the INPUT.



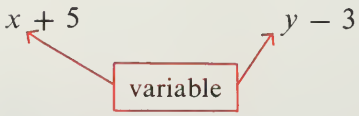
INPUT	OUTPUT
4	9
7	12
20	25
x	$x + 5$

Here is a machine which subtracts 3. Let's use the letter y to represent the INPUT this time.



INPUT y	OUTPUT $y - 3$
3	0
8	5
9	6
20	17

When we use a letter to represent a number we call the letter a **variable.**
We call $x + 5$ and $y - 3$ **variable expressions.**



Here's a hint to help you remember the word *variable:* A variable may stand for one of *various* numbers.

We find the value of an expression in the following way.

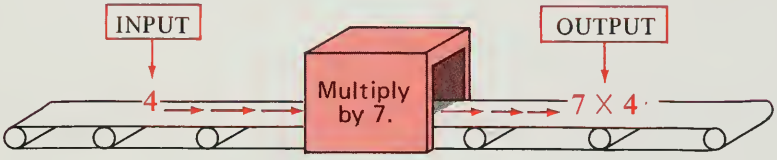
EXAMPLE 1 If $n = 10$, find the value of $n + 2$.

$$\begin{aligned} n + 2 &= 10 + 2 \\ &= 12 \end{aligned}$$

EXAMPLE 2 If $x = 14$ and $y = 3$, find the value of $x - y$.

$$\begin{aligned} x - y &= 14 - 3 \\ &= 11 \end{aligned}$$

Suppose you had a machine which multiplied by 7.



There is another way to show multiplication, too. It is with a dot. You'll see this used very often.

$7 \times 4 \longleftarrow$ means the same as $\longrightarrow 7 \cdot 4$

In algebra, the times sign and the dot are usually left out when a variable is used.

$2n \longleftarrow$ means the same as $\longrightarrow 2 \cdot n$

$ax \longleftarrow$ means the same as $\longrightarrow a \cdot x$

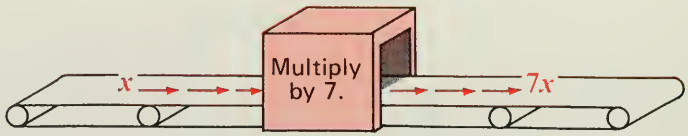
EXAMPLE 3 If $n = 5$, find the value of $7n$.

$$\begin{aligned} 7n &= 7 \cdot 5 \\ &= 35 \end{aligned}$$

Classroom Practice

Complete the table for the machine shown.

	INPUT x	OUTPUT $7x$
Sample	3	21
1.	4	? 28
2.	8	? 56
3.	5	? 35



Find the value of the expression.

4. $6 \cdot 7$ 42 **5.** $8 \cdot 3$ 24 **6.** $3 \cdot 5$ 15 **7.** $4 \cdot 8$ 32

If $x = 10$, find the value of the expression.

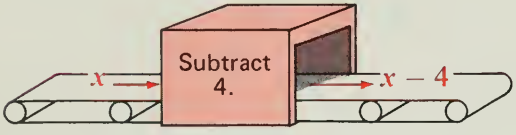
8. $x + 5$ 15 **9.** $x - 5$ 5 **10.** $4x$ 40 **11.** $x \div 2$ 5

If $s = 12$ and $t = 4$, find the value of the expression.

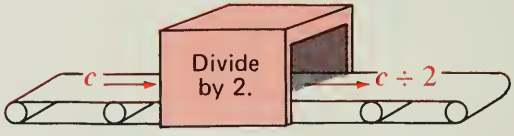
12. $s + t$ 16 **13.** $s - t$ 8 **14.** st 48 **15.** $s \div t$ 3

Written Exercises

Complete the table for the machine shown.



		INPUT x	OUTPUT $x - 4$
A	1.	5	? 1
	2.	7	? 3
	3.	14	? 10
	4.	27	? 23



	INPUT c	OUTPUT $c \div 2$
5.	6	? 3
6.	16	? 8
7.	20	? 10
8.	30	? 15

If $s = 4$, find the value of the expression.

9. $4s$ 16
10. $4 + s$ 8
11. $4 - s$ 0
12. $4 \div s$ 1

If $n = 12$, find the value of the expression.

13. $n - 3$ 9
14. $n + 7$ 19
15. $2n$ 24
16. $n \div 4$ 3

If $k = 8$, find the value of the expression.

17. $3k$ 24
18. $4k$ 32
19. $k \div 2$ 4
20. $k \div 4$ 2

If $r = 15$, find the value of the expression.

21. $r + 2$ 17
22. $r \div 3$ 5
23. $r - 5$ 10
24. $2r$ 30

If $y = 20$, find the value of the expression.

25. $y - 10$ 10
26. $5y$ 100
27. $y \div 2$ 10
28. $30 - y$ 10

If $p = 12$ and $q = 8$, find the value of the expression.

29. $p + q + 6$ 26
30. $p - q + 6$ 10
31. pq 96
32. $p + 4 - q$ 8

Tell what is missing in the table.

	INPUT	Operation	OUTPUT
Sample	x	Add 2.	$x + 2$
33.	x	Add 7.	? $x + 7$
34.	x	Subtract 4.	? $x - 4$
35.	n	Multiply by 2.	? $2n$
36.	n	Divide by 2.	? $n \div 2$
37.	a	Multiply by b.	? ba
38.	a	Subtract 7.	? $a - 7$
39.	x	? Subtract 4.	$x - 4$
40.	y	? Multiply by 3.	$3y$
41.	z	? Divide by 8.	$z \div 8$
42.	? x	Add 3.	$x + 3$
43.	? t	Multiply by s.	st

2 • Parentheses, Order of Operations

When you have several numbers to add, subtract, multiply, or divide, it helps to show which operation to do first. Parentheses are used.

Here we add first.

$$(3 + 5) \cdot 2$$
$$8 \cdot 2$$
$$16$$

Multiply first.

$$7 - (2 \cdot 3)$$
$$7 - 6$$
$$1$$

Subtract first.

$$5 + (8 - 2)$$
$$5 + 6$$
$$11$$

When there are no parentheses, there are only two steps to follow.

Step 1: Do all multiplications and divisions in order from left to right.
Step 2: Do all additions and subtractions in order from left to right.

$$3 + 5 \cdot 2$$
$$3 + 10$$
$$13$$

$$5 + 8 - 2 \cdot 3$$
$$5 + 8 - 6$$
$$13 - 6$$
$$7$$

Now let's see how to use these ideas when there are variables.

EXAMPLES If $x = 4$, find the value of $2(x + 5)$ and $2x + 5$.

This means that you multiply here.

$$2(x + 5) = 2(4 + 5)$$
$$= 2 \cdot 9$$
$$= 18$$

$$2x + 5 = 2 \cdot 4 + 5$$
$$= 8 + 5$$
$$= 13$$

Classroom Practice

Find the value of the expression.

1. $(7 - 3) \cdot 2$ 8 **2.** $2 + 3 \cdot 4$ 14 **3.** $3 + (4 - 1)$ 6 **4.** $7 - (3 \cdot 2)$ 1

If $x = 3$, find the value of the expression.

5. $2(x + 5)$ 16 **6.** $2x + 5$ 11 **7.** $3(x - 1)$ 6 **8.** $3x - 1$ 8

Written Exercises

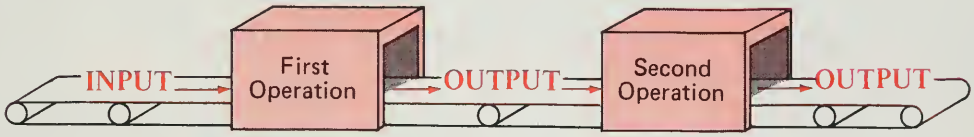
Find the value of the expression.

A

1. $(16 - 4) \cdot 2$ 24
2. $16 - (4 \cdot 2)$ 8
3. $3 \cdot (5 + 7)$ 36
4. $(16 - 4) \div (3 + 1)$ 3
5. $(8 + 4) \div (4 + 2)$ 2
6. $9 \div 1 + 3$ 12
7. $3 \cdot 4 + 5$ 17
8. $6 + 8 \cdot 4$ 38
9. $(8 + 6) \div (7 - 5)$ 7
10. $2 \cdot 2 - 1$ 3
11. $3 + 7 \cdot 4$ 31
12. $8 \cdot 2 + 5 \cdot 4$ 36
13. $8 + (2 \cdot 3)$ 14
14. $(2 + 3) \cdot (4 + 1)$ 25
15. $4 \cdot 4 + 4 \div 2$ 18
16. $4 + 8 \div 2$ 8
17. $6 + 2 \div 2 + 2$ 9
18. $(3 - 2) \cdot (9 - 3)$ 6

19. If $x = 4$, find the value of $2(x + 6)$ and $2x + 6$. 20, 14
20. If $x = 7$, find the value of $5(x + 1)$ and $5x + 1$. 40, 36
21. If $b = 9$, find the value of $8(b - 4)$ and $8b - 4$. 40, 68
22. If $y = 8$, find the value of $3(y + 2)$ and $3y + 2$. 30, 26
23. If $a = 5$, find the value of $4(a - 2)$ and $4a - 2$. 12, 18
24. If $s = 3$, find the value of $6(s + 4)$ and $6s + 4$. 42, 22

Tell what is missing in the table.



	INPUT →	First Operation →	OUTPUT →	Second Operation →	OUTPUT
25.	3	Multiply by 2.	? 6	Add 7.	? 13
26.	4	Multiply by 2.	? 8	Add 7.	? 15
27.	x	Multiply by 2.	? $2x$	Add 7.	? $2x + 7$
28.	3	Add 7.	? 10	Multiply by 2.	? 20
29.	4	Add 7.	? 11	Multiply by 2.	? 22
30.	x	Add 7.	? $x + 7$	Multiply by 2.	? $2(x + 7)$

For more practice, see page 406.

3 • Coefficients and Terms

In arithmetic you know that $5 + 5 = 2 \cdot 5$, that $5 + 5 + 5 = 3 \cdot 5$, and so on. Working with variables is very similar.

$$x = 1x$$
$$x + x = 2x$$
$$x + x + x = 3x$$
$$x + x + x + x = 4x$$

Note that x and $1x$ mean the same.

The numbers 1, 2, 3, and 4 are called **coefficients** of x.

Now you can rewrite expressions.

$$x + x + y + y + y = (x + x) + (y + y + y)$$
$$= 2x + 3y$$

In the expression $2x + 3y$, we call $2x$ and $3y$ the **terms.**

$2x + 3y$ — terms of the expression

$3a + 4b - 8c$ — terms of the expression

Sometimes the terms of an expression can be combined. We can only combine *like* terms. We cannot combine *unlike* terms. Here are some examples of like and unlike terms.

like terms: $2x + 3x = 5x$

unlike terms: $2x + 3y$

like terms: $6a - 4a = 2a$

unlike terms: $6ab - 4b$

like terms: $6c + c = 7c$

unlike terms: $6c + 6$

When you combine like terms you **simplify** the expression.

EXAMPLES Simplify $2a + 6a + 3$.
$2a + 6a + 3 = 8a + 3$

Simplify $3ab + 4ab - 9$.
$3ab + 4ab - 9 = 7ab - 9$

Simplify $4x + 5x + 7 - 2$.
$4x + 5x + 7 - 2 = 9x + 5$

Simplify $5a + 3xy - xy$.
$5a + 3xy - xy = 5a + 2xy$

Classroom Practice

Complete the statement.

1. In the expression $5x$ the coefficient of x is ___?5___.
2. In the expression $6x + 7y$ the coefficient of x is ___?6___.
3. In the expression $4a + b$ the coefficient of b is not written. It is understood to be ___?1___.

Simplify.

4. $x + x$ — $2x$
5. $y + y + y$ — $3y$
6. $2ab + 3ab$ — $5ab$
7. $3a + a + 3 + 2$ — $4a + 5$

Written Exercises

Simplify.

A

1. $2x + 4x$ — $6x$
2. $3x + 5x$ — $8x$
3. $3a + 7a$ — $10a$
4. $5b - 2b$ — $3b$
5. $9y - 6y$ — $3y$
6. $6xy - 2xy$ — $4xy$
7. $2a + 5a + 6$ — $7a + 6$
8. $5y + 3y + 4y$ — $12y$
9. $3a + 5a + 4a$ — $12a$
10. $6b + 3b - b$ — $8b$
11. $4 + 5y + 6y$ — $4 + 11y$
12. $2x + 4x + 7$ — $6x + 7$
13. $8s - 2s - 2$ — $6s - 2$
14. $3a + a - 2$ — $4a - 2$
15. $3 + 4y + 5y$ — $3 + 9y$
16. $3a + 3a + 3$ — $6a + 3$
17. $3a + a - 4$ — $4a - 4$
18. $2a + 5a + 3a$ — $10a$
19. $6 - 2 + 4x$ — $4 + 4x$
20. $y + 2y - 7$ — $3y - 7$
21. $6xy - xy + 7$ — $5xy + 7$
22. $9c - 2c + 4c$ — $11c$
23. $5d + 6d - 3d$ — $8d$
24. $8y + 3y - 4y$ — $7y$
25. $7y - 2y + 9x$ — $5y + 9x$
26. $2x + x + 4y$ — $3x + 4y$
27. $7ab - 2ab + 5b$ — $5ab + 5b$
28. $cd + 4cd - 2a$ — $5cd - 2a$

Simplify if possible. If you cannot simplify, write *not possible*.

29. $3n + 3$ — not possible
30. $4s - 4s$ — 0
31. $3ab + 3b$ — not possible
32. $2a + 5a$ — $7a$
33. $ar + 4ar$ — $5ar$
34. $r + 4s$ — not possible
35. $8n + 2$ — not possible
36. $9a - 3$ — not possible

4 • Properties of Addition

Addition has some properties which are probably familiar to you. Here are two of them.



THE COMMUTATIVE PROPERTY
In addition, the order of the terms makes no difference.

Arithmetic Example	*Algebra Example*
$1 + 2 = 2 + 1$	$a + b = b + a$



THE ASSOCIATIVE PROPERTY
In addition, the grouping of the terms makes no difference.

Arithmetic Example	*Algebra Example*
$(1 + 2) + 3 = 1 + (2 + 3)$	$(a + b) + c = a + (b + c)$

You don't need to know the names of the properties. You should, however, be able to use them to simplify your work.

EXAMPLE 1 Find the value of $97 + 54 + 3$.

$$\begin{aligned} 97 + 54 + 3 &= 97 + 3 + 54 \\ &= (97 + 3) + 54 \\ &= 100 + 54 \\ &= 154 \end{aligned}$$

EXAMPLE 2 Simplify $3n + 5 + 6n - 2$.

$$\begin{aligned} 3n + 5 + 6n - 2 &= (3n + 6n) + (5 - 2) \\ &= 9n + 3 \end{aligned}$$

Classroom Practice

Simplify.

1. $2x + 3 + 4x + 5$
$6x + 8$

2. $3a + 4 + a - 2$
$4a + 2$

3. $8 + 5xy - 2xy + 3$
$3xy + 11$

4. $9r + 5s + 3s + 7r$
$16r + 8s$

5. $5mn + 3 + 4mn - 2$
$9mn + 1$

6. $8c + 5d - 4c - 3d$
$4c + 2d$

7. $5m + 6 - 3m + 9$
$2m + 15$

8. $10 + 2x - x + 6$
$16 + x$

9. $3a + 2b - a + 4b$
$2a + 6b$

Written Exercises

Find the value of the expression. Show how you use the properties.

A

1. $26 + 5 + 15$ — 46
2. $7 + 63 + 3$ — 73
3. $49 + 93 + 1$ — 143
4. $7 + 25 + 5 + 3$ — 40
5. $998 + 571 + 2$ — 1571
6. $187 + 38 - 87$ — 138

Simplify.

7. $3n + 6 + 5n$ — $8n + 6$
8. $2n + 3 + 4n$ — $6n + 3$
9. $5c + 7 - 3 + 2c$ — $7c + 4$
10. $4x + 3 - 2 + 4x$ — $8x + 1$
11. $30 + 5a + a + 1$ — $31 + 6a$
12. $9 + 2ac + 3ac + 7$ — $16 + 5ac$
13. $8m + 5 + 6 + 2m$ — $10m + 11$
14. $4xy + 5 - xy + 2$ — $3xy + 7$
15. $7ab + 7 - 6ab + 3$ — $ab + 10$
16. $12m + 3 - 5m + 8m$ — $15m + 3$
17. $6s + 5 + 2s - 4s$ — $4s + 5$
18. $12a - 3a + 5 - 2a$ — $7a + 5$
19. $6k + 9n - 5n - 2k$ — $4k + 4n$
20. $4p - 2p + 9a + 3p$ — $5p + 9a$
21. $4n - 9b + 3n - 2n$ — $5n - 9b$
22. $3c + 7d + 5c + 5d$ — $8c + 12d$
23. $8a - 3b + 9b - a$ — $7a + 6b$
24. $3p + 4q + 7p - 2q$ — $10p + 2q$
25. $8a + 2b + 4 + 3b$ — $8a + 5b + 4$
26. $7x + 3y - 2y + 5$ — $7x + y + 5$
27. $4d - 7a - 2d + 5d$ — $7d - 7a$
28. $5m + 3m - 3 + 4n$ — $8m - 3 + 4n$
29. $6a - 2a + 4b - a$ — $3a + 4b$
30. $cd + 4a + 5cd - 2$ — $6cd + 4a - 2$

For more practice, see page 406.

SELF-TEST

If $x = 14$, find the value of the expression.

1. $x + 6$ — 20
2. $x - 6$ — 8
3. $3x$ — 42
4. $x \div 7$ — 2

Find the value of the expression.

5. $3 \cdot 2 + 5$ — 11
6. $(8 - 4) \cdot 2$ — 8
7. $8 - 4 \cdot 2$ — 0
8. $4 + 6 \div 2$ — 7

Simplify if possible. If you cannot simplify, write *not possible*.

9. $4xy - 2xy$ — $2xy$
10. $5s - 4$ — not possible
11. $3y + 4y + y$ — $8y$
12. $7w + 2w - 5$ — $9w - 5$

Simplify.

13. $3a - 5 - a$ — $2a - 5$
14. $4m + 2m - 3 - m$ — $5m - 3$
15. $4x - y + 3y - x$ — $3x + 2y$
16. $9k + j - 3k + 4j$ — $6k + 5j$
17. $10x - 4x + 3x$ — $9x$
18. $5a - 3 + 8a - b$ — $13a - b - 3$

5 • Exponents and Factors

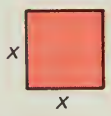
In both addition and multiplication we can simplify the expressions.

Adding	Multiplying
$x = 1x$	$x = x^1$
$x + x = 2x$	$x \cdot x = x^2$
$x + x + x = 3x$	$x \cdot x \cdot x = x^3$
coefficients	exponents

x^1, x^2, and x^3 are called **powers** of x.

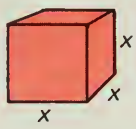
x to the first power $\longrightarrow x^1$ ← Usually written as just x.
x to the second power $\longrightarrow x^2$
x to the third power $\longrightarrow x^3$

Often the second power of x is called x **squared.** To help you remember this, think of a square like this.



Area $= x^2$ or
x squared

The third power of x is often called x **cubed.** Think of a cube like this one.



Volume $= x^3$ or
x cubed

EXAMPLE 1 Find the value of 2^5.

$$\begin{aligned} 2^5 &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \\ &= 32 \end{aligned}$$

EXAMPLE 2 Simplify $x \cdot x \cdot y \cdot y \cdot y$.

$$\begin{aligned} x \cdot x \cdot y \cdot y \cdot y &= (x \cdot x) \cdot (y \cdot y \cdot y) \\ &= x^2y^3 \end{aligned}$$

Each term in $3x - 5y + 4xy$ expresses multiplication. Each part of the term is called a **factor** of the term.

The factors of the first term are 3 and x.

$3x - 5y + 4xy$

The factors of the third term are 4, x, and y.

The factors of the second term are 5 and y.

Classroom Practice

Find the value of the expression.

1. 2^2 — 4
2. 2^3 — 8
3. 2^4 — 16
4. 2^1 — 2
5. 6^2 — 36

Simplify.

6. $x \cdot x \cdot x$ — x^3
7. $a \cdot a \cdot a \cdot a$ — a^4
8. $x \cdot x \cdot y \cdot y$ — x^2y^2
9. $6 \cdot c \cdot c \cdot c$ — $6c^3$

10. What is the third power of 10? — $10^3 = 1000$
11. What is the third term of the expression $3a + 4b + 7ab$? — $7ab$
12. Name the factors in the third term of $3a + 4b + 7ab$. — 7, a, b

Written Exercises

Find the value of the expression.

A

1. 3^2 — 9
2. 3^3 — 27
3. 4^1 — 4
4. 5^2 — 25
5. 10^2 — 100

Simplify.

6. $x \cdot x \cdot x$ — x^3
7. $y \cdot y \cdot y \cdot y$ — y^4
8. $a \cdot a \cdot a \cdot a$ — a^4
9. $x \cdot x \cdot y \cdot y \cdot y$ — x^2y^3
10. $p \cdot q \cdot q$ — pq^2
11. $r \cdot s \cdot s \cdot s$ — rs^3
12. $c \cdot c \cdot c \cdot c \cdot c$ — c^5
13. $t \cdot t \cdot m \cdot m$ — t^2m^2
14. $n \cdot r \cdot r \cdot r$ — nr^3
15. $f \cdot f \cdot f \cdot g \cdot g \cdot h$ — f^3g^2h
16. $a \cdot a \cdot b \cdot b \cdot b \cdot c$ — a^2b^3c
17. $d \cdot e \cdot e \cdot f \cdot f$ — de^2f^2
18. $5 \cdot n \cdot n$ — $5n^2$
19. $7 \cdot m \cdot m \cdot m$ — $7m^3$
20. $6 \cdot a \cdot b \cdot b$ — $6ab^2$
21. $2 \cdot a \cdot a \cdot m \cdot m$ — $2a^2m^2$
22. $c \cdot c \cdot c \cdot m \cdot m$ — c^3m^2
23. $5 \cdot n \cdot n \cdot n \cdot p$ — $5n^3p$
24. $7 \cdot s \cdot s \cdot t \cdot t \cdot t$ — $7s^2t^3$
25. $3 \cdot 2 \cdot a \cdot a \cdot b$ — $6a^2b$
26. $9 \cdot r \cdot r \cdot s \cdot s$ — $9r^2s^2$



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6 • Properties of Multiplication

Multiplication has some properties which are like the properties of addition.



THE COMMUTATIVE PROPERTY
In multiplication, the order of the factors makes no difference.

Arithmetic Example	*Algebra Example*
$2 \cdot 3 = 3 \cdot 2$	$ab = ba$



THE ASSOCIATIVE PROPERTY
In multiplication, the grouping of the factors makes no difference.

Arithmetic Example	*Algebra Example*
$(2 \cdot 3) \cdot 4 = 2 \cdot (3 \cdot 4)$	$(ab)c = a(bc)$

As with the addition properties, you do not need to memorize the names. Just be sure you can use the properties. They can make your work easier.

EXAMPLE 1 Evaluate $5 \cdot 82 \cdot 2$.

$$\begin{aligned} 5 \cdot 82 \cdot 2 &= 5 \cdot 2 \cdot 82 \\ &= (5 \cdot 2) \cdot 82 \\ &= 10 \cdot 82 \\ &= 820 \end{aligned}$$

EXAMPLE 2 Simplify $2 \cdot (5x)$.

$$\begin{aligned} 2 \cdot (5x) &= (2 \cdot 5) \cdot x \\ &= 10x \end{aligned}$$

EXAMPLE 3 Simplify $(2r)(4r)$.

$$\begin{aligned} (2r)(4r) &= (2 \cdot 4)(r \cdot r) \\ &= 8r^2 \end{aligned}$$

EXAMPLE 4 Simplify $6 \cdot a \cdot a \cdot 5 \cdot a$.

$$\begin{aligned} 6 \cdot a \cdot a \cdot 5 \cdot a &= (6 \cdot 5)(a \cdot a \cdot a) \\ &= 30a^3 \end{aligned}$$

Classroom Practice

Find the value of the expression. Show how you use the properties.

1. $17 \cdot 25 \cdot 4$ — 1700
2. $(2 \cdot 43) \cdot 5$ — 430
3. $2 \cdot 2 \cdot 5 \cdot 5$ — 100

Simplify.

4. $3 \cdot (4x)$ — $12x$
5. $5 \cdot 6a$ — $30a$
6. $3n \cdot 2$ — $6n$
7. $a \cdot (4a)$ — $4a^2$
8. $(3x) \cdot x$ — $3x^2$
9. $(2x)(3x)$ — $6x^2$
10. $(3a)(4b)$ — $12ab$
11. $4 \cdot x \cdot x$ — $4x^2$

Written Exercises

Find the value of the expression.

A

1. $4 \cdot 29 \cdot 25$ — 2900
2. $92 \cdot 5 \cdot 2$ — 920
3. $(33 \cdot 25) \cdot 4$ — 3300
4. $(50 \cdot 83) \cdot 2$ — 8300
5. $2 \cdot (73 \cdot 5)$ — 730
6. $10 \cdot 3 \cdot 10 \cdot 2 \cdot 10$ — 6000

Simplify.

7. $2 \cdot (3x)$ — $6x$
8. $3 \cdot (4x)$ — $12x$
9. $5 \cdot (3a)$ — $15a$
10. $4 \cdot (4a)$ — $16a$
11. $b \cdot 2b$ — $2b^2$
12. $c \cdot 3c$ — $3c^2$
13. $(3y) \cdot 6$ — $18y$
14. $(5k) \cdot 2$ — $10k$
15. $(4n) \cdot 5$ — $20n$
16. $2s \cdot s$ — $2s^2$
17. $8 \cdot 9f$ — $72f$
18. $m \cdot 6n$ — $6mn$
19. $(2a)(2a)$ — $4a^2$
20. $(3x)(5y)$ — $15xy$
21. $(2a)(4b)$ — $8ab$
22. $(6s)(3s)$ — $18s^2$
23. $8 \cdot x \cdot x \cdot 2$ — $16x^2$
24. $3 \cdot b \cdot b \cdot 2$ — $6b^2$
25. $5 \cdot a \cdot 7 \cdot a \cdot a$ — $35a^3$
26. $4 \cdot y \cdot y \cdot y \cdot 2$ — $8y^3$
27. $2 \cdot b \cdot 5 \cdot b \cdot b$ — $10b^3$
28. $3 \cdot a \cdot a \cdot 4 \cdot a$ — $12a^3$

Tell what is missing in the table.

	INPUT	→ Operation →	OUTPUT
29.	$3x$	Multiply by 2.	? $6x$
30.	$5a$	Multiply by 3.	? $15a$
31.	$2x$	Multiply by x.	? $2x^2$
32.	? $3c$	Multiply by 5.	$15c$
33.	$5x$	? Multiply by 4.	$20x$
34.	$3b$	? Multiply by 6.	$18b$

For more practice, see page 407.

7 • The Distributive Property



Jan is buying two felt-tip pens at \$1 each and two books at \$3 each. Her total bill can be figured in two ways.

$2(\$1 + \$3)$	$2 \cdot \$1 + 2 \cdot \$3$
$2 \cdot \$4$	$\$2 + \$6$
$\$8$	$\$8$

The work above shows that the following is true.

$$2(1 + 3) = 2 \cdot 1 + 2 \cdot 3$$

Let's check a similar statement to see if it is also true.

Is $3(4 + 6) = 3 \cdot 4 + 3 \cdot 6$ true?

Work one side at a time. Do they check?

$3(4 + 6) \stackrel{?}{=} 3 \cdot 4 + 3 \cdot 6$

$3 \cdot 10$	$12 + 18$
30	30 ✓

Yes, they check.

This example shows the **distributive property.**

THE DISTRIBUTIVE PROPERTY

Arithmetic Examples	*Algebra Examples*
$2(1 + 3) = 2 \cdot 1 + 2 \cdot 3$	$a(b + c) = ab + ac$
$3(4 - 1) = 3 \cdot 4 - 3 \cdot 1$	$a(b - c) = ab - ac$

You can use the distributive property to change an expression with parentheses to one without them.

EXAMPLE 1 $3(x + 6) = 3x + 3 \cdot 6$
$= 3x + 18$

EXAMPLE 2 $2(a - 3b) = 2a - 2 \cdot 3b$
$= 2a - 6b$

Classroom Practice

1. Check to see if $3(2 + 1) = (3 \cdot 2) + (3 \cdot 1)$ is true. true

2. Is $2(4 \cdot 3) = (2 \cdot 4) \cdot (2 \cdot 3)$ true? no

3. Once you've checked to see if the statement in Exercise 2 is true, you should be able to tell if $2(ab) = (2a) \cdot (2b)$ is true. Is it? no

True or false?

4. $3(a + b) = 3a + 3b$ true

5. $3(cd) = (3c) \cdot (3d)$ false

State the expression without parentheses.

6. $2(x + 5)$ $2x + 10$
7. $3(x + 2)$ $3x + 6$
8. $4(a + 1)$ $4a + 4$
9. $5(7 + a)$ $35 + 5a$
10. $3(x - 4)$ $3x - 12$
11. $a(a - 3)$ $a^2 - 3a$
12. $b(b - 3)$ $b^2 - 3b$
13. $y(2 - y)$ $2y - y^2$

Written Exercises

Check to see if the statement is true.

A

1. $3(4 + 5) = (3 \cdot 4) + (3 \cdot 5)$ true
2. $4(8 + 3) = (4 \cdot 8) + (4 \cdot 3)$ true
3. $5(7 - 3) = (5 \cdot 7) - (5 \cdot 3)$ true
4. $6(21 - 11) = (6 \cdot 21) - (6 \cdot 11)$ true

State the expression without parentheses.

5. $2(x + 3)$ $2x + 6$
6. $3(x + 5)$ $3x + 15$
7. $4(x + 2)$ $4x + 8$
8. $5(a + 4)$ $5a + 20$
9. $7(a + 3)$ $7a + 21$
10. $8(b + 1)$ $8b + 8$
11. $9(x + 4)$ $9x + 36$
12. $6(y + 2)$ $6y + 12$
13. $3(x - 2)$ $3x - 6$
14. $4(y - 7)$ $4y - 28$
15. $c(c - 3)$ $c^2 - 3c$
16. $d(d - 4)$ $d^2 - 4d$
17. $7(2x + 1)$ $14x + 7$
18. $4(3x + 2)$ $12x + 8$
19. $5(3x - 2)$ $15x - 10$
20. $3(7x - 3)$ $21x - 9$
21. $2(3x + 4y)$ $6x + 8y$
22. $b(4a + 2b)$ $4ab + 2b^2$
23. $x(3x - y)$ $3x^2 - xy$
24. $a(2a - 3b)$ $2a^2 - 3ab$

Use the distributive property. Then combine like terms.

25. $2(x + 5) + 3$ $2x + 13$
26. $3(x - 1) + 5x$ $8x - 3$
27. $5(x + 3) + 2x$ $7x + 15$
28. $3(2x - 1) + 4x$ $10x - 3$
29. $4(9 + 2y) - 3y$ $36 + 5y$
30. $7(1 + 2c) - 8c$ $7 + 6c$

B

31. $5(3x + 5) - 7x - 2$ $8x + 23$
32. $2(1 + 3x) + 7x - 1$ $1 + 13x$
33. $5(2 + 7x) - 3x - 2$ $8 + 32x$
34. $4(a + 3) + 3(a - 2)$ $7a + 6$
35. $8(2b + 3) + 4(b - 5)$ $20b + 4$
36. $6(c + 3) + 4(1 - c)$ $2c + 22$

For more practice, see page 407.

8 • Properties of 0

- Any number times 0 is 0.

$$7 \cdot 0 = 0 \qquad a \cdot 0 = 0$$

- No number can be divided by 0.

$\frac{6}{0}$ IMPOSSIBLE $\qquad \frac{n}{0}$ IMPOSSIBLE

- Zero divided by any number (except 0) is 0.

$$\frac{0}{6} = 0 \qquad \frac{0}{n} = 0$$

Divisions are often written with a bar.

- Adding 0 to a number gives that number.

$$6 + 0 = 6 \qquad n + 0 = n$$

- Adding and subtracting the same number is like adding 0.

$$n + 5 - 5 = n + 0 = n$$

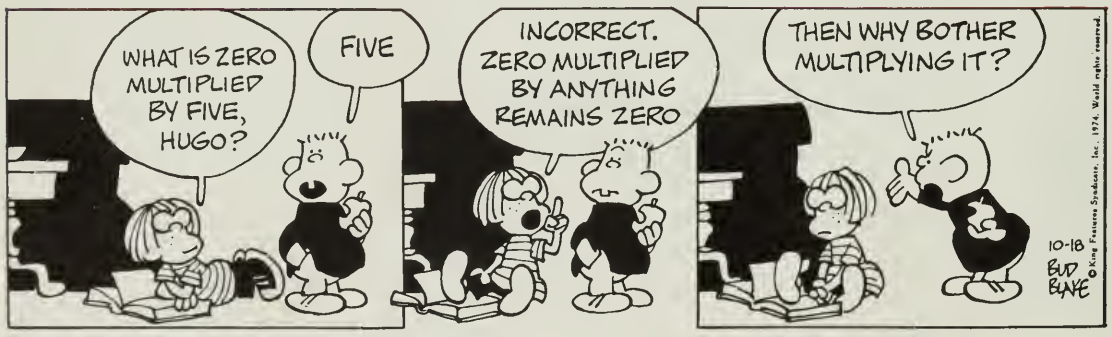
Let's put these ideas to work.

EXAMPLE 1 Let $x = 4$. Find the value of $9(x - 4)$.

$$\begin{aligned} 9(x - 4) &= 9(4 - 4) \\ &= 9 \cdot 0 \\ &= 0 \end{aligned}$$

EXAMPLE 2 Simplify $2 + 3x - 2$.

$$\begin{aligned} 2 + 3x - 2 &= 3x + 2 - 2 \\ &= 3x + 0 \\ &= 3x \end{aligned}$$



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Classroom Practice

Find the value if possible. If not, say *impossible*.

1. $5 \cdot 0$ — 0
2. $\frac{0}{7}$ — 0
3. $\frac{7}{0}$ — impossible
4. $\frac{8 \cdot 0}{9}$ — 0

Let $x = 5$. Find the value of the expression.

5. $\frac{0}{x}$ — 0
6. $x + 0$ — 5
7. $\frac{x - 5}{11}$ — 0
8. $35(x - 5)$ — 0

Written Exercises

Find the value if possible. If not, write *impossible*.

A

1. $12 \cdot 0$ — 0
2. $0 \cdot 31$ — 0
3. $18 + 35 - 18$ — 35
4. $16 + 0$ — 16
5. $\frac{0}{16}$ — 0
6. $\frac{31}{0}$ — impossible
7. $\frac{5}{5 \cdot 0}$ — impossible
8. $16 \cdot 8 \cdot 0$ — 0

9. Let $x = 1$. Find the value of $5(x - 1)$. 0
10. Let $x = 2$. Find the value of $(x - 2) \div 4$. 0
11. Let $x = 3$. Find the value of $(2x - 6) \div 4$. 0
12. Let $x = 0$. Find the value of $x \div 3$. 0

Simplify.

13. $3x + 6 + 5x - 6$ — $8x$
14. $5a + 3 + 8a - 3$ — $13a$
15. $6 + 4c + 2c - 6$ — $6c$
16. $4(x + 2) - 8$ — $4x$
17. $3(2 + 5a) - 6$ — $15a$
18. $2(c + 5) - 10$ — $2c$

Think of an INPUT-OUTPUT machine. Tell what is missing in the table.

	Sample	19.	20.	21.	22.	23.	24.
INPUT	$x - 4$	$x - 5$	$x - 9$	$x - 1$	$x - 3$	$x + 2$	$x + 6$
↓ Operation	Add 4.	Add 5.	Add 9.	Add 1.	Add 3.	Subtract 2.	Subtract 6.
↓ OUTPUT	x	? x	? x	? x	? x	? x	? x

	25.	26.	27.	28.	29.	30.	31.
INPUT	$y - 7$	$x - 13$	$y + 9$	$y + 4$	$x - 16$	$14 + n$	$n - 11$
↓ Operation	? Add 7.	? Add 13.	? Sub. 9.	? Sub. 4.	? Add 16.	? Sub. 14.	? Add 11.
↓ OUTPUT	y	x	y	y	x	n	n

9 • Properties of 1

Whether you add 0 to a number or multiply the number by 1, you get the same result—the original number.

$$\text{original number} + 0 = \text{original number}$$
$$\text{original number} \times 1 = \text{original number}$$

The second sentence above states a property of 1.



- Multiplying a number by 1 gives that number.

$$6 \cdot 1 = 6 \qquad n \cdot 1 = n$$

- Any number (except 0) divided by itself is 1.

$$\frac{4}{4} = 1 \qquad \frac{n}{n} = 1$$

Remember, you can write a division with a bar.

- Any number divided by 1 is that number.

$$\frac{8}{1} = 8 \qquad \frac{n}{1} = n$$

Here are a couple of examples to show these properties.

EXAMPLE 1 Simplify $8 \cdot \frac{x}{8}$.

This means $\frac{8}{8}x$ or $1x$.

$1x = x$

$$8 \cdot \frac{x}{8} = \frac{8}{8}x$$
$$= x$$

EXAMPLE 2 Simplify $4x \div 4$.

$$4x \div 4 = \frac{4x}{4}$$

This gives you $\frac{4}{4}x$, or x.

$$= \frac{4}{4}x$$
$$= x$$

Written Exercises

Let $x = 4$. Find the value of the expression.

A **1.** $\frac{x}{4}$ 1 **2.** $4x$ 16 **3.** $(4 \div x) \cdot 1$ 1 **4.** $\frac{x+2}{6}$ 1

Simplify.

5. $b \div b$ 1 **6.** $5x \div 5$ x **7.** $3x \div x$ 3 **8.** $10x \div 10$ x

9. $\frac{5c}{5}$ c **10.** $\frac{3}{3}x$ x **11.** $\frac{4n}{n}$ 4 **12.** $\frac{6}{6}y$ y

13. $5 \cdot \frac{x}{5}$ x **14.** $\frac{x}{7} \cdot 7$ x **15.** $\frac{2a}{2}$ a **16.** $\frac{c}{9} \cdot 9$ c

Think of an INPUT-OUTPUT machine. Tell what is missing in the table.

	17.	18.	19.	20.	21.
INPUT	$4x$	$5x$	$3x$	$7x$	$6x$
↓ Operation	Divide by 4.	Divide by 5.	Divide by 3.	?	?
↓ OUTPUT	? x	? x	? x	x	x
				Div. by 7.	Div. by 6.

	22.	23.	24.	25.	26.
INPUT	$\frac{x}{8}$	$\frac{x}{4}$	$\frac{x}{9}$	$\frac{x}{7}$	$\frac{x}{10}$
↓ Operation	Multiply by 8.	Multiply by 4.	Multiply by 9.	?	?
↓ OUTPUT	? x	? x	? x	x	x
				Multiply by 7.	Multiply by 10.

For more practice, see page 408.

SELF-TEST

Simplify.

1. $r \cdot r \cdot r \cdot s \cdot s$ r^3s^2 **2.** $6 \cdot m \cdot m \cdot n \cdot p$ $6m^2np$ **3.** $5 \cdot (6x)$ $30x$

4. $(4a)(7a)$ $28a^2$ **5.** $2(a + 4) - 8$ $2a$ **6.** $13m \div 13$ m

State the expression without parentheses.

7. $3(x - 2)$ $3x - 6$ **8.** $y(3y + 1)$ $3y^2 + y$ **9.** $2(4a - 6b)$ $8a - 12b$ **10.** $6(3d + 5)$ $18d + 30$

10 • Introduction to Equations



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Consider whether the following statements are true or false.

Hank Aaron is a baseball hero.....true
Charlie Brown is a baseball hero.....false
He is a baseball hero.....maybe true, maybe false
(It depends on who *he* is.)

The sentences below are like the ones above.

$6 + 4 = 10$ true
$5 + 1 = 8$ false
$x + 2 = 7$ maybe true, maybe false
(It depends on what x is.)

These number sentences are called **equations.** Equations always have an $=$ sign, but are not always true.

Is $x + 2 = 7$ true if $x = 6$?	Is $x + 2 = 7$ true if $x = 5$?
$x + 2 = 7$	$x + 2 = 7$
$6 + 2 \mid 7$	$5 + 2 \mid 7$
$8 \mid$ No!	$7 \mid$ √ Yes!

We call 5 a **solution** or **root** of the equation $x + 2 = 7$ because it is a value for x which makes the equation true.

EXAMPLE Is 4 a solution of $9 - x = 3$? Is 6 a solution?

Check 4:

$9 - x = 3$
$9 - 4 \mid 3$
$5 \mid$ No

Check 6:

$9 - x = 3$
$9 - 6 \mid 3$
$3 \mid$ √ Yes

Classroom Practice

Tell which of the numbers shown in color is a solution.

1. $x + 3 = 9$ 6 — 7 or 6?
2. $y - 7 = 11$ 18 — 18 or 4?
3. $4 + x = 10$ 6 — 9 or 6?
4. $3n = 12$ 4 — 4 or 9?
5. $3x - 1 = 8$ 3 — 2 or 3?
6. $4x - 12 = 20$ 8 — 7 or 8?
7. $m - 9 = 25$ 34 — 16 or 34?
8. $5a = 35$ 7 — 30 or 7?

Written Exercises

Tell which of the numbers shown in color is a solution.

A 1. $x + 9 = 12$ 3 — 3 or 21?
2. $y - 7 = 15$ 22 — 8 or 22?
3. $3n = 36$ 12 — 12 or 33?
4. $4x = 24$ 6 — 6 or 20?
5. $3a + 7 = 13$ 2 — 2 or 4?
6. $2a - 6 = 4$ 5 — 4 or 5?
7. $5(r + 2) = 30$ 4 — 3 or 4?
8. $7(s - 3) = 21$ 6 — 5 or 6?
9. $x + 9 = 4x$ 3 — 2, 3, or 4?
10. $2b + 5 = b + 7$ 2 — 1, 2, or 3?
11. $5a - 1 = 3a + 9$ 5 — 5, 6, or 7?
12. $4(a - 5) = a + 1$ 7 — 6, 7, or 8?
13. $n^2 = 25$ 5 — 0, 5, 10?
14. $a^3 = 1$ 1 — 0, 1, or 2?

B 15. Write an equation which has 9 as a solution.
16. Write an equation which has 11 as a solution.
17. Write an equation which has 0 as a solution.
18. Write an equation which has no number as a solution.

Answers to exercises 15–18 may vary.

Find at least one solution. You may need to try many numbers for some exercises.

19. $x + 5 = 13$ 8
20. $y - 17 = 20$ 37
21. $5n = 30$ 6
22. $6x = 36$ 6
23. $b \div 4 = 2$ 8
24. $b \div 5 = 5$ 25
25. $n \cdot 0 = 0$ Answers may vary.
26. $x + 5 = 11$ 6
27. $y + 1 = 41$ 40
28. $x + 5 = 25$ 20
29. $x + 3 = x + 3$ Answers may vary.
30. $m - 9 = 27$ 36

C 31. $3a \cdot 7 = 42$ 2
32. $2m + 9 = 21$ 6
33. $2b + 4 = 4$ 0
34. $x^2 = x$ 0, 1

11 • Introduction to Inequalities

Not all number sentences are equations. Some number sentences are statements that two numbers are not equal. We call a number sentence like this an **inequality.** Here are some examples of inequalities.

4 is less than 7.	8 is greater than 2.	6 is not equal to 3.
$4 < 7$	$8 > 2$	$6 \neq 3$

Some inequalities have variables. A **solution** of the inequality is a number that makes the statement true.

EXAMPLE 1 Is 4 a solution of $x < 6$? Is 9 a solution?

Check 4: $x < 6$

$4 \overset{?}{<} 6$ ✓ Yes

Check 9: $x < 6$

$9 \overset{?}{<} 6$ No

EXAMPLE 2 Is 2 a solution of $y > 6$? Is 4 a solution?

Check 2: $y > 6$

$2 \overset{?}{>} 6$ No

Check 4: $y > 6$

$4 \overset{?}{>} 6$ No

Classroom Practice

Read the statement aloud.

1. $17 \neq 7$ **2.** $6 > 3$ **3.** $7 < 8$ **4.** $5 > 2$

5. $9 < 10$ **6.** $4 \neq 9$ **7.** $2x > 7$ **8.** $2x + 1 < 5$

Tell which of the numbers shown in color are solutions.

9. $x < 3$ 2, 1, 0 5, 2, 1, 0

10. $a > 6$ 9, 10, 12 4, 9, 10, 12

11. $y > 12$ 15 3, 5, 7, 15

12. $y < 21$ 12, 15 12, 15, 25, 35

13. $s > 13$ 15 9, 11, 13, 15

14. $m < 2$ 0, 1 0, 1, 2, 3

15. For Exercises 9–14, are there more solutions which are not given? yes

Written Exercises

Compare. Write $<$ or $>$.

A

1. $6 \,?\, 7$ $<$
2. $0 \,?\, 4$ $<$
3. $3 \,?\, 1$ $>$
4. $4 \,?\, 9$ $<$
5. $7 \,?\, 4$ $>$
6. $5 \,?\, 11$ $<$
7. $6 \,?\, 3$ $>$
8. $5 \,?\, 0$ $>$

Tell which of the numbers shown in color are solutions.

9. $x < 6$ 4 — 4, 6, 7, 10
10. $c > 2$ 3, 5, 7 — 0, 3, 5, 7
11. $x > 0$ 1, 3, 7 — 0, 1, 3, 7
12. $y > 9$ 10, 14 — 4, 6, 10, 14
13. $x > 14$ 20 — 7, 8, 10, 20
14. $y < 9$ 6, 2, 4 — 6, 2, 10, 4
15. $a < 7$ 6, 2 — 7, 6, 2, 12
16. $c > 10$ 12, 35 — 4, 8, 12, 35
17. $x < 9$ 4, 6 — 4, 6, 9, 10
18. $y < 12$ 6, 7, 11 — 6, 12, 7, 11
19. $x + 7 > 16$ 10, 11 — 8, 9, 10, 11
20. $3y - 1 < 9$ 3, 2 — 4, 3, 6, 2
21. $3x + 3 < 24$ 2, 5 — 2, 5, 7, 9
22. $4y - 7 > 13$ 7 — 2, 4, 5, 7
23. $2y - 8 < 6$ 4, 5, 6 — 4, 5, 6, 7
24. $4a > a + 6$ 5 — 1, 0, 2, 5

Compare. Write $<$ or $>$ or $=$.

25. $3(8 + 4) \,?\, 7 \cdot 6 - 5$ $<$
26. $2 \cdot 3 + 7 \,?\, 10 + 6 \div 2$ $=$
27. $83 \cdot 5 \cdot 0 \div 9 \,?\, 0$ $=$
28. $791 \cdot (17 - 17) \,?\, 2$ $<$
29. $(24 \div 8) \cdot 8 \,?\, 3^3$ $<$
30. $(5 \cdot 4 - 2) \div 6 \,?\, 4^2$ $<$

For more practice, see page 408.

SELF-TEST

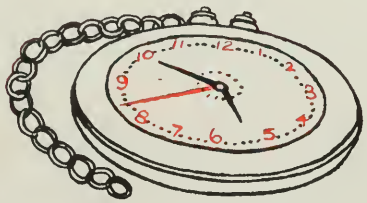
Compare. Write $<$ or $>$.

1. $5 \,?\, 6$ $<$
2. $0 \,?\, 2$ $<$
3. $9 \,?\, 7$ $>$
4. $4 \,?\, 8$ $<$

Tell which of the numbers shown in color are solutions.

5. $x - 4 = 9$ 13 — 13 or 15?
6. $5m = 35$ 7 — 7 or 30?
7. $2a + 4 = 18$ 7 — 7 or 8?
8. $9(s - 3) = 36$ 7 — 4 or 7?
9. $b \neq 13$ 3, 5, 21 — 5, 3, 13, 21
10. $y + 6 < 12$ 4, 5 — 4, 6, 5, 7

Reviewing Arithmetic Skills



Take time out to work some arithmetic problems. See if you can solve them quickly, but *correctly*. Maybe you can time yourself. Ten minutes is very good!

Add.

1. 24 + 32 = 56

2. 146 + 253 = 399

3. 566 + 427 = 993

4. 824 + 56 = 880

5. 137 + 92 = 229

6. 270 + 125 = 395

7. 452 + 76 + 184 = 712

8. 760 + 135 + 85 = 980

9. 621 + 6 + 58 + 137 = 822

10. 584 + 861 + 76 + 35 = 1556

Subtract.

11. 58 − 24 = 34

12. 67 − 35 = 32

13. 126 − 49 = 77

14. 376 − 298 = 78

15. 620 − 85 = 535

16. 704 − 367 = 337

17. 491 − 137 = 354

18. 852 − 68 = 784

19. 266 − 49 = 217

20. 306 − 87 = 219

Multiply.

21. 36 × 8 = 288

22. 59 × 4 = 236

23. 73 × 25 = 1825

24. 87 × 42 = 3654

25. 120 × 17 = 2040

26. 311 × 54 = 16,794

27. 672 × 65 = 43,680

28. 247 × 132 = 32,604

29. 721 × 105 = 75,705

30. 630 × 116 = 73,080

Divide.

31. $6\overline{)102}$ = 17

32. $4\overline{)632}$ = 158

33. $7\overline{)478}$ = 68R2

34. $9\overline{)367}$ = 40R7

35. $2\overline{)149}$ = 74R1

36. $7\overline{)492}$ = 70R2

37. $6\overline{)1427}$ = 237R5

38. $4\overline{)8562}$ = 2140R2

39. $57\overline{)5941}$ = 104R13

40. $24\overline{)675}$ = 28R3

41. $38\overline{)8412}$ = 221R14

42. $72\overline{)5629}$ = 78R13

CAREER NOTEBOOK

Behind the Scenes in a TV Studio

Not all the jobs in a TV studio are in front of the camera. There are opportunities in advertising, writing, producing, directing, and in many technical fields.



Film editors prepare film that will be shown on the air. They trim the film segments to specified lengths, and cut and splice the film to insert commercials.



Broadcast technicians operate equipment that controls the sounds and pictures being shown.



Meteorologists study data from weather satellites and from observers all over the world.

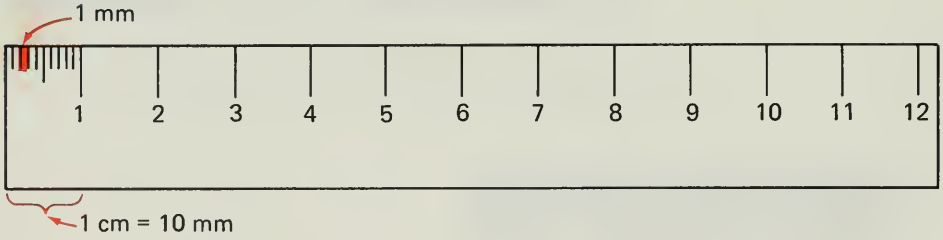
CONSUMER CORNER



The metric system is becoming more and more a part of our lives. This system of measurement is used around the world.

The base unit of length in the metric system is the **meter.** (The symbol is m.) A meter is a little more than a yard.

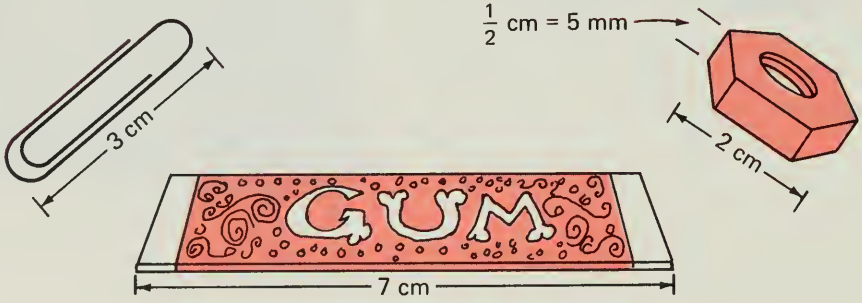
The meter can be divided into 100 **centimeters** (symbol, cm) or into 1000 **millimeters** (symbol, mm). This ruler shows the length of a centimeter and millimeter.



100 cm = 1 m

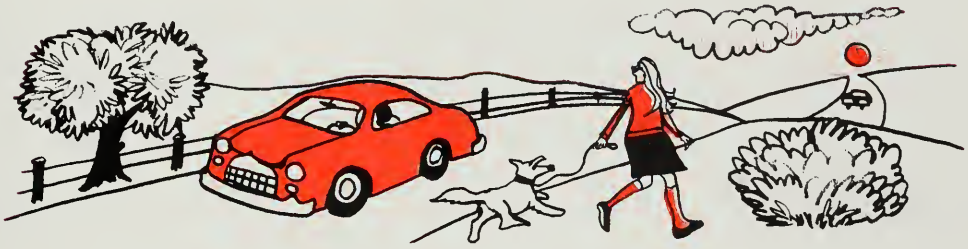
1000 mm = 1 m

Here are a few familiar objects drawn to actual size.



For measuring long distances, the **kilometer** (symbol, km) is used. A kilometer is 1000 meters. It's a little more than half a mile. If you walk quickly, you can walk 1 kilometer in about 10 minutes. If you drive on a highway, you can go about 85 kilometers in an hour.

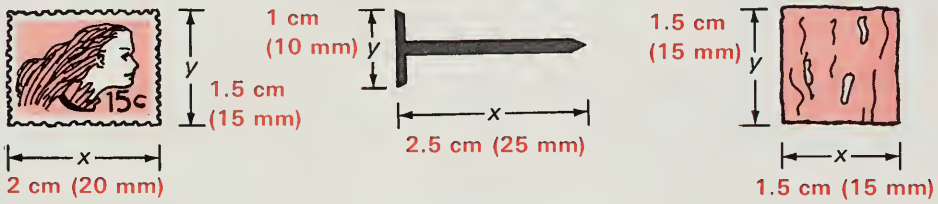
1000 m = 1 km



Try these exercises.

1. Use a metric ruler, or make a copy of the one on the last page. Find the length and width of this page. about 19 cm by 23 cm

2. Find the missing measurements in the figures below. Give your answer in two ways—in centimeters and in millimeters.



3. What is your height in centimeters?
 Answers may vary, for example 150 cm.

4. Find the Olympic records for the following events.
 These records were set before 1976.
 a. 400-meter dash (Record Time = ? seconds) men, 43.8; women, 52
 b. 1500-meter run (Record Time = ? minutes, ? seconds) men, 3:34.9; women, 4:01.4
 c. Shot put (Record Distance = ? meters) men, 21.18; women, 21.03
 d. Javelin throw (Record Distance = ? meters) men, 90.47; women, 63.88

Reviewing the Chapter

If $x = 8$, find the value of the expression. *(See pp. 2–7.)*

1. $x + 3$ 11
2. $2x$ 16
3. $x \div 4$ 2
4. $2x + 5$ 21
5. $2(x + 5)$ 26
6. $3x - 3$ 21
7. $3(x - 3)$ 15
8. $5x - 2$ 38
9. $x - 5$ 3
10. $4(x + 3)$ 44
11. $4x + 3$ 35
12. $4(x - 3)$ 20

Find the value of the expression. *(See pp. 6–7.)*

13. $3 \cdot 4 + 2$ 14
14. $5 + 8 \cdot 2$ 21
15. $(3 + 4)(5 - 2)$ 21
16. $(8 + 4) \div 4 - 2$ 1
17. $8 + 4 \div 4 - 2$ 7
18. $8 + 4 \div (4 - 2)$ 10
19. $(8 + 4) \div (4 - 2)$ 6
20. $13 - 4 \cdot 2$ 5
21. $4 \cdot 3 + 6 \cdot 2$ 24
22. $(2 + 3) \cdot 6 - 4$ 26
23. $2 + 3 \cdot 6 - 4$ 16
24. $(2 + 3)(6 - 4)$ 10

Simplify. *(See pp. 8–11.)*

25. $2x + 5x$ $7x$
26. $8a - 6a$ $2a$
27. $3b + 4b + 2$ $7b + 2$
28. $7c - c + 3c$ $9c$
29. $7c - c + 3$ $6c + 3$
30. $8y - 2y + 7 + 5$ $6y + 12$
31. $5x + 7 + 3x - 4$ $8x + 3$
32. $4a - 4 + 6a - a$ $9a - 4$
33. $3a + 2b + 7a - b$ $10a + b$
34. $7s - 4 + 5s$ $12s - 4$
35. $x + 3y + 7x - 3$ $8x + 3y - 3$
36. $9t + t - 5 - 3t$ $7t - 5$

Find the value of the expression. *(See pp. 12–13.)*

37. 2^4 16
38. 3^5 243
39. 5^3 125
40. 3 to the third power 27
41. 5^2 25
42. 4^3 64
43. 7^2 49
44. 1 to the first power 1

Simplify. *(See pp. 14–15.)*

45. $2 \cdot (5x)$ $10x$
46. $3 \cdot 4a$ $12a$
47. $8 \cdot 2b$ $16b$
48. $4 \cdot x \cdot x \cdot x$ $4x^3$
49. $3 \cdot b \cdot b \cdot b \cdot 2$ $6b^3$
50. $a \cdot b \cdot b \cdot a \cdot a$ a^3b^2
51. $(2x)(3x)$ $6x^2$
52. $(4y)(3y)$ $12y^2$
53. $n(2n)(3n)$ $6n^3$
54. $9 \cdot c \cdot c \cdot d \cdot d$ $9c^2d^2$
55. $(5s)(6s)$ $30s^2$
56. $(6x) \cdot 9$ $54x$

State the expression without parentheses. *(See pp. 16–17.)*

57. $2(x + 5)$ $2x + 10$

58. $3(2a - 7)$ $6a - 21$

59. $5(4x - 2y)$ $20x - 10y$

60. $5(3y - 4)$ $15y - 20$

61. $4(2c - 3d)$ $8c - 12d$

62. $2(9x + 7y)$ $18x + 14y$

63. $3(a - 6b)$ $3a - 18b$

64. $6(4x + 3)$ $24x + 18$

65. $3(a + 2b)$ $3a + 6b$

66. $7(2s - 4t)$ $14s - 28t$

67. $9(2x - 4y)$ $18x - 36y$

68. $4(6 - 2a)$ $24 - 8a$

69. $5(3m + n)$ $15m + 5n$

70. $6(a + 2b)$ $6a + 12b$

71. $4(5m + 7k)$ $20m + 28k$

72. $3(9c + d)$ $27c + 3d$

Let $x = 4$. Find the value if possible. If not, write *impossible*. *(See pp. 18–21.)*

73. $6(x - 4)$ 0

74. $\frac{x - 4}{x + 4}$ 0

75. $\frac{x - 4}{x - 4}$ impossible

76. $\frac{x - 4}{x}$ 0

77. $\frac{x}{x - 4}$ impossible

78. $\frac{x}{4}$ 1

79. $0 \div x$ 0

80. $x \div 0$ impossible

Simplify. *(See pp. 18–19.)*

81. $3x + 7 + 5x - 7$ $8x$

82. $9 - 4a + 4a - 3$ 6

83. $4a + 7b - 3a - 7b$ a

84. $2b + 5c - 5c - 2b$ 0

85. $3(2x - 3) + 9$ $6x$

86. $3(x - 4) + 2(x + 6)$ $5x$

87. $4s - 9 + 6s + 9$ $10s$

88. $8t + 6 - 8t - 5$ 1

89. $5(m + n) - 5m$ $5n$

90. $8x - 4(9 + 2x)$ ~~$4x = 36$~~

91. $4(2x + 4) - 8(x + 2)$ 0

92. $10 + 5(3a - 2)$ $15a$

Tell which of the numbers shown in color is a solution. *(See pp. 22–25.)*

93. $5x - 7 = 13$ 4 — 3, 4, or 5?

94. $3x + 9 = 36$ 9 — 5, 7, or 9?

95. $(2b - 6) \div 5 = 0$ 3 — 3, 6, or 9?

96. $x + 3 = 4x$ 1 — 0, 1, or 3?

97. $x + 8 > 15$ 8 — 6, 7, or 8?

98. $3y - 4 < 6$ 3 — 3, 4, or 5?

99. $2a + 3 < 24$ 10 — 10, 11, or 12?

100. $5x + 7 > 27$ 6 — 2, 4, or 6?

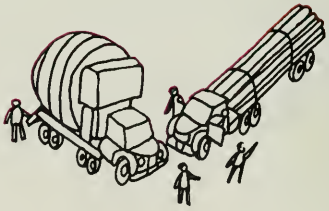
101. $2x - x > 1$ 2 — 0, 1, or 2?

102. $9x \div 9 < 1$ 0 — 0, 1, or 2?

A permission-to-reproduce chapter test can be found on page T11.

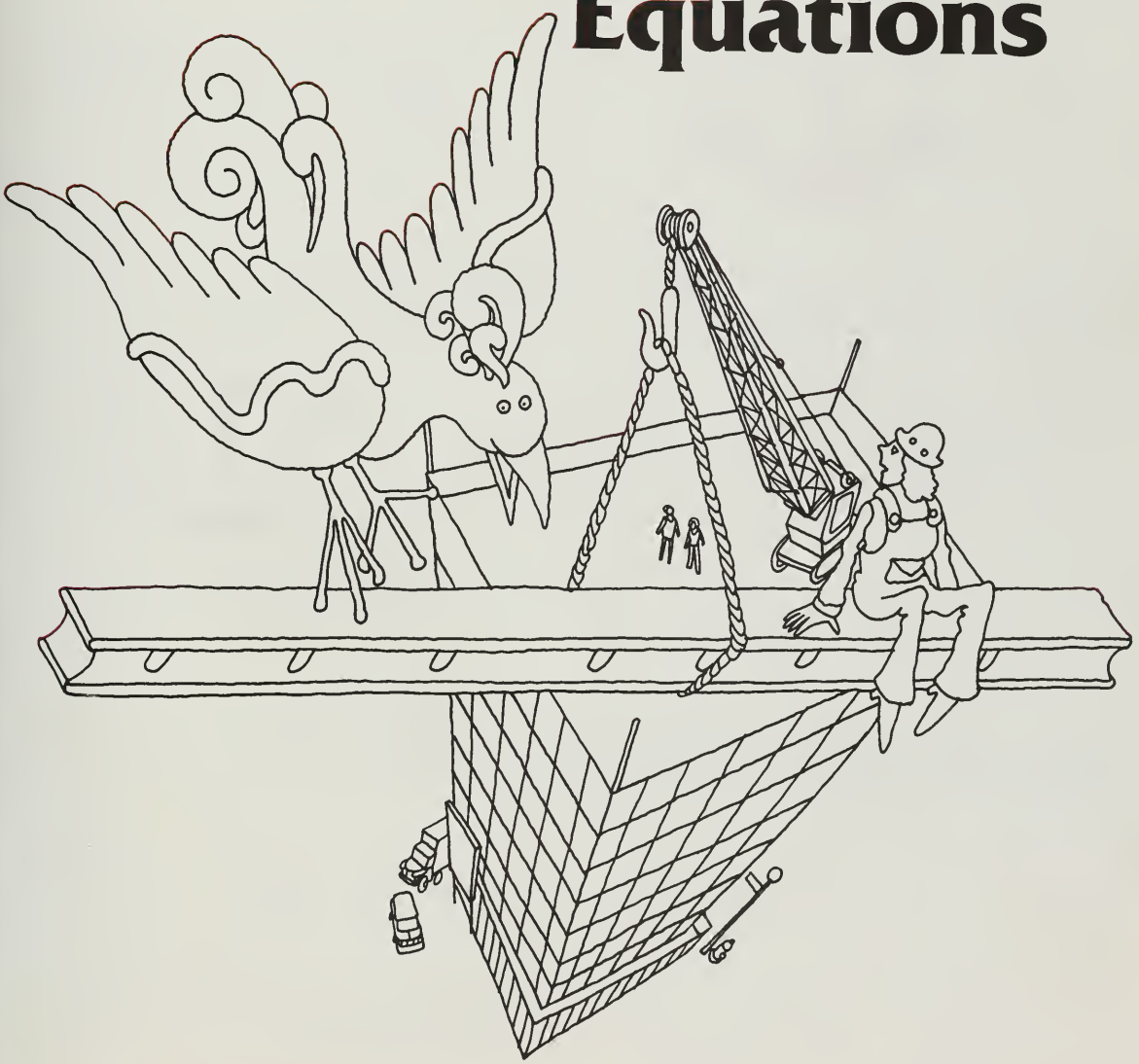
Here's what you'll learn in this chapter:

1. To solve equations with one variable.
2. To write algebraic expressions for everyday situations.
3. To use equations to solve word problems.



Chapter 2

Solving Equations



1 • Solving Equations by Addition



The figure at the left shows a balanced scale.



If you add the same weight to both sides, the scale will remain balanced.

You can use this idea with equations too. For example, the equation $x - 4 = 19$ is like a balanced scale.

If you add 4 to both sides, the equation still "balances."

$$x - 4 = 19$$
$$x - 4 + 4 = 19 + 4$$
$$x + 0 = 23$$
$$x = 23$$

You know that subtracting and adding 4 is like adding 0.



The basic plan in solving any equation is to get the variable by itself on one side of the equation. For the equations in this section, you do this by adding the same number to both sides of the equation.

EXAMPLE 1

$$x - 5 = 12$$
$$x - 5 + 5 = 12 + 5$$ ← Add 5 to both sides.
$$x = 17$$

EXAMPLE 2

$$50 = n - 8$$
$$50 + 8 = n - 8 + 8$$ ← Add 8 to both sides.
$$58 = n$$

Classroom Practice

You want to solve for x. Tell what must be added to both sides of the equation. Solve.

1. $x - 5 = 7$
 Add 5; 12
2. $x - 11 = 19$
 Add 11; 30
3. $x - 3 = 21$
 Add 3; 24
4. $x - 27 = 4$
 Add 27; 31
5. $x - 2 = 19$
 Add 2; 21
6. $x - 4 = 11$
 Add 4; 15
7. $9 = x - 6$
 Add 6; 15
8. $25 = x - 7$
 Add 7; 32

Written Exercises

Solve.

A

1. $x - 5 = 11$ 16
2. $x - 7 = 5$ 12
3. $x - 8 = 2$ 10
4. $y - 3 = 17$ 20
5. $x - 3 = 9$ 12
6. $y - 4 = 7$ 11
7. $a - 7 = 6$ 13
8. $y - 6 = 2$ 8
9. $b - 9 = 15$ 24
10. $a - 12 = 12$ 24
11. $x - 9 = 10$ 19
12. $x - 3 = 12$ 15
13. $y - 2 = 13$ 15
14. $x - 5 = 2$ 7
15. $y - 7 = 2$ 9
16. $a - 9 = 4$ 13
17. $n - 1 = 8$ 9
18. $x - 8 = 20$ 28
19. $x - 9 = 21$ 30
20. $x - 7 = 19$ 26
21. $x - 15 = 10$ 25
22. $a - 9 = 25$ 34
23. $6 = x - 2$ 8
24. $9 = y - 4$ 13
25. $10 = x - 3$ 13
26. $6 = x - 5$ 11
27. $0 = y - 5$ 5
28. $15 = x - 5$ 20
29. $21 = b - 14$ 35
30. $43 = x - 21$ 64
31. $36 = y - 15$ 51
32. $x - 52 = 100$ 152

Solve. Do a step in your head, if you can.

Sample

All steps

$$x - 5 = 8$$
$$x - 5 + 5 = 8 + 5$$
$$x = 13$$

Can you do this step in your head?

A step left out

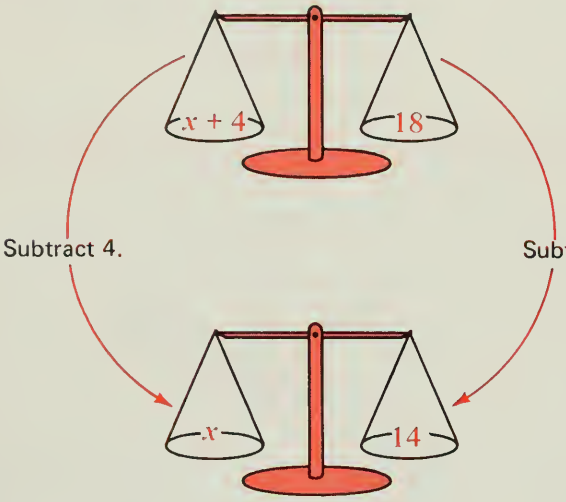
$$x - 5 = 8$$
$$x = 13$$

B

33. $x - 3 = 7$ 10
34. $x - 4 = 13$ 17
35. $x - 2 = 4$ 6
36. $y - 5 = 3$ 8
37. $y - 2 = 9$ 11
38. $n - 6 = 4$ 10
39. $n - 10 = 11$ 21
40. $x - 3 = 0$ 3
41. $a - 5 = 10$ 15
42. $x - 7 = 20$ 27
43. $7 = y - 2$ 9
44. $10 = x - 7$ 17
45. $12 = x - 3$ 15
46. $20 = n - 5$ 25
47. $22 = x - 3$ 25
48. $14 = y - 9$ 23

2 • Solving Equations by Subtraction

Suppose you have an equation like $x + 4 = 18$ to solve. You want to get the variable alone on one side of the equation. How do you do it? *Subtract* 4 from both sides.



$$\begin{aligned} x + 4 &= 18 \\ x + 4 - 4 &= 18 - 4 \\ x + 0 &= 14 \\ x &= 14 \end{aligned}$$

Here are some other examples.

EXAMPLE 1

$$\begin{aligned} x + 5 &= 28 \\ x + 5 - 5 &= 28 - 5 \\ x &= 23 \end{aligned}$$

← Subtract 5 from both sides.

EXAMPLE 2

$$\begin{aligned} 19 &= a + 7 \\ 19 - 7 &= a + 7 - 7 \\ 12 &= a \end{aligned}$$

← Subtract 7 from both sides.

Classroom Practice

You want to solve for x. Tell what to subtract from each side of the equation. Solve.

1. $x + 9 = 14$
Subtract 9; 5

2. $x + 3 = 7$
Subtract 3; 4

3. $x + 5 = 9$
Subtract 5; 4

4. $x + 2 = 12$
Subtract 2; 10

5. $x + 7 = 9$
Subtract 7; 2

6. $x + 4 = 34$
Subtract 4; 30

7. $20 = x + 11$
Subtract 11; 9

8. $60 = x + 45$
Subtract 45; 15

Written Exercises

Solve.

A

1. $x + 5 = 15$ 10	**2.** $x + 8 = 20$ 12	**3.** $x + 2 = 10$ 8	**4.** $a + 7 = 25$ 18
5. $y + 7 = 11$ 4	**6.** $n + 6 = 14$ 8	**7.** $n + 4 = 16$ 12	**8.** $y + 6 = 21$ 15
9. $x + 20 = 34$ 14	**10.** $8 + y = 15$ 7	**11.** $5 + x = 20$ 15	**12.** $28 + y = 42$ 14
13. $24 = x + 7$ 17	**14.** $30 = y + 8$ 22	**15.** $60 = b + 48$ 12	**16.** $88 = x + 7$ 81

Solve. Do a step in your head, if you wish.

Sample

$$x + 7 = 15$$
$$x + 7 - 7 = 15 - 7$$
$$x = 8$$

You might do this step in your head.

B

17. $x + 4 = 9$ 5	**18.** $x + 7 = 14$ 7	**19.** $y + 3 = 20$ 17	**20.** $n + 8 = 12$ 4
21. $x + 3 = 15$ 12	**22.** $y + 6 = 10$ 4	**23.** $n + 4 = 17$ 13	**24.** $y + 9 = 25$ 16
25. $6 + x = 16$ 10	**26.** $9 + y = 10$ 1	**27.** $8 + x = 21$ 13	**28.** $4 + x = 27$ 23
29. $18 = x + 3$ 15	**30.** $20 = x + 4$ 16	**31.** $14 = x + 6$ 8	**32.** $24 = x + 6$ 18

Mixed Practice Exercises

Solve.

1. $x - 2 = 7$ 9	**2.** $x - 5 = 4$ 9	**3.** $x - 9 = 9$ 18	**4.** $y - 7 = 3$ 10
5. $y - 6 = 5$ 11	**6.** $x - 4 = 6$ 10	**7.** $y - 7 = 11$ 18	**8.** $x - 2 = 14$ 16
9. $x + 5 = 11$ 6	**10.** $x + 6 = 13$ 7	**11.** $y + 7 = 12$ 5	**12.** $n + 2 = 17$ 15
13. $n + 8 = 14$ 6	**14.** $x + 3 = 22$ 19	**15.** $n + 9 = 26$ 17	**16.** $x + 12 = 19$ 7
17. $y - 2 = 17$ 19	**18.** $x - 12 = 42$ 54	**19.** $y + 6 = 35$ 29	**20.** $n + 17 = 31$ 14
21. $x - 6 = 27$ 33	**22.** $y + 7 = 40$ 33	**23.** $x - 18 = 12$ 30	**24.** $y + 9 = 37$ 28
25. $20 = 16 + x$ 4	**26.** $12 = x - 7$ 19	**27.** $15 = x - 2$ 17	**28.** $25 = y + 6$ 19
29. $36 = y + 9$ 27	**30.** $40 = x - 16$ 56	**31.** $29 = x - 17$ 46	**32.** $48 = n + 26$ 22

For more practice, see page 409.

3 • Solving More Equations

Solving a multiplication equation like $2x = 16$ should be no problem. All you do is *divide* both sides by 2.

This is the same as $\frac{2}{2}x$, or $1x$.

$1x = x$

$$2x = 16$$
$$\frac{2x}{2} = \frac{16}{2}$$
$$x = 8$$

This means $16 \div 2$.

EXAMPLES

$$3x = 21 \qquad\qquad 5x = 30$$
$$\frac{3x}{3} = \frac{21}{3} \qquad\qquad \frac{5x}{5} = \frac{30}{5}$$
$$x = 7 \qquad\qquad x = 6$$

You might do these steps in your head.

When you see a division equation like $\frac{x}{3} = 12$, you can solve it by *multiplying* each side by 3.

This gives you $\frac{3}{3}x$, or x.

$$\frac{x}{3} = 12$$
$$3 \cdot \frac{x}{3} = 3 \cdot 12$$
$$x = 36$$

EXAMPLES

$$\frac{x}{2} = 5 \qquad\qquad \frac{x}{4} = 6$$
$$2 \cdot \frac{x}{2} = 2 \cdot 5 \qquad\qquad 4 \cdot \frac{x}{4} = 4 \cdot 6$$
$$x = 10 \qquad\qquad x = 24$$

Classroom Practice

You want to solve for x. By what number would you divide each side of the equation? Solve.

1. $2x = 10$
Divide by 2; 5

2. $4x = 24$
Divide by 4; 6

3. $3x = 18$
Divide by 3; 6

4. $5x = 40$
Divide by 5; 8

5. $7x = 21$
Divide by 7; 3

6. $8x = 88$
Divide by 8; 11

7. $9x = 36$
Divide by 9; 4

8. $12x = 36$
Divide by 12; 3

You want to solve for y. By what number would you multiply each side of the equation? Solve.

9. $\frac{y}{2} = 8$ Mult. by 2; 16

10. $\frac{y}{5} = 9$ Mult. by 5; 45

11. $\frac{y}{4} = 3$ Mult. by 4; 12

12. $\frac{y}{6} = 5$ Mult. by 6; 30

13. $\frac{y}{3} = 6$ Mult. by 3; 18

14. $\frac{y}{8} = 7$ Mult. by 8; 56

15. $\frac{y}{10} = 4$ Mult. by 10; 40

16. $\frac{y}{7} = 12$ Mult. by 7; 84

Written Exercises

Solve. Do one step in your head, if you wish.

Sample $2x = 18$

$\frac{2x}{2} = \frac{18}{2}$ ← You might do this step in your head.

$x = 9$

A **1.** $3x = 12$ 4

2. $5x = 15$ 3

3. $6x = 42$ 7

4. $4a = 32$ 8

5. $6b = 72$ 12

6. $4y = 16$ 4

7. $7x = 35$ 5

8. $5x = 25$ 5

9. $2x = 20$ 10

10. $3x = 15$ 5

11. $6y = 18$ 3

12. $7y = 28$ 4

13. $8m = 24$ 3

14. $6x = 48$ 8

15. $7n = 49$ 7

16. $4a = 36$ 9

17. $9a = 81$ 9

18. $8x = 72$ 9

19. $9x = 108$ 12

20. $10a = 100$ 10

21. $7x = 91$ 13

22. $12a = 84$ 7

23. $8x = 112$ 14

24. $9b = 513$ 57

25. $3b = 540$ 180

26. $11n = 231$ 21

27. $7y = 175$ 25

28. $4m = 244$ 61

Solve. Do one step in your head, if you wish.

29. $\frac{x}{2} = 9$ 18

30. $\frac{x}{3} = 8$ 24

31. $\frac{x}{4} = 11$ 44

32. $\frac{x}{5} = 7$ 35

33. $\frac{a}{6} = 9$ 54

34. $\frac{a}{3} = 7$ 21

35. $\frac{n}{2} = 6$ 12

36. $\frac{x}{6} = 4$ 24

37. $\frac{n}{5} = 3$ 15

38. $\frac{x}{2} = 10$ 20

39. $\frac{n}{7} = 8$ 56

40. $\frac{x}{3} = 11$ 33

41. $\frac{x}{6} = 8$ 48

42. $\frac{a}{9} = 9$ 81

43. $\frac{a}{9} = 8$ 72

44. $\frac{x}{10} = 13$ 130

Mixed Practice Exercises

Think of an INPUT-OUTPUT machine. Tell what is missing in the table.

	Sample	1.	2.	3.	4.
INPUT	$x + 4 = 11$	$x + 2 = 13$	$7x = 14$	$9x = 45$	$x - 6 = 7$
↓ Operation	Subtract 4 from both sides.	Subtract 2 from both sides.	Divide both sides by 7.	Divide both sides by 9.	Add 6 to both sides.
↓ OUTPUT	$x = 7$	? 11	? 2	? 5	? 13

	5.	6.	7.	8.	9.
INPUT	$x + 6 = 10$	$y - 4 = 8$	$5x = 35$	$8y = 32$	$x - 7 = 13$
↓ Operation	Subtract 6 from both sides.	Add 4 to both sides.	Divide both sides by 5.	Divide both sides by 8.	Add 7 to both sides.
↓ OUTPUT	? 4	? 12	? 7	? 4	? 20

	10.	11.	12.	13.	14.
INPUT	$y - 3 = 11$	$x + 6 = 12$	$7x = 84$	$x - 8 = 15$	$n + 7 = 22$
↓ Operation	Add 3 to both sides.	Subtract 6 from both sides.	Divide both sides by 7.	Add 8 to both sides.	Subtract 7 from both sides.
↓ OUTPUT	? 14	? 6	? 12	? 23	? 15

Solve.

15. $x - 4 = 10$ 14 **16.** $y + 2 = 7$ 5 **17.** $x + 4 = 10$ 6 **18.** $y - 6 = 3$ 9

19. $n - 7 = 4$ 11 **20.** $n + 8 = 17$ 9 **21.** $x - 7 = 14$ 21 **22.** $y + 9 = 11$ 2

23. $x - 9 = 12$ 21 **24.** $4x = 20$ 5 **25.** $8y = 16$ 2 **26.** $n + 6 = 32$ 26

27. $\frac{a}{7} = 3$ 21 **28.** $\frac{x}{4} = 9$ 36 **29.** $\frac{n}{5} = 2$ 10 **30.** $\frac{n}{8} = 9$ 72

31. $8 + y = 13$ 5 **32.** $a - 6 = 18$ 24 **33.** $x + 12 = 26$ 14 **34.** $y + 17 = 30$ 13

35. $a - 15 = 12$ 27 **36.** $12x = 48$ 4 **37.** $m - 14 = 37$ 51 **38.** $n + 36 = 52$ 16

39. $\frac{x}{6} = 3$ 18 **40.** $\frac{n}{8} = 1$ 8 **41.** $\frac{x}{5} = 6$ 30 **42.** $\frac{a}{7} = 9$ 63

43. $y + 10 = 15$ 5 **44.** $x - 3 = 20$ 23 **45.** $9x = 27$ 3 **46.** $7n = 63$ 9

47. $3y = 63$ 21 **48.** $7x = 42$ 6 **49.** $5a = 100$ 20 **50.** $9x = 90$ 10

51. $7 + x = 20$ 13 **52.** $a - 12 = 14$ 26 **53.** $x + 9 = 28$ 19 **54.** $x - 15 = 22$ 37

55. $x - 3 = 18$ 21 **56.** $y + 7 = 26$ 19 **57.** $3x = 57$ 19 **58.** $2n = 104$ 52

59. $35 = x + 7$ 28 **60.** $29 = y - 6$ 35 **61.** $54 = 9x$ 6 **62.** $7 = 7x$ 1

For more practice, see page 409.

Puzzles & Things

What's wrong here?



A glass $\frac{1}{2}$ full = A glass $\frac{1}{2}$ empty

Multiply by 2. Multiply by 2.

A glass full = A glass empty

See page 2 of Answers to Odd-Numbered Exercises in the back of the book.

SELF-TEST

Solve.

1. $x - 3 = 19$ 22 **2.** $a - 14 = 6$ 20 **3.** $m - 4 = 17$ 21

4. $m + 2 = 15$ 13 **5.** $n + 7 = 18$ 11 **6.** $x + 9 = 21$ 12

7. $5 = n - 5$ 10 **8.** $17 = s + 7$ 10 **9.** $13 = b - 10$ 23

10. $5b = 45$ 9 **11.** $9x = 63$ 7 **12.** $7a = 56$ 8

13. $4x = 12$ 3 **14.** $8n = 96$ 12 **15.** $12x = 132$ 11

16. $\frac{n}{3} = 5$ 15 **17.** $\frac{a}{6} = 7$ 42 **18.** $\frac{x}{11} = 7$ 77

4 • Equations with More Steps

In the last sections you studied equations that needed just one operation to solve. In this section you'll learn to solve equations which need more steps to solve.



$$2x - 5 = 3$$

You'll want to get x alone on one side of the equation. First get rid of the 5. Then get rid of the 2.

$$\begin{aligned} 2x - 5 &= 3 \\ 2x - 5 + 5 &= 3 + 5 \quad \leftarrow \text{Add 5 to both sides.} \\ 2x &= 8 \\ \frac{2x}{2} &= \frac{8}{2} \quad \leftarrow \text{Next divide both sides by 2.} \\ x &= 4 \end{aligned}$$

It's easy if you just take one step at a time.

EXAMPLE 1

$$\begin{aligned} 3x + 7 &= 19 \\ 3x + 7 - 7 &= 19 - 7 \quad \leftarrow \text{First subtract 7 from both sides.} \\ 3x &= 12 \\ x &= 4 \quad \leftarrow \text{Divide both sides by 3 to get } x = 4. \end{aligned}$$

EXAMPLE 2

$$\begin{aligned} 6 + 2x &= 10 \\ 6 - 6 + 2x &= 10 - 6 \quad \leftarrow \text{Subtract 6 from both sides.} \\ 2x &= 4 \\ x &= 2 \quad \leftarrow \text{Divide both sides by 2 to get } x = 2. \end{aligned}$$

Classroom Practice

Tell what steps you would take to solve the equation.

Sample $2x - 5 = 9$ *What you say:* Add 5 to both sides. Then divide both sides by 2.

1. $2x - 4 = 16$
Add 4; div. by 2.

2. $5x - 2 = 8$
Add 2; div. by 5.

3. $2x - 1 = 9$
Add 1; div. by 2.

4. $7x - 2 = 12$
Add 2; div. by 7.

5. $2a + 4 = 12$
Sub. 4; div. by 2.

6. $3b + 5 = 5$
Sub. 5; div. by 3.

7. $4s + 7 = 11$
Sub. 7; div. by 4.

8. $8p + 3 = 11$
Sub. 3; div. by 8.

Solve.

9. $2a - 3 = 3$ 3 **10.** $2m + 1 = 9$ 4 **11.** $3x - 4 = 11$ 5 **12.** $5b - 4 = 6$ 2

13. $3r + 2 = 11$ 3 **14.** $4y + 2 = 14$ 3 **15.** $5a + 1 = 36$ 7 **16.** $2x - 9 = 3$ 6

17. $4b - 3 = 13$ 4 **18.** $6x - 1 = 23$ 4 **19.** $8m + 2 = 10$ 1 **20.** $6y + 3 = 33$ 5

Written Exercises

Think of two INPUT-OUTPUT machines. Tell what is missing in the table.

A

	Sample	1.	2.	3.	4.
INPUT	$2x - 1 = 5$	$5x - 3 = 7$	$2a + 1 = 3$	$3b + 5 = 8$	$4y + 1 = 9$
↓ First Operation	Add 1 to both sides.	Add 3 to both sides.	Subtract 1 from both sides.	Subtract 5 from both sides.	Subtract 1 from both sides.
OUTPUT	$2x = 6$	$5x \; ? = 10$	$2a \; ? = 2$	$3b \; ? = 3$	$4y \; ? = 8$
↓ Second Operation	Divide both sides by 2.	Divide both sides by 5.	Divide both sides by 2.	Divide both sides by 3.	Divide both sides by 4.
OUTPUT	$x = 3$	$?x = 2$	$?a = 1$	$?b = 1$	$?y = 2$

	5.	6.	7.	8.	9.
INPUT	$2a - 5 = 13$	$5b - 9 = 31$	$4c + 3 = 15$	$8a + 7 = 55$	$6n - 3 = 15$
↓ First Operation	Add 5 to both sides.	Add 9 to both sides.	Subtract 3 from both sides.	Subtract 7 from both sides.	Add 3 to both sides.
OUTPUT	$2a \; ? = 18$	$5b \; ? = 40$	$4c \; ? = 12$	$8a \; ? = 48$	$6n \; ? = 18$
↓ Second Operation	Divide both sides by 2.	Divide both sides by 5.	Divide both sides by 4.	Divide both sides by 8.	Divide both sides by 6.
OUTPUT	$?a = 9$	$?b = 8$	$?c = 3$	$?a = 6$	$?n = 3$

Solve.

10. $2x - 4 = 0$ 2 **11.** $5x - 8 = 7$ 3 **12.** $2x + 1 = 5$ 2

13. $5x - 1 = 14$ 3 **14.** $3x - 2 = 10$ 4 **15.** $2x - 1 = 7$ 4

16. $3x - 5 = 4$ 3

17. $4x - 7 = 1$ 2

18. $2x - 4 = 8$ 6

19. $4x - 8 = 12$ 5

20. $2x - 5 = 1$ 3

21. $4x - 5 = 15$ 5

22. $2x + 5 = 11$ 3

23. $3x + 4 = 22$ 6

24. $4x + 1 = 1$ 0

25. $4y + 7 = 27$ 5

26. $3y + 2 = 8$ 2

27. $2a + 5 = 25$ 10

28. $5n - 1 = 9$ 2

29. $4n - 7 = 13$ 5

30. $2y + 6 = 22$ 8

31. $4y + 7 = 15$ 2

32. $6y + 4 = 34$ 5

33. $8 + 3r = 23$ 5

34. $2x - 3 = 5$ 4

35. $8y + 9 = 33$ 3

36. $2a - 5 = 11$ 8

37. $6s - 1 = 17$ 3

38. $3c - 7 = 14$ 7

39. $3t - 8 = 13$ 7

40. $7y + 2 = 23$ 3

41. $5b - 9 = 21$ 6

42. $6r - 3 = 21$ 4

43. $4c + 1 = 13$ 3

44. $8a + 5 = 53$ 6

45. $8x - 5 = 35$ 5

46. $3y + 11 = 41$ 10

47. $4x - 5 = 23$ 7

48. $8a - 5 = 75$ 10

49. $4b + 6 = 30$ 6

50. $3x - 5 = 28$ 11

51. $3b - 12 = 45$ 19

52. $7s - 9 = 68$ 11

53. $8y + 5 = 37$ 4

54. $8t - 14 = 26$ 5

Write each sentence as an equation. Then solve.

Sample 7 more than twice a number is 15.

$7 + 2n = 15$

$2n = 8$

$n = 4$

B 55. 8 more than twice a number is 32.
$2n + 8 = 32$; 12

56. 7 more than 3 times a number is 25.
$3n + 7 = 25$; 6

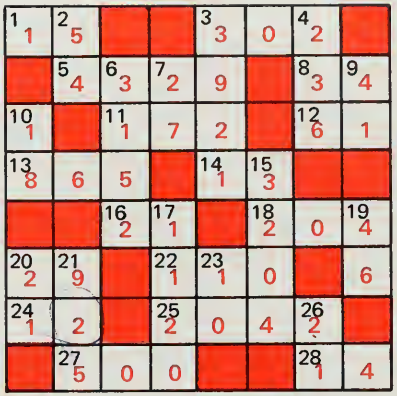
57. Twice a number increased by 4 is 16.
$2n + 4 = 16$; 6

58. Five times a number decreased by 17 is 53.
$5n - 17 = 53$; 14

59. When 5 is added to 4 times a number, the result is 17.
$4n + 5 = 17$; 3

60. When 6 is subtracted from 3 times a number, the result is 21.
$3n - 6 = 21$; 9

C **Can you solve this cross-number puzzle?**



Across

1. $x + 7 = 22$

3. $m - 6 = 296$

5. $w - 34 = 4295$

8. $a + 15 = 49$

11. $\frac{x}{2} = 86$

12. $52 = n - 9$

13. $b + 9 = 874$

14. $3x = 39$

16. $5x = 105$

18. $\frac{w}{4} = 51$

20. $2m - 19 = 39$

22. $3a + 11 = 341$

24. $5y + 9 = 69$

25. $n - 50 = 1992$

27. $3x - 21 = 1479$

28. $9b + 24 = 150$

Down

2. $b + 5 = 59$

3. $x - 12 = 3909$

4. $m + 6 = 242$

6. $\frac{a}{8} = 394$

7. $n + 5 = 32$

9. $3x = 123$

10. $2y = 36$

15. $b - 56 = 3148$

17. $2a + 9 = 2249$

19. $4x - 9 = 175$

20. $3m + 4 = 67$

21. $\frac{n}{25} = 37$

23. $9y - 47 = 43$

26. $6m + 11 = 137$

For more practice, see page 410.

SELF-TEST

Solve.

1. $3x - 6 = 0$ 2

2. $4n - 5 = 7$ 3

3. $2a + 3 = 11$ 4

4. $5r + 4 = 34$ 6

5. $3t - 4 = 17$ 7

6. $6x - 1 = 35$ 6

7. $7s + 3 = 66$ 9

8. $5a + 4 = 59$ 11

9. $9n - 4 = 32$ 4

10. $4t - 15 = 13$ 7

11. $2r - 9 = 31$ 20

12. $6x + 5 = 35$ 5

5 • A Mind-Reading Trick



Here's a trick to play on a friend. You will ask your friend to think of a number, then give some steps to compute. Then you'll ask for the result. Like magic you'll be able to find your friend's original number!

		This is how your friend's arithmetic might go.	This is your algebra.
Step 1:	Think of a whole number.	4	n
Step 2:	Add 8.	12	$n + 8$
Step 3:	Multiply by 5.	60	$5(n + 8)$ *or* $5n + 40$
Step 4:	Subtract 37.	23	$5n + 3$
Step 5:	Multiply by 20.	460	$20(5n + 3)$ *or* $100n + 60$
Step 6:	Tell me your answer.	460	$100n + 60$

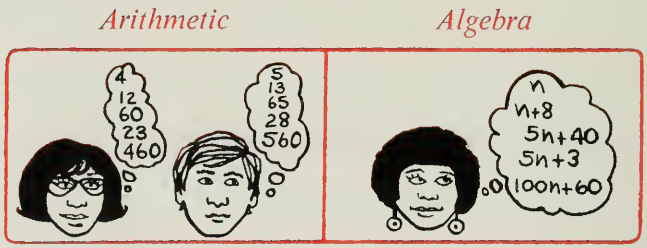
To find your friend's original number, solve.

$$460 = 100n + 60$$
$$400 = 100n$$
$$4 = n$$

This is it! →

COMPARING ALGEBRA AND ARITHMETIC

1. Algebra is more powerful than arithmetic.



2. Algebra is often easier than arithmetic.



Written Exercises

Copy and complete.

A **1.**

Steps	Arithmetic	Algebra
Think of a number.	8	n
Multiply it by 5.	? 40	$5n$?
Add 4.	? 44	$5n$? + 4
Multiply by 20.	? 880	$100n$? + 80
Subtract 80.	? 800	$100n$?

2.

Steps	Arithmetic	Algebra
Think of a number.	6	n
Add 9.	? 15	n ? + 9
Multiply by 4.	? 60	$4n$? + 36
Subtract 30.	? 30	$4n$? + 6
Multiply by 25.	? 750	$100n$? + 150

Using algebra, find the original number.

3.

Steps
Think of a number.
Add 8. $n + 8$
Multiply by 2. $2n + 16$
Subtract 6. $2n + 10$
Divide by 2. $n + 5$
Subtract 5. n

Result: 9
Original number: ? 9

4.

Steps
Think of a number.
Add 5. $n + 5$
Multiply by 2. $2n + 10$
Subtract 6. $2n + 4$
Multiply by 5. $10n + 20$
Subtract 4. $10n + 16$

Result: 56
Original number: ? 4

5.

Steps
Think of a number.
Multiply by 5. $5n$
Add 6. $5n + 6$
Multiply by 4. $20n + 24$
Add 9. $20n + 33$
Multiply by 5. $100n + 165$

Result: 1365
Original number: ?12

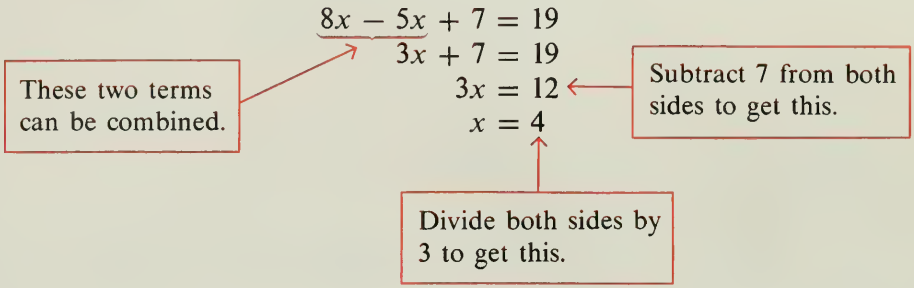
6. Make up a mind-reading trick of your own.

B **7.** Try this mind-reading trick on a friend. The steps are given at the right. After all the steps are computed, tell your friend that the final answer is 20. It will always be 20. Why?
$n + 20 - n$ always equals 20.

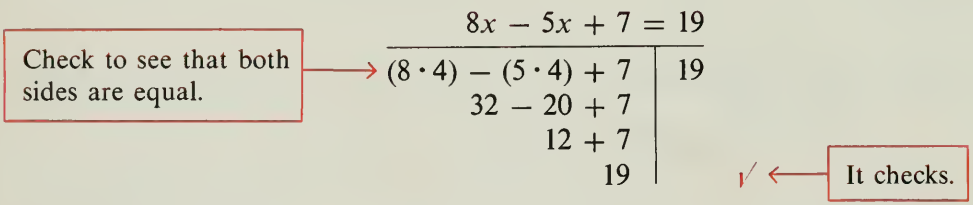
Steps
Think of a number. n
Add 10. $n + 10$
Multiply by 2. $2n + 20$
Add 5. $2n + 25$
Multiply by 4. $8n + 100$
Add 60. $8n + 160$
Divide by 8. $n + 20$
Subtract the original number. 20

6 • Combining Terms

Sometimes you have to combine like terms to solve an equation.



It is a good idea to check your work. Substitute your solution, 4, for x in the equation.



Written Exercises

Solve. Check.

A

1. $4a + 9a - 6a = 42$ 6 **2.** $6y - 4y + y = 18$ 6 **3.** $9c + 4c - c = 48$ 4

4. $9 + 6a - 2a = 21$ 3 **5.** $8a + 7 - 5a = 31$ 8 **6.** $9a - 4 + 3a = 32$ 3

7. $13 + 5b - 2b = 37$ 8 **8.** $8b + 4b - b = 55$ 5 **9.** $12c - 8 - 5c = 27$ 5

10. $8x + 4 - 3x = 24$ 4 **11.** $7n + 2 + 3n = 12$ 1 **12.** $13x + 5 - 4x = 23$ 2

13. $4x + 5x - 3 = 15$ 2 **14.** $4x - x + 1 = 22$ 7 **15.** $7x + 3x - 5 = 15$ 2

16. $12 = 3c - 12 + 5c$ 3 **17.** $9 = 13x - 7 - 5x$ 2 **18.** $35 = 19m - 4m + 5$ 2

19. $18 = 8y - 7 - 3y$ 5 **20.** $21 = 4a - 3a + 6a$ 3 **21.** $16 = 3x + 2x - 9$ 5

22. $21 = 4n - 3 - n$ 8 **23.** $32 = 8x + 4 - 4x$ 7 **24.** $7 = 3a + 5a - a + 7$ 0

Write an equation. Solve.

Sample Twice a number plus 3 times the number is 55.

$$\underbrace{2x}_{\text{twice a number}} + \underbrace{3x}_{\text{3 times the number}} = 55$$

$$5x = 55$$

$$x = 11$$

25. Four times a number plus 3 times the same number is 63. What is the number? $4x + 3x = 63$; 9

26. The difference between $7y$ and $3y$ is 20. Find the value of y. $7y - 3y = 20$; 5

27. A number is doubled, then 5 is added. The result is 73. Find the number. $2x + 5 = 73$; 34

28. A number is multiplied by 3 and then 7 is subtracted. The result is 83. Find the number. $3x - 7 = 83$; 30

B **29.** I am thinking of a number. When I double it and add 9, I get 25. What is the number? $2x + 9 = 25$; 8

30. I am thinking of a number. When I multiply it by 3 and subtract 11, I get 37. What is the number? $3x - 11 = 37$; 16

31. Eight more than twice a number is 24. What is the number? $2x + 8 = 24$; 8

For more practice, see page 410.

SELF-TEST

Solve. Check.

1. $x + 3x - 2x = 12$ 6

2. $4a - 2a + a = 42$ 14

3. $s - 3 + 4s = 12$ 3

4. $8 = n + 3n - 24$ 8

5. $46 = 5x + x + 16$ 5

6. $9a + 2 - 6a = 35$ 11

Write an equation. Solve.

7. Three times a number plus five times the same number is 40. What is the number? $3x + 5x = 40$; 5

8. A number is doubled, then 9 is added. The result is 23. What is the number? $2x + 9 = 23$; 7

7 • Writing Algebraic Expressions

Before you can use algebra to solve real-life problems you will need practice in using variables to translate sentences into equations.



The Sox won 15 more games than they lost.
Let x be the number of games lost.
Write an expression to show the number of games won.

The number won is 15 more than the number lost.

The number won $=$ 15 $+$ x

So, the number won is $15 + x$.

Here are some more examples.

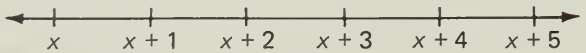
EXAMPLE 1 Joanna has twice as much money as Barb.
Let x be Barb's amount of money.
Then $\underline{?}$ is Joanna's amount.

Joanna's amount is twice Barb's amount.

Joanna's amount $=$ 2 $\cdot$ x

So, Joanna's amount $= 2x$.

EXAMPLE 2 Let x be a whole number.
Then the next two whole numbers are $\underline{?}$ and $\underline{?}$.



The next two whole numbers are $x + 1$ and $x + 2$.

Classroom Practice



1. The Cardinals won 6 more games than they lost.
 Let x = number of games lost.
 Then $\underset{x+6}{?}$ = number of games won.

2. The Tigers had twice as many hits as the Yanks.
 Let x = number of hits by the Yanks.
 Then $\underset{2x}{?}$ = number of hits by the Tigers.

3. The length of a rectangle is 3 units more than the width.
 Let x = the width.
 Then $\underset{x+3}{?}$ = the length.

4. Jo is 5 cm taller than Kathy.
 Let k = Kathy's height.
 Then $\underset{k+5}{?}$ = Jo's height.

5. Jo is 5 cm taller than Kathy.
 Let j = Jo's height.
 Then $\underset{j-5}{?}$ = Kathy's height.

6. Alma is 3 years older than Liz.
 Let l = Liz's age.
 Then $\underset{l+3}{?}$ = Alma's age.

7. Alma is 3 years older than Liz.
 Let a = Alma's age.
 Then $\underset{a-3}{?}$ = Liz's age.

8. Howard strikes out 5 times as often as Lucy.
 And Lucy strikes out twice as often as Raymond.
 Let K = number of times Raymond strikes out.
 Then $\underset{2k}{?}$ = number of times Lucy strikes out,
 and $\underset{10k}{?}$ = number of times Howard strikes out.

Written Exercises

A

1. Ted is 5 years older than Ned.
 Let n = Ned's age.
 Then $\underset{n+5}{?}$ = Ted's age.

2. Ted is 5 years older than Ned.
 Let t = Ted's age.
 Then $\underset{t-5}{?}$ = Ned's age.

3. Rose is 2 cm taller than Joan.
 Let j = Joan's height.
 Then $\underset{j+2}{?}$ = Rose's height.

4. Rose is 2 cm taller than Joan.
 Let r = Rose's height.
 Then $\underset{r-2}{?}$ = Joan's height.

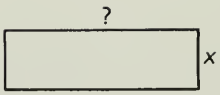
5. There are 5 more girls here than boys.
Let b = number of boys.
Then $\underset{b+5}{?}$ = number of girls.

6. There are 5 more girls here than boys.
Let g = number of girls.
Then $\underset{g-5}{?}$ = number of boys.

7. Pete has three times as much money as Mike.
Let x = Mike's amount of money.
Then $\underset{3x}{?}$ = Pete's amount of money.

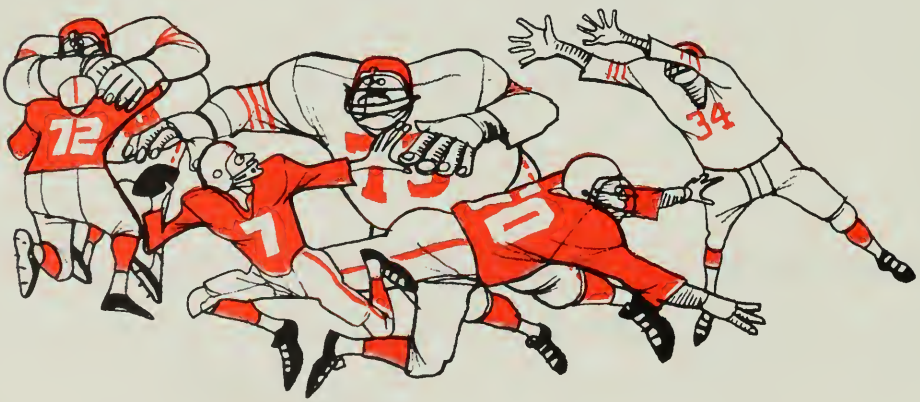
8. There are 200 more students than teachers.
Let x = number of teachers.
Then $\underset{x+200}{?}$ = number of students.

9. The length of a rectangle is four times the width.
Let x = the width.
Then $\underset{4x}{?}$ = the length.



10. The temperature today is 5° higher than yesterday.
Let t = yesterday's temperature.
Then $\underset{t+5}{?}$ = today's temperature.

11. The Trojans scored 3 times as many touchdowns as the Warriors.
Let x = number of touchdowns scored by the Warriors.
Then $\underset{3x}{?}$ = number of touchdowns scored by the Trojans.



12. The Rams have won 4 more games than they have lost.
Let n = number of games lost.
Then $\underset{n+4}{?}$ = number of games won.

13. Sam is x years old.
How old will he be next year?
$x + 1$

14. Judy is x years old.
How old will she be in 4 years?
$x + 4$

15. Roger is r years old.
How old was he last year?
$r - 1$

16. Kate is k years old.
How old was she 3 years ago?
$k - 3$

B **17.** Al has \$5 more than Ben.
Ben has \$6 more than Carl.
Let x = Carl's amount of money.
Then ___?___ = Ben's amount, $x + 6$
and ___?___ = Al's amount. $x + 11$

18. Denny is 3 cm taller than Ed.
Franco is 2 cm shorter than Ed.
Let x = Ed's height in cm.
Then ___?___ = Denny's height, $x + 3$
and ___?___ = Franco's height. $x - 2$

19. Cindy is 2 years younger than Dale.
Let c = Cindy's age.
Then ___?___ = Dale's age. $c + 2$
Next year, Cindy's age will be ___?___ ($c + 1$) and Dale's age will be ___?___ ($c + 3$).

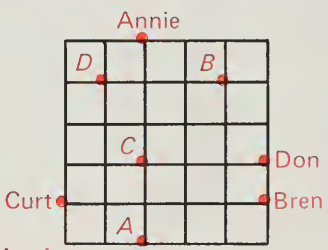
20. Let n = an odd number.
Then the next two odd numbers are ___?___ ($n + 2$) and ___?___ ($n + 4$).

21. Let x = an even number.
The next two even numbers are ___?___ ($x + 2$) and ___?___ ($x + 4$).

22. Frances won one game more than Keith.
Will won twice as many as Frances.
Let k = the number of games Keith won.
Then ___?___ = the number Frances won, $k + 1$
and ___?___ = the number Will won. $2(k + 1)$ or $2k + 2$

Puzzles & Things

This is a street map of a certain town. Annie lives at A, Bren lives at B, Curt lives at C, and Don lives at D. How can each person get home without running into another person's path? (They must walk along the streets. No shortcuts!)



Answers may vary. See page 2 of Answers to Odd-Numbered Exercises in the back of the book.

8 • Applied Problems and Puzzles

Now you can use what you have just learned to solve problems.



A basketball team played 35 games.
They won 7 more games than they lost.
How many games did they win and lose?

Let x = number of games lost.
Then $x + 7$ = number of games won.

number lost + number won = 35

$$x + (x + 7) = 35$$
$$2x + 7 = 35$$
$$2x = 28$$
$$x = 14$$

Answer: $x = 14 \longrightarrow$ 14 games lost
$x + 7 = 21 \longrightarrow$ 21 games won

Check: games lost = 14
games won = 21
total games = 35 ✓

A Guide for Problem Solving

1. Read the problem. Read it more than once if you wish.
2. Let a variable stand for one of the unknowns. Show the other unknowns in terms of that variable.
3. Write an equation and solve it.
4. Answer the question.
5. Check your answer.

EXAMPLE

1. *Read the problem.*
 Al has twice as much money as Chip.
 Together they have \$78.
 How much money does each have?

2. Let x = amount Chip has.
 Then $2x$ = amount Al has.

3. Chip's amount + Al's amount = \$78

$$\begin{aligned} x \quad + \quad 2x &= 78 \\ 3x &= 78 \\ x &= 26 \end{aligned}$$

4. *Answer:* $x = 26 \longrightarrow$ Chip has \$26.
 $2x = 52 \longrightarrow$ Al has \$52.

5. *Check:* $26 + 52 = 78$ ✓

Classroom Practice

1. There are 15 more girls than boys in a summer camp.
 There are 135 campers in all.
 How many boys and how many girls are there? **60 boys, 75 girls**

 let b = boys
 $b+15$ = girls
 $b + b + 15 = 135$

2. Janet has three times as much money as Peter.
 Together they have \$40.
 How much money does each have? **Peter has \$10. Janet has \$30.**

 let P = Peter's \$
 $3P$ = Janet's \$
 $P + 3P = \$40$
 $4P = 40$
 $P = 10$

Written Exercises

A

1. There are 800 students in Central School.
 There are 32 more girls than boys.
 How many boys are there? **384 boys**

 let b = boys
 $b+32$ = girls
 $b + b + 32 = 800$

2. Carmen's golf score is 6 strokes less than Linda's.
 Their two scores total 168.
 What is each girl's score? **Linda: 87; Carmen: 81**

 L = Linda
 $L - 6$ = Carmen
 $L + L - 6 = 168$

3. South High School has 85 students more than North High School.
The total number for both schools is 1495.
How many students are at South High? 790 students



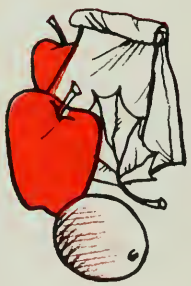
4. The Flyers played 78 games.
They won 16 more than they lost.
How many games did they win and lose?
won 47; lost 31

5. A baseball team played 23 games.
They won 5 games more than they lost.
How many games did they win? 14 games

6. Luis has $48 more than Ed.
Together they have $200.
How much money does each man have?
Ed: $76; Luis: $124

7. A hamburger has twice as many calories as a cup of soup.
Together they have 495 calories.
How many calories are in each?
hamburger: 330 calories; soup: 165 calories

8. An apple has twice as many calories as a peach.
The two fruits together have 105 calories.
How many calories are in each fruit?
peach: 35 calories; apple: 70 calories



9. The A's played 153 ball games.
They won twice as many as they lost.
How many did they win? 102 games

10. The Bears scored a total of 63 points in their first two games.
They scored twice as many points in the second game as in the first.
How many points did they score in their first game? 21 points

Consecutive numbers are numbers which follow each other in order. Solve these problems about consecutive numbers.

11. Find two consecutive whole numbers which total 75. 37, 38

12. Find three consecutive whole numbers which total 75. 24, 25, 26

B 13. Book *B* is twice as long as book *A*.
Book *C* is 30 pages longer than book *B*.
Together the three books have 280 pages.
How long is book *C*? 130 pages

14. There were 90 people at a party.
There were 4 more men than women.
There were 10 more children than adults.
How many children were there? 50 children

C **15.** "We made $150 profit," said Liz. "How do we split it up?"
"Well," said Sue, "You worked twice as long as I did,
and Wendy worked three times as long as I."
How much should each girl get? Sue: $25; Liz: $50; Wendy: $75



The Impollutable Pogo (Chapter 6, "Uncle Beanfellow Is Dead Sure"), © 1970, by Walt Kelly. Reprinted by permission of Simon & Schuster, Inc.

For more practice, see page 410.

SELF-TEST

1. Jan is 3 cm shorter than Tom.
Let m = Jan's height.
Then $\underset{m+3}{?}$ = Tom's height.

2. Vera is 3 years older than Len.
Let x = Len's age.
Then $\underset{x+3}{?}$ = Vera's age.

3. Peg has $37 more than Hal.
Together they have $175.
How much does Hal have? $69

4. The Reds played 162 games.
They won twice as many as they lost.
How many games did they win? 108 games

9 • Variable on Both Sides of the Equation

In this section we shall consider equations with the variable on both sides. You'll use several steps to get the variable alone on one side of the equation.

$$\begin{aligned} 8x &= 10 + 3x \\ 8x - 3x &= 10 + 3x - 3x \\ 5x &= 10 \\ x &= 2 \end{aligned}$$

Check:

$8x = 10 + 3x$	
$8 \cdot 2$	$10 + 3 \cdot 2$
16	$10 + 6$
	16 ✓

Sometimes it's easier to get the variable alone on the right side of the equation.

$$\begin{aligned} 15 - 4x &= x \\ 15 - 4x + 4x &= x + 4x \\ 15 &= 5x \\ 3 &= x \end{aligned}$$

Check:

$15 - 4x = x$	
$15 - 4 \cdot 3$	3
$15 - 12$	
3	✓

Sometimes you may want to skip some steps when solving an equation. Remember to skip *only* those steps that are easy for you to do in your head.

All steps given

$$\begin{aligned} 6x - 7 &= 4x + 13 \\ 6x - 7 + 7 &= 4x + 13 + 7 \\ 6x &= 4x + 20 \\ 6x - 4x &= 4x + 20 - 4x \\ 2x &= 20 \\ x &= 10 \end{aligned}$$

Some steps skipped

$$\begin{aligned} 6x - 7 &= 4x + 13 \\ 6x &= 4x + 20 \\ 2x &= 20 \\ x &= 10 \end{aligned}$$

Classroom Practice

Tell what steps you would take to solve the equation.

Sample $6x = 8 + 2x$ *What you say:* Subtract $2x$ from both sides, then divide both sides by 4.

1. $5x = 8 + 3x$
Sub. $3x$, div. by 2.

2. $8x = 2x + 6$
Sub. $2x$, div. by 6.

3. $4y = 2y + 10$
Sub. $2y$, div. by 2.

4. $5 - 3y = 2y$
Add $3y$, div. by 5.

5. $12 - a = 5a$
Add a, div. by 6.

6. $6 + 3b = 5b$
Sub. $3b$, div. by 2.

7. $2a + 4 = 4a$
Sub. $2a$, div. by 2.

8. $4c + 4 = 5c$
Sub. $4c$.

Written Exercises

Solve.

A

1. $2x = 14 + x$ 14
2. $7x = 2x + 5$ 1
3. $9x = 5x + 16$ 4
4. $6y = 18 - 3y$ 2
5. $4n = 21 - 3n$ 3
6. $6x = 20 - 4x$ 2
7. $8x = 25 + 3x$ 5
8. $9x = 26 - 4x$ 2
9. $12x = 10 + 7x$ 2
10. $6 - 2x = x$ 2
11. $16 - 3x = x$ 4
12. $20 - 6a = 4a$ 2
13. $7 - 3x = 4x$ 1
14. $10 + x = 6x$ 2
15. $12 + 3y = 9y$ 2
16. $16 + 5a = 13a$ 2
17. $14 - 6y = 8y$ 1
18. $25 - 2n = 3n$ 5
19. $6y - 3 = 3y$ 1
20. $4x - 2 = 3x$ 2
21. $8n - 6 = 5n$ 2
22. $5x - 9 = 2x$ 3
23. $12n - 20 = 2n$ 2
24. $15x - 12 = 12x$ 4
25. $6x - 5 = 4x + 9$ 7
26. $13x - 7 = 5x + 1$ 1
27. $8a + 6 = 3a + 21$ 3
28. $4x + 4 = 2x + 6$ 1
29. $7a - 3 = 3a + 5$ 2
30. $10n - 8 = 3n - 1$ 1
31. $5b + 8 = 3b + 26$ 9
32. $18 + 3x = x + 30$ 6
33. $7 - 4x = 5x - 2$ 1
34. $5y + 9 = 8y - 15$ 8
35. $7c + 9 = 11c - 31$ 10
36. $8a - 5 = 3a + 25$ 6

Write an equation. Solve for the variable.

Sample 5 times a number is 12 more than 2 times the number.

$$\begin{aligned} 5n &= 12 + 2n \\ 5n - 2n &= 12 + 2n - 2n \\ 3n &= 12 \\ n &= 4 \end{aligned}$$

B

37. 7 times a number is 12 more than 3 times the number. $7n = 3n + 12$; 3
38. 4 times a number is 18 more than the number. $4n = n + 18$; 6
39. If you double a number and add 24, the result is 8 times the number. $2n + 24 = 8n$; 4
40. If you subtract a certain number from 30, the result is 4 times the number. $30 - n = 4n$; 6

For more practice, see page 411.

10 • Equations with Parentheses

You can get rid of parentheses in an equation by using the distributive property. The equation will then be one you already know how to solve.

EXAMPLE 1 $4(x + 2) = 20$

$$\begin{aligned} 4x + 8 &= 20 \\ 4x + 8 - 8 &= 20 - 8 \\ 4x &= 12 \\ x &= 3 \end{aligned}$$

Check: $4(x + 2) = 20$

$4(3 + 2)$	20
$4 \cdot 5$	
20	

✓

EXAMPLE 2 $3(x - 1) = 2x$

$$\begin{aligned} 3x - 3 &= 2x \\ 3x - 3 + 3 &= 2x + 3 \\ 3x &= 2x + 3 \\ 3x - 2x &= 2x + 3 - 2x \\ x &= 3 \end{aligned}$$

Check: $3(x - 1) = 2x$

$3(3 - 1)$	$2 \cdot 3$
$3 \cdot 2$	6
6	

✓

Of course you may want to skip some steps after you have solved a few equations like this, but do that *only* if you are ready.

Classroom Practice

Use the distributive property to simplify.

1. $3(x - 2)$ — $3x - 6$
2. $6(x - 1)$ — $6x - 6$
3. $5(x + 4)$ — $5x + 20$
4. $2(3x - 7)$ — $6x - 14$
5. $4(2x + 3)$ — $8x + 12$
6. $2(a + 4) + a$ — $3a + 8$
7. $5(a - 1) + 5$ — $5a$
8. $4(1 + x) + 5x$ — $4 + 9x$

Tell the first step you would take to solve.

9. $2(x - 3) = x$ — $2x - 6 = x$
10. $2(x - 2) = 4$ — $2x - 4 = 4$
11. $2(x + 3) = 3(x - 4)$ — $2x + 6 = 3x - 12$

Solve.

12. $3(x - 1) = 6$ — 3
13. $5(x + 2) = 15$ — 1
14. $4(a - 2) = 3(a + 1)$ — 11

Written Exercises

Use the distributive property to simplify.

A

1. $3(2x - 5)$ — $6x - 15$
2. $5(6 - 2a)$ — $30 - 10a$
3. $8(2x - 5)$ — $16x - 40$
4. $7(3 - 5a)$ — $21 - 35a$
5. $6(8y + 4)$ — $48y + 24$
6. $7(4x + 12)$ — $28x + 84$
7. $8(3 + 6b)$ — $24 + 48b$
8. $9(2 + 5y)$ — $18 + 45y$

Solve.

9. $2(x + 3) = 10$ 2
10. $3(2x - 1) = 9$ 2
11. $2(6 + 2a) = 24$ 3
12. $5(x - 2) = 0$ 2
13. $8(y - 4) = 0$ 4
14. $6(x + 1) = 36$ 5
15. $2(x - 5) = x$ 10
16. $3(x - 2) = 2x$ 6
17. $4(x - 1) = 2x$ 2
18. $3(y + 4) = 5y$ 6
19. $6(y - 2) = 3y$ 4
20. $2(3 - y) = 4y$ 1
21. $4(x - 3) = 0$ 3
22. $2(2x + 2) = 6x$ 2
23. $7(2 + y) = 14$ 0
24. $5(x - 3) = 2x + 3$ 6
25. $5(x - 1) = 7 + x$ 3
26. $7(x + 1) = 9 + 5x$ 1
27. $4(y - 1) = 2y + 6$ 5
28. $3(x - 3) = x + 1$ 5
29. $5(y + 4) = 2y + 20$ 0

Solve and check.

B

30. $6(x - 1) = 3(x + 1)$ 3
31. $2(x + 3) = 3(x - 3)$ 15
32. $8(x - 1) = 4(x + 4)$ 6
33. $7(2a - 4) = 2(a + 4)$ 3
34. $3(a - 2) + a = 2(a + 1)$ 4
35. $5(x + 2) = x + 6(x - 3)$ 14
36. $4(m + 3) - 2m = 3(m - 3)$ 21
37. $2(a + 4) = 2(a - 4) + 4a$ 4

For each equation tell which of the following is true.

a. No number is a solution.

b. One number is a solution.

c. All numbers are solutions.

C

38. $2(x + 5) = 3x + 4 - x$ a
39. $6(2x + 3) = 2(9 + 6x)$ c
40. $7(x - 5) = 2(x + 5)$ b
41. $4(x + 1) = 4x + 1$ a
42. $6(x + 3) = 2(3x + 1)$ a
43. $2(8x + 6) = 4(4x + 3)$ c

For more practice, see page 411.

11 • Puzzles



A piece of pizza has 50 more calories than a can of cola. Three pieces of pizza have as many calories as four cans of cola. How many calories are in each?

Let x = number of calories in a can of cola.
Then $x + 50$ = number of calories in a piece of pizza.

number of calories in 3 pieces of pizza = number of calories in 4 cans of cola

$$3(x + 50) = 4x$$
$$3x + 150 = 4x$$
$$150 = x$$

Subtract $3x$ from both sides to get this.

Answer: $x = 150 \longrightarrow$ 150 calories in a can of cola
$x + 50 = 200 \longrightarrow$ 200 calories in a piece of pizza

Check: number of calories in 3 pieces of pizza $= 3 \cdot 200 = 600$
number of calories in 4 cans of cola $= 4 \cdot 150 = 600$ ✓

Written Exercises

A **1.** An apple has 30 more calories than a peach.
Five peaches have as many calories as 3 apples.
How many calories are in each?
peach: 45 calories; apple: 75 calories

2. A donut has 50 fewer calories than a glass of milk.
Four donuts have as many calories as 3 glasses of milk.
How many calories are in each?
milk: 200 calories; doughnut: 150 calories

3. One number is 7 more than another.
Twice the larger is 22 less than 4 times the smaller.
Find the numbers. 18, 25

4. One number is 5 less than another.
Five times the smaller number is 1 less than 3 times the larger.
Find the numbers. 7, 12

5. Maria has twice as much money as Paul.
Paul has $8 more than Rocky.
Together they have $104.
How much does each have? Paul: $28; Maria: $56; Rocky: $20

6. Rich is 3 years older than Carla.
Ruth is twice as old as Rich.
Their ages total 33 years.
How old is each person? Carla: 6 yrs. old; Rich: 9 yrs. old; Ruth: 18 yrs. old

B 7. Warren is 14 years older than Chuck.
Next year Warren will be 3 times as old as Chuck.
How old is Warren? 20 years old

8. Francine is 6 years older than Carol.
Last year Francine was three times as old as Carol.
How old is Carol? 4 years old

C 9. See if you can solve this problem.



© 1972 United Feature Syndicate, Inc. The man is 41 years old.

For more practice, see page 411.

SELF-TEST

Solve.

1. $3x = 48 - x$ 12

2. $6(a - 3) = 12$ 5

3. $4(m - 3) = m + 15$ 9

4. Lou has twice as much money as Jo.
Jo has $11 more than Sherry.
Together they have $89.
How much does each have? Jo: $25; Lou: $50; Sherry: $14

5. Al has twice as much money as Vic.
Vic has $5 less than Connie.
Together they have $125.
How much does each have? Vic: $30; Al: $60; Connie: $35

CONSUMER CORNER

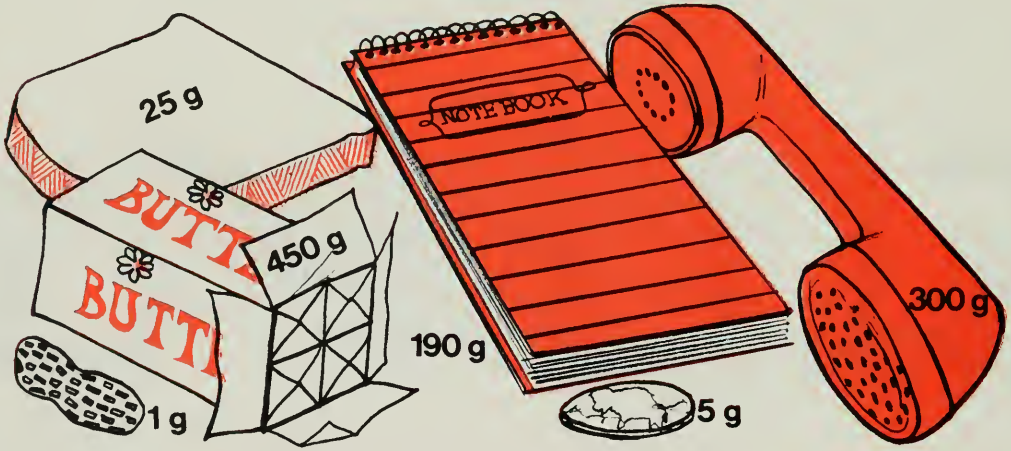
Metric units such as the **milligram** (mg), **gram** (g), and **kilogram** (kg) are used to measure quantity.

The prefix *milli-* means "$\frac{1}{1000}$ of."

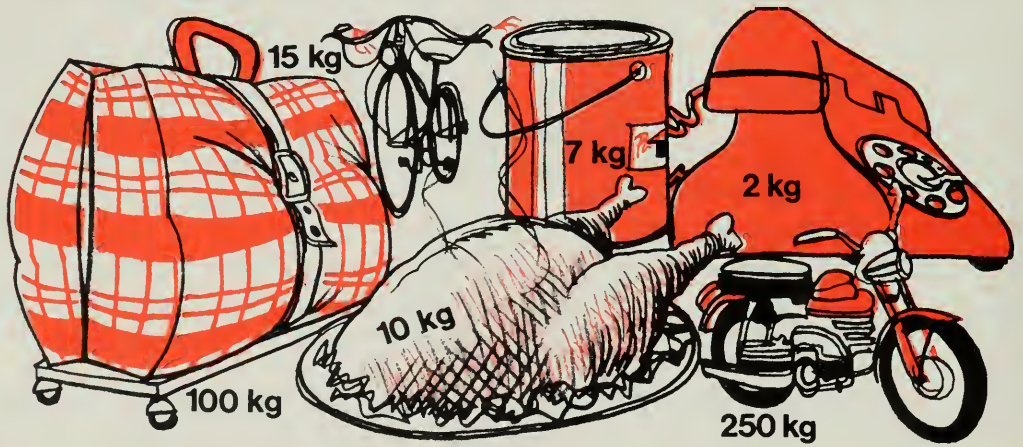
1000 milligrams = 1 gram
1000 grams = 1 kilogram

The prefix *kilo-* means "1000."

Here are some objects usually measured in grams or milligrams.



Here are some objects measured in kilograms.



1. Check yourself against the following tables.

AVERAGE NUMBER OF KILOGRAMS FOR STUDENTS OF GIVEN HEIGHTS

Girls

Height (centimeters)	147	152	157	162	167	172	177
Number of kilograms	45	49	51	56	59	65	62

Boys

Height (centimeters)	152	157	162	167	172	177	182
Number of kilograms	45	50	53	58	62	66	72

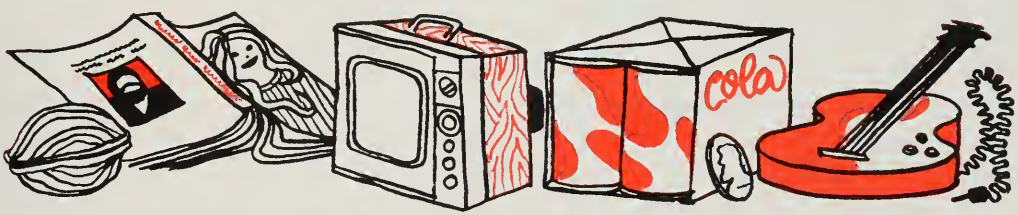
Remember, these are only *average* measures. People can vary a great deal because of bone structure too.

2. Study the first table below. Estimate the objects in the table at the right. Check with the answers at the foot of the page.

Ping-Pong Ball	2.5 g
Hockey Puck	160 g
Soccer Ball	450 g
Baseball Bat	900 g
Bowling Ball	4.5–7 kg

Baseball	?
Basketball	?
Tennis Racket	?
Badminton Shuttle	?
Heavy Shot Put	?

3. Estimate the following items as best as you can. The answers are given at the foot of the page.



Answers: **2.** Baseball 150 g; Basketball 600 g; Tennis Racket 350 g; Badminton Shuttle 5 g; Heavy Shot Put 7 kg **3.** Walnut 10 g; Book 200 g; TV 8 kg; Cola 2 kg; Penny 3 g; Guitar 5 kg

Reviewing the Chapter

Solve. *(See pp. 34–45.)*

1. $a - 5 = 13$ 18
2. $x + 9 = 24$ 15
3. $3b = 24$ 8
4. $5m = 35$ 7
5. $2x - 7 = 9$ 8
6. $3y + 4 = 19$ 5
7. $\frac{b}{2} = 7$ 14
8. $\frac{x}{3} = 13$ 39
9. $\frac{m}{4} = 7$ 28
10. $5a + 9 = 29$ 4
11. $3x - 11 = 52$ 21
12. $8y - 15 = 41$ 7
13. $12a - 7 = 17$ 2
14. $19 = x + 7$ 12
15. $25 = b - 9$ 34
16. $3n + 4 = 40$ 12
17. $6x - 9 = 21$ 5
18. $4s - 11 = 41$ 13
19. $\frac{a}{5} = 14$ 70
20. $\frac{m}{7} = 21$ 147
21. $\frac{x}{6} = 12$ 72
22. $9n + 4 = 58$ 6
23. $5x + 7 = 52$ 9
24. $3m - 7 = 32$ 13

Write an equation. Solve for the variable. *(See pp. 42–45.)*

25. 7 more than twice a number is 25. $2n + 7 = 25$; $n = 9$

26. The difference between $9y$ and $3y$ is 42. $9y - 3y = 42$; $y = 7$

Complete. *(See pp. 50–53.)*

27. Lea is 5 cm taller than Jose.
Let x = Jose's height in centimeters.
Then $\underset{x + 5}{?}$ = Lea's height.

28. Amy is 3 years older than Becky.
Let a = Amy's age.
Then $\underset{a - 3}{?}$ = Becky's age.

29. Jake has \$3 more than Marie.
Marie has \$6 more than Hank.
Let h = Hank's amount of money.
Then $\underline{?}$ = Marie's amount of money, $h + 6$
and $\underline{?}$ = Jake's amount of money. $h + 9$

Solve. *(See pp. 54–57, 62–63.)*

30. Marta has 3 times as much money as Phil.
Together they have $44.
How much does each have? **Phil: $11; Marta: $33**

31. The Tigers played 48 games.
They won twice as many as they lost.
How many did they win? **32 games**

32. There are 900 students in Wayne High School.
There are 20 more girls than boys.
How many girls are there? **460 girls**

33. A board 400 cm long is cut into 2 pieces.
One piece is 66 cm longer than the other.
Find the length of the shorter piece. **167 cm**

34. Mona has $8 more than Cindy.
Cindy has twice as much money as Bert.
Together they have $63.
How much does each have? **Bert: $11; Cindy: $22; Mona: $30**

35. I'm thinking of a number.
If I add 7 to it, then multiply by 3, I get 36.
What is my number? **5**

36. I'm thinking of a number.
If I subtract 8 from it, then multiply by 6, I get 24.
What is my number? **$6(n - 8) = 24$; $n = 12$**

Solve and check. *(See pp. 58–61.)*

37. $8x = 25 + 3x$ **5**

38. $9t = 40 - t$ **4**

39. $7x - 10 = 2x$ **2**

40. $3y + 12 = 7y$ **3**

41. $11a - 6 = 5a + 24$ **5**

42. $3(a - 4) = a$ **6**

43. $4(m - 4) = 2m - 2$ **7**

44. $5m + 7 = 3(m + 5)$ **4**

45. $x - 3 = 4(3 - x)$ **3**

46. $6(y + 2) = 5y + 18$ **6**

Cumulative Review

Simplify.

1. $9a - 5a + 7$ $4a + 7$
2. $4x + 3y - y + 5x$ $9x + 2y$
3. $5 \cdot b \cdot b \cdot c \cdot b \cdot 2$ $10b^3c$
4. $m(3m)(9m)$ $27m^3$
5. $13 + 9t - 9t + 6$ 19
6. $3(a + 4) - 12$ $3a$
7. $4n + 11 - n + 13$ $3n + 24$
8. $7(2a)(4a)$ $56a^2$
9. $2x + 9y - x - 5y$ $x + 4y$
10. $\frac{x}{12} \cdot 12$ x
11. $\frac{9m}{9}$ m
12. $6 \cdot \frac{a}{6} \cdot 0$ 0

State the expression without parentheses.

13. $5(x - 13)$ $5x - 65$
14. $9(2a + 7b)$ $18a + 63b$
15. $6(7m - n)$ $42m - 6n$

Tell which of the numbers shown in color is a solution.

16. $x - 11 = 18$ 29 — 7, 9, 29
17. $b + 12 > 23$ 12 — 10, 11, 12
18. $5m + 10 = 55$ 9 — 9, 11, 13
19. $4x - 3 < 5$ 0 — 0, 2, 4

Complete.

20. Anthony is 5 years older than Leona.
Let a = Anthony's age.
Then ___?___ = Leona's age. $a - 5$

21. Rhoda is 2 years younger than Stacy.
Let s = Stacy's age.
Then ___?___ = Rhoda's age. $s - 2$

Solve.

22. $x + 9 = 35$ 26
23. $a - 15 = 7$ 22
24. $23 = y + 8$ 15
25. $7n = 84$ 12
26. $8b = 120$ 15
27. $6b - 7 = 47$ 9
28. $\frac{n}{12} = 0$ 0
29. $\frac{x}{5} = 6$ 30
30. $\frac{a}{19} = 2$ 38
31. $5y + 7 - 3y = 53$ 23
32. $3(x + 5) = 8x$ 3
33. $9(m - 3) = 5m + 9$ 9

34. Lucas has \$13 more than Brenda.
Together they have \$93.
How much does Lucas have? \$53

UNIT B

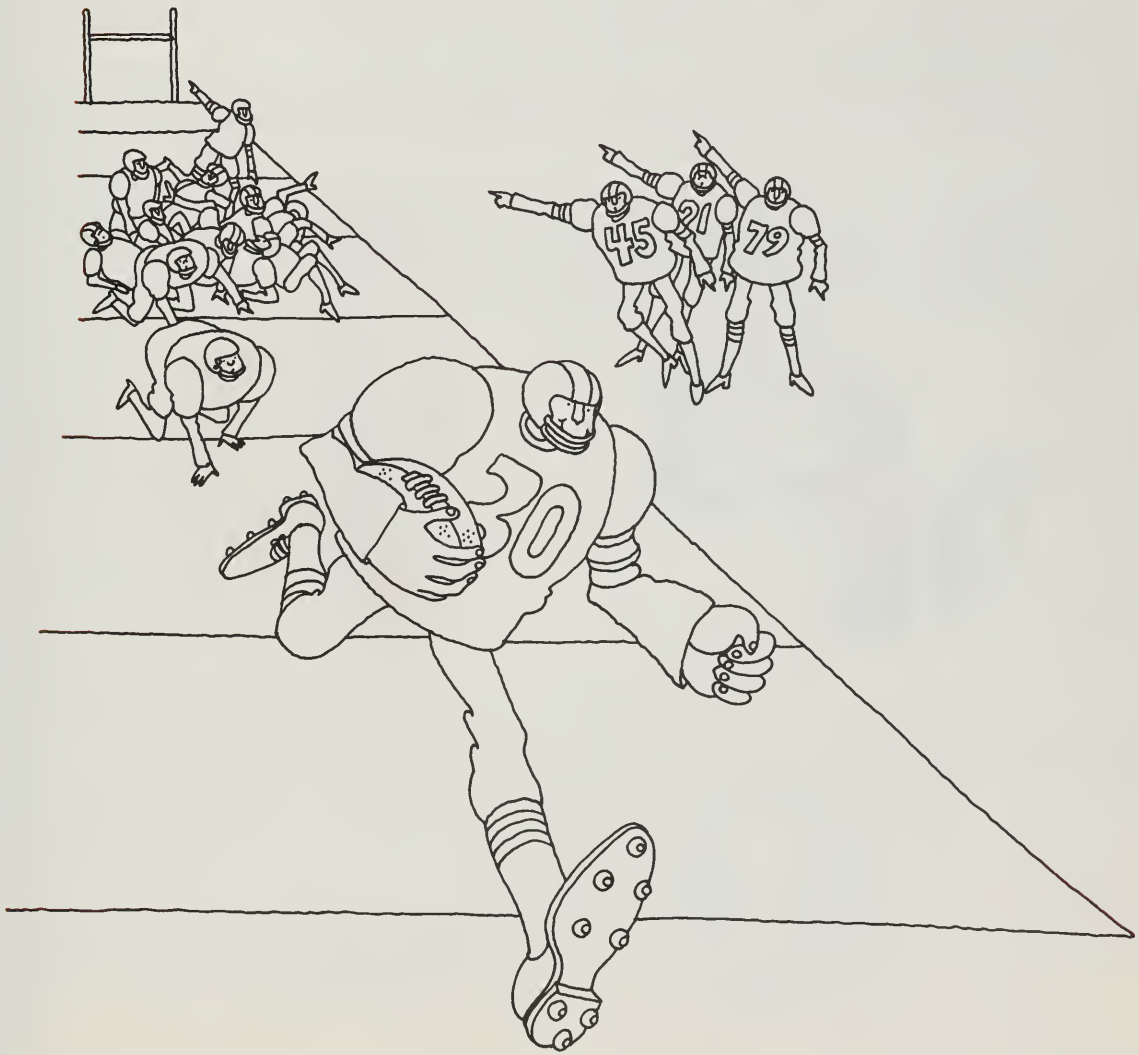
Here's what you'll learn in this chapter:

1. To compare integers.
2. To draw the graph of the solution of a simple equation or inequality.
3. To add, subtract, multiply, and divide positive and negative numbers.
4. To simplify expressions.
5. To find powers of positive and negative numbers.
6. To solve equations involving negative numbers.



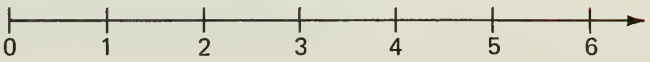
Chapter 3

Positive and Negative Numbers

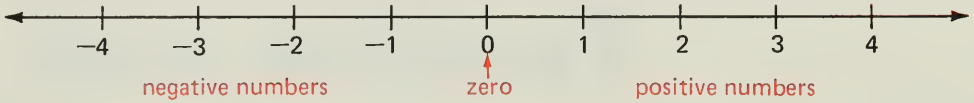


1 • Positive and Negative Numbers

The number line is often used to picture numbers.



In algebra we use numbers to the left of zero too.



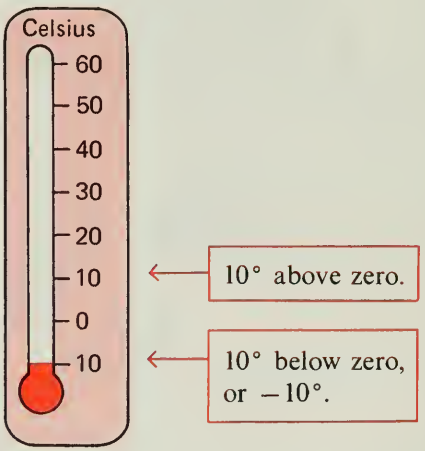
The numbers shown on the number line above form the set of **integers.**

Positive numbers can be written in two ways.

We write	We say
+5 or 5	positive five
−5	negative five

Positive and negative numbers are used in talking about temperatures. You can see that a thermometer is really like a number line.





A number line helps to show relationships between numbers.



$-4 < -2$

-4 is less than -2

$1 > -3$

1 is greater than -3

EXAMPLES Compare -3 and -2. $-3 < -2$

Compare 3 and -4. $3 > -4$

Classroom Practice

1. $+30$ is read ? positive 30
2. -4 is read ? negative 4
3. $+30°$ temperature means ? $30°$ above zero
4. -16 is read ? negative 16
5. If $+20$ means a gain, -20 means ? a loss
6. If -5 means a fall, $+5$ means ? a rise

Read aloud.

7. $4 > 3$ 4 is greater than 3
8. $1 < 5$ 1 is less than 5
9. $-4 < -2$ -4 is less than -2
10. $-2 > -4$ -2 is greater than -4

Written Exercises

Write a positive or negative number.

A

1. a temperature 25° above zero $+25$
2. a gain of \$20 $+20$
3. a loss of \$10 -10
4. a gain of 5 points $+5$
5. an ocean depth of 200 m -200
6. a loss of 7 points -7

Compare the numbers. Write $>$ or $<$.

7. -3 ? 0 $<$
8. -2 ? 3 $<$
9. 5 ? 0 $>$
10. 3 ? -3 $>$
11. -1 ? -4 $>$
12. -2 ? 5 $<$
13. -4 ? 4 $<$
14. -2 ? -3 $>$
15. -6 ? 3 $<$
16. -2 ? -1 $<$
17. 7 ? -8 $>$
18. -5 ? -2 $<$

19. A diving bell is 274 meters below the surface of the water. Write this depth as a positive or negative number. -274

B

20. A plane flew from a city 30° North latitude to a city 30° South latitude. Through how many degrees did it fly? 60 degrees

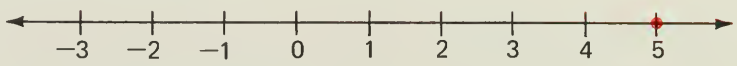


21. Find the latitude of your home town or city. Express the latitude with a + or a − sign. Answers will vary with location.

2 • Inequalities and Graphs

The number line is useful for picturing a solution of an equation.

$x - 3 = 2$
$x = 5$



Think about inequalities. Sometimes there are an unlimited number of solutions. Take the inequality $x < 8$.

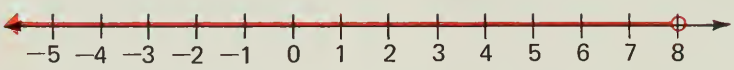
Some solutions: 7, 6, 5, 4, 3, 2, 1, 0, −1, −2, −3, . . .

means "and so on"

More solutions: all numbers *between* 8 and 7, *between* 7 and 6, . . .

In order to show *all* the solutions, we make a graph like this.

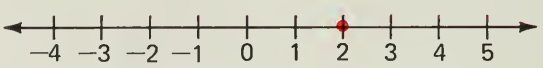
$x < 8$



Study these examples.

EXAMPLE 1 Graph the solution of $x + 3 = 5$.

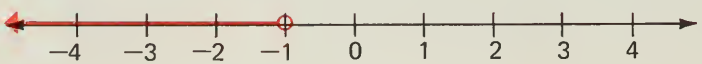
$x + 3 = 5$
$x = 2$



EXAMPLE 2 Graph the solutions of $x > -2$.



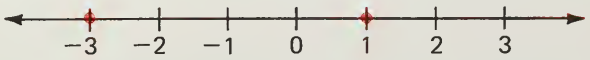
EXAMPLE 3 Graph the solutions of $x < -1$.



Written Exercises

Graph the given numbers. Write an inequality to compare them.

Sample $-3, 1$



$-3 < 1$

A **1.** $-2, 3$ $-2 < 3;\ 3 > -2$ **2.** $0, -5$ $-5 < 0;\ 0 > -5$ **3.** $-1, -6$ $-6 < -1;\ -1 > -6$ **4.** $7, -2$ $-2 < 7;\ 7 > -2$ **5.** $5, -4$ $-4 < 5;\ 5 > -4$

6. $-7, -1$ $-7 < -1;\ -1 > -7$ **7.** $-1, -7$ $-7 < -1;\ -1 > -7$ **8.** $0, -4$ $-4 < 0;\ 0 > -4$ **9.** $8, -8$ $-8 < 8;\ 8 > -8$ **10.** $-5, -2$ $-5 < -2;\ -2 > -5$

Graph the solution of the equation on the number line.

11. $x + 1 = 7$ 6 **12.** $y - 2 = 5$ 7 **13.** $x + 6 = 9$ 3 **14.** $5 + x = 10$ 5

15. $y - 6 = 2$ 8 **16.** $a + 3 = 12$ 9 **17.** $x - 6 = 4$ 10 **18.** $y - 2 = 7$ 9

Graph the solutions of the inequality on the number line. Check students' drawings.

19. $x \geq 2$ **20.** $x > 0$ **21.** $x < -2$ **22.** $x < 3$

23. $x < 0$ **24.** $x > -3$ **25.** $x < 2$ **26.** $x > -2$

27. $y > -1$ **28.** $x < 4$ **29.** $y < -1$ **30.** $x < 7$

Write an inequality to compare the three numbers.

Sample Compare -3, 1, and -4. $-4 < -3 < 1$

B **31.** 0, -4, and 4 $-4 < 0 < 4$ **32.** -1, 2, and -6 $-6 < -1 < 2$ **33.** -3, -5, and 0 $-5 < -3 < 0$

34. 3, -5, and -6 $-6 < -5 < 3$ **35.** -4, 6, and -2 $-4 < -2 < 6$ **36.** -2, -1, and -6 $-6 < -2 < -1$

For more practice, see page 412.

SELF-TEST

Compare the numbers.

1. $6 \ ?\ -7$ $>$ **2.** $3 \ ?\ 5$ $<$ **3.** $-6 \ ?\ 9$ $<$ **4.** $-4 \ ?\ 0$ $<$

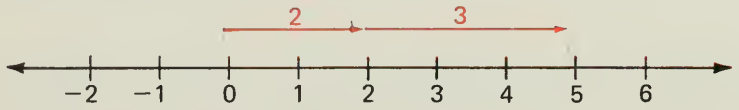
5. $-2 \ ?\ 4$ $<$ **6.** $1 \ ?\ -8$ $>$ **7.** $-9 \ ?\ -5$ $<$ **8.** $-7 \ ?\ -10$ $>$

Graph the solutions of the inequality. Check students' drawings.

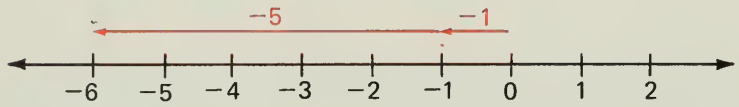
9. $x > 5$ **10.** $x > -6$ **11.** $x < -4$ **12.** $x < 6$

3 • Addition

Addition can easily be shown on the number line.



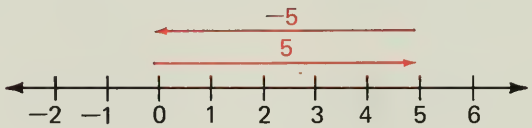
$2 + 3 = 5$



$(-1) + (-5) = -6$

The sum of two positive numbers is positive.
The sum of two negative numbers is negative.

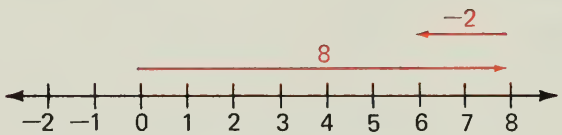
Suppose in a football game you gain 5 yards, then lose 5 yards.



$5 + (-5) = 0$

Suppose you gain 8 yards, then lose 2 yards.

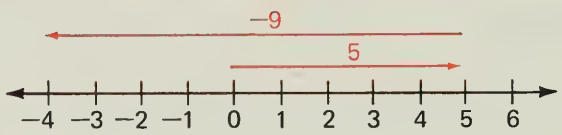
You gained more than you lost. The sum is positive.



$8 + (-2) = 6$

Now imagine that you gained 5 yards but lost 9 yards on the next play.

You lost more than you gained. The sum is negative.



$5 + (-9) = -4$

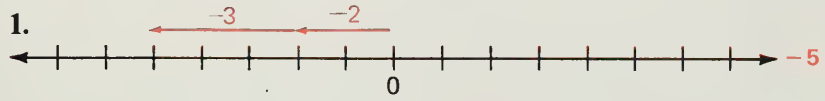
The sum of a positive number and a negative number may be zero, positive, or negative.

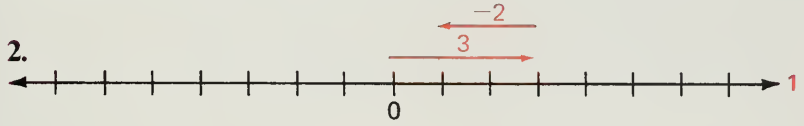
EXAMPLES $(-6) + 4 = -2$ $\quad 8 + (-3) = 5$

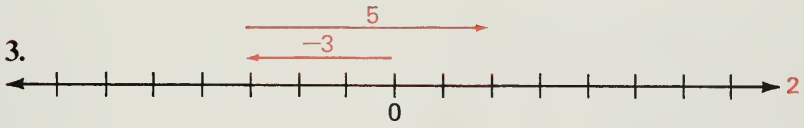
$(-3) + (-2) = -5$ $\quad (-5) + 5 = 0$

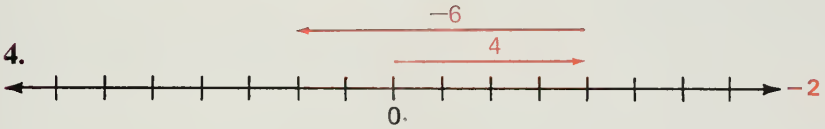
Classroom Practice

Read the sum from the number-line drawing.

1. 

2. 

3. 

4. 

Written Exercises

Add. Think of a number line.

A

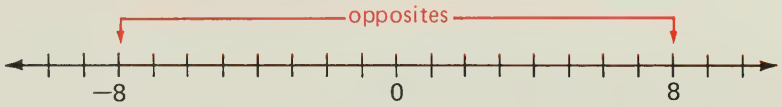
1. $-2 + (-3)$ −5
2. $-7 + (-4)$ −11
3. $-4 + 3$ −1
4. $2 + (-4)$ −2
5. $-6 + (-2)$ −8
6. $2 + (-2)$ 0
7. $-8 + 0$ −8
8. $-1 + (-2)$ −3
9. $6 + (-3)$ 3
10. $-5 + (-4)$ −9
11. $-4 + 6$ 2
12. $3 + (-5)$ −2
13. $-2 + (-4)$ −6
14. $0 + (-9)$ −9
15. $-7 + 4$ −3
16. $-3 + 7$ 4
17. $-5 + (-1)$ −6
18. $-6 + (-4)$ −10
19. $-8 + 8$ 0
20. $-2 + 2$ 0
21. $4 + (-4)$ 0
22. $6 + (-6)$ 0
23. $3 + (-9)$ −6
24. $-5 + (-8)$ −13
25. $-7 + (-2)$ −9
26. $-5 + (-12)$ −17
27. $6 + (-11)$ −5
28. $12 + (-6)$ 6

Add.

29. $5 + (-2) + (-6)$ −3
30. $-10 + 2 + (-5)$ −13
31. $1 + (-5) + 3$ −1
32. $-6 + 6 + (-2)$ −2
33. $9 + (-6) + 7$ 10
34. $12 + (-12) + 0$ 0

4 • Subtraction

Every number has an opposite.

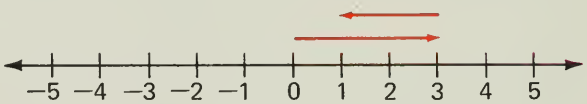


In Words	In Symbols
The opposite of 8 is −8. The opposite of −8 is 8.	−(8) = −8 −(−8) = 8

You'll use this idea of opposites in subtraction.

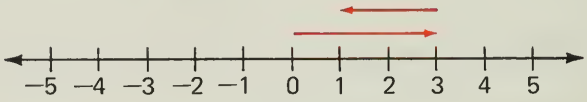
If you have 3 and subtract 2, you can show this on the number line.

$3 - 2 = 1$



You know that the sum of 3 and −2 is also 1.

$3 + (-2) = 1$



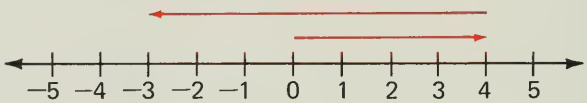
You can then write the following equation.

$$3 - 2 = 3 + (-2)$$

Let's consider another subtraction.

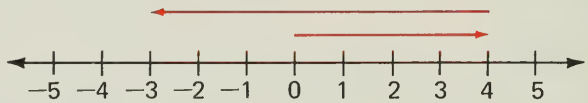
If you have 4 and subtract 7, you can show this on the number line.

$4 - 7 = -3$



The sum of 4 and −7 is also −3.

$4 + (-7) = -3$



$$4 - 7 = 4 + (-7)$$

We can write a rule for any subtraction.



To subtract any number, we add its opposite.

EXAMPLE 1 $5 - 2 = 5 + (-2)$
$= 3$

EXAMPLE 2 $-3 - 5 = -3 + (-5)$
$= -8$

EXAMPLE 3 $6 - (-3) = 6 + (3)$
$= 9$

EXAMPLE 4 $-4 - (-6) = -4 + (6)$
$= 2$

Classroom Practice

1. The opposite of 2 is ___?___. −2

2. The opposite of −2 is ___?___. 2

3. The opposite of ___?___ is 6. −6

4. The opposite of ___?___ is −6. 6

Complete.

5. $2 - 10 = 2 + (_?_)$ −10
$= _?_$ −8

6. $-2 - 10 = -2 + (_?_)$ −10
$= _?_$ −12

7. $2 - (-10) = 2 + (_?_)$ 10
$= _?_$ 12

8. $-2 - (-10) = -2 + (_?_)$ 10
$= _?_$ 8

9. $8 - 17 = 8 + (_?_)$ −17
$= _?_$ −9

10. $-8 - 17 = -8 + (_?_)$ −17
$= _?_$ −25

Subtract.

11. $6 - 7$ −1

12. $-6 - (-7)$ 1

13. $4 - 9$ −5

14. $7 - (-2)$ 9

15. $3 - (-5)$ 8

16. $-2 - 8$ −10

17. $-7 - (-1)$ −6

18. $-2 - (-10)$ 8

Written Exercises

Complete.

A **1.** $15 - 10 = 15 + (\underline{?}) \; -10$
$= \underline{?} \; 5$

2. $10 - 15 = 10 + (\underline{?}) \; -15$
$= \underline{?} \; -5$

3. $-16 - 7 = -16 + (\underline{?}) \; -7$
$= \underline{?} \; -23$

4. $-3 - 7 = -3 + (\underline{?}) \; -7$
$= \underline{?} \; -10$

5. $4 - (-2) = 4 + (\underline{?}) \; 2$
$= \underline{?} \; 6$

6. $6 - (-1) = 6 + (\underline{?}) \; 1$
$= \underline{?} \; 7$

Subtract.

7. $2 - 3$ −1

8. $1 - 5$ −4

9. $6 - 9$ −3

10. $8 - 10$ −2

11. $-3 - 4$ −7

12. $-5 - 2$ −7

13. $-7 - 6$ −13

14. $-3 - 7$ −10

15. $-3 - 9$ −12

16. $-1 - 6$ −7

17. $6 - (-2)$ 8

18. $7 - (-1)$ 8

19. $3 - (-5)$ 8

20. $8 - (-3)$ 11

21. $5 - (-8)$ 13

22. $9 - (-4)$ 13

23. $-2 - (-1)$ −1

24. $-7 - (-3)$ −4

25. $-10 - (-4)$ −6

26. $-3 - (-2)$ −1

27. $-6 - (-5)$ −1

28. $-2 - (-5)$ 3

29. $-8 - (-6)$ −2

30. $-10 - (-1)$ −9

31. $5 - (-3)$ 8

32. $8 - (-4)$ 12

33. $7 - 10$ −3

34. $-4 - 6$ −10

35. $-2 - 5$ −7

36. $3 - 9$ −6

37. $-9 - (-4)$ −5

38. $6 - (-5)$ 11



Solve.

39. Temperatures in Chicago range from about 8°C below zero in the winter to 29°C above zero in the summer. What is the difference in temperatures?
37 degrees

40. Temperatures in one place in the Arctic range from 60°C below zero in the winter to 70°C above zero in the summer. What is the difference between these temperatures? 130 degrees

41. Add 7 and 3. From the sum subtract 16. −6

42. Add −4 and 9. From the sum subtract 10. −5

43. From the sum of −7 and −17 subtract −77. 53

44. From the sum of 35 and -50 subtract -3. -12

45. A number is 12 less than -18. What is the number? -30

46. A number is 100 less than 10. What is the number? -90

For more practice, see page 412.

Mixed Practice Exercises

Add or subtract.

1. $2 + (-3)$ -1	**2.** $-5 + (-4)$ -9	**3.** $2 - 7$ -5	**4.** $-3 - (-2)$ -1
5. $8 - (-1)$ 9	**6.** $-7 + (-2)$ -9	**7.** $-3 - 4$ -7	**8.** $6 - (-4)$ 10
9. $8 + (-4)$ 4	**10.** $-2 + (-5)$ -7	**11.** $6 - (-2)$ 8	**12.** $-5 + (-5)$ -10
13. $-2 + 2$ 0	**14.** $-8 - (-6)$ -2	**15.** $4 - 10$ -6	**16.** $6 + (-2)$ 4
17. $-5 + 4$ -1	**18.** $-7 + 0$ -7	**19.** $-1 + 1$ 0	**20.** $2 - (-3)$ 5
21. $8 - (-7)$ 15	**22.** $-10 - (2)$ -12	**23.** $-7 + (-2)$ -9	**24.** $-3 - (-4)$ 1
25. $5 - (-6)$ 11	**26.** $-12 - (-3)$ -9	**27.** $8 - 12$ -4	**28.** $7 - (-1)$ 8
29. $9 - (-2)$ 11	**30.** $8 + (-6)$ 2	**31.** $4 - (-6)$ 10	**32.** $-12 - (-9)$ -3

Puzzles & Things

A Dot Puzzle



Copy these nine dots on a piece of paper.

Can you draw 4 straight lines through the dots so that each dot is touched?

You must not lift your pencil from the paper. And be sure you do not retrace a line or go through any dot more than once. See page 3 of Answers to Odd-Numbered Exercises in the back of the text.

5 • Simplifying Expressions

Do you remember when you studied the distributive property in Chapter 1? It allowed you to do the following.

$$2(x + y) = 2x + 2y$$

If you turn the equation around, you'll still have a true statement.

$$2x + 2y = 2(x + y)$$

The following equation is also true.

$$\begin{aligned} 3a + 5a &= (3 + 5)a \\ &= 8a \end{aligned}$$

The distributive property helps to explain why you can combine like terms. Of course you can combine like terms whether the coefficients are positive or negative numbers.

EXAMPLE 1 $2x - 5x = (2 - 5)x$
$= -3x$

All you have to do is to combine the coefficients of x for both terms.

$2x + (-5)x$
coefficients of x

EXAMPLE 2 $-4x - 5x = (-4 - 5)x$
$= -9x$

Combine the coefficients of x.

Once you understand how like terms are combined, it is easy to do one step in your head, if you wish.

Think: $3 - 7 = -4$

EXAMPLE 3 $3a - 7a = -4a$

Think: $-5 + 2 = -3$

Note that $-5c^2$ and $2c^2$ are like terms.

EXAMPLE 4 $-5c^2 + 2c^2 = -3c^2$

Written Exercises

Simplify.

A

1. $x + 3x$ — $4x$
2. $6x + 7x$ — $13x$
3. $3x + 7x$ — $10x$
4. $x - 3x$ — $-2x$
5. $6y - 7y$ — $-y$
6. $a - 4a$ — $-3a$
7. $-x^2 + 3x^2$ — $2x^2$
8. $-b + 5b$ — $4b$
9. $-2c^2 + 8c^2$ — $6c^2$
10. $-6y + 3y$ — $-3y$
11. $a - 2a$ — $-a$
12. $7c^2 - 9c^2$ — $-2c^2$
13. $-2x + 8x$ — $6x$
14. $-5b - 6b$ — $-11b$
15. $-8a - 2a$ — $-10a$
16. $-x + 5x$ — $4x$
17. $2n^2 - 5n^2$ — $-3n^2$
18. $-10x + 4x$ — $-6x$
19. $y - 10y + 10$ — $-9y + 10$
20. $x + 3 - 3x$ — $-2x + 3$
21. $-3y + 3y + 1$ — 1
22. $-7x - 8x - 2$ — $-15x - 2$
23. $-6x - (-7x)$ — x
24. $10x - (-7x)$ — $17x$
25. $-4x - (-5x)$ — x
26. $2a - (-a) + 4$ — $3a + 4$
27. $x^2 - 3x^2 - (-y)$ — $-2x^2 + y$
28. $y - 2x^2 - (-3y)$ — $4y - 2x^2$

Solve.

B

29. Subtract $2r$ from $10 - r$. Then decrease the result by $4r$. $10 - 7r$

30. Anita rode $(6n - 2)$ kilometers. Her sister rode n kilometers less. What is the difference? $5n - 2$ kilometers

For more practice, see page 412.

SELF-TEST

Add or subtract.

1. $(-3) + 9$ — 6
2. $(-4) + (-2)$ — -6
3. $6 + (-7)$ — -1
4. $(-5) + 3$ — -2
5. $5 - 3$ — 2
6. $-8 - 3$ — -11
7. $7 - (-5)$ — 12
8. $-4 - (-6)$ — 2

Simplify.

9. $5x - 4x$ — x
10. $-6n + 7n$ — n
11. $-3c - 8c$ — $-11c$
12. $2m^2 - 4m^2$ — $-2m^2$

6 • Multiplication

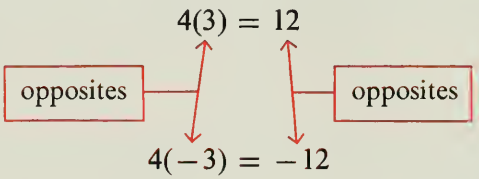
You know that multiplication and addition are related.

$$\begin{aligned} 4 \times 3 &= 3 + 3 + 3 + 3 \\ &= 12 \end{aligned}$$

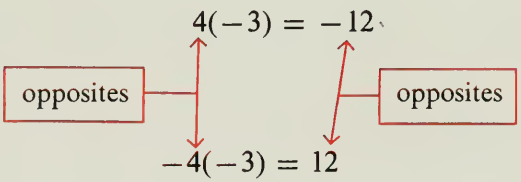
In the same way you can show multiplication with a negative number.

$$\begin{aligned} 4(-3) &= (-3) + (-3) + (-3) + (-3) \\ &= -12 \end{aligned}$$

Notice: When the sign of *one* factor is changed, the sign of the product is changed.



We use this pattern to multiply two negative numbers.



These examples lead to some rules about signs in multiplication.



When two numbers have *like* signs, their product is a *positive* number.

positive × positive = positive ⟶ $6 \cdot 2 = 12$
negative × negative = positive ⟶ $-6(-2) = 12$



When two numbers have *unlike* signs, their product is a *negative* number.

positive × negative = negative ⟶ $6(-2) = -12$
negative × positive = negative ⟶ $-6(2) = -12$

Classroom Practice

Multiply.

1. $6 \cdot 3$ 18
2. $-6 \cdot 3$ -18
3. $-3 \cdot 6$ -18
4. $-3(-6)$ 18
5. $3 \cdot 0$ 0
6. $12 \cdot 1$ 12
7. $(-3) \cdot 0$ 0
8. $-1 \cdot 14$ -14
9. $-3 \cdot -9$ 27
10. $-4 \cdot -6$ 24
11. $-1(1)$ -1
12. $-1(-2)$ 2

Simplify.

Samples $(-3)^2 = (-3)(-3) = 9$ $\quad -(3)^2 = -(3)(3) = -9$

13. $(-1)^2$ 1
14. $(-3)^2$ 9
15. $(-4)^2$ 16
16. $-(2)^2$ -4
17. $-(-2)^2$ -4

Written Exercises

Multiply.

A

1. $4 \cdot 8$ 32
2. $-2(100)$ -200
3. $-3(-40)$ 120
4. $6 \cdot 8$ 48
5. $-2 \cdot 7$ -14
6. $-16 \cdot 0$ 0
7. $-4(-5)$ 20
8. $5(-2)$ -10
9. $-6(4)$ -24
10. $-9(-1)$ 9
11. $6(-1)$ -6
12. $-7(-3)$ 21
13. $5 \cdot 6$ 30
14. $-8(3)$ -24
15. $-1(7)$ -7
16. $-2(4)$ -8
17. $3(-4)$ -12
18. $(0)(-2)$ 0
19. $4(-1)$ -4
20. $8(-6)$ -48
21. $-1(-4)$ 4
22. $2(-10)$ -20
23. $7(-4)$ -28
24. $-3 \cdot 9$ -27
25. $8(-2)$ -16
26. $-7(-7)$ 49
27. $5(-5)$ -25
28. $9(-2)$ -18

Simplify.

29. $(-5)^2$ 25
30. 6^2 36
31. $(-6)^2$ 36
32. 4^2 16
33. $(-4)^2$ 16
34. $-(-1)^2$ -1
35. $-(10)^2$ -100
36. $-(7)^2$ -49
37. $-(-5)^2$ -25
38. $-(-3)^2$ -9
39. $(-10)^2$ 100
40. $(-8)^2$ 64
41. $-(6)^2$ -36
42. $-(-8)^2$ -64
43. $-(-10)^2$ -100

B

44. $3(-10) + 3(6)$ -12
45. $-1(6) + (-1)(-6)$ 0
46. $-7(-1) + (-4)(-1)$ 11
47. $8(-7) - 4(-2)$ -48
48. $3(-2) - 7(-4)$ 22
49. $(4)(-2)(-3) + (-1)(-1)$ 25
50. $(-3)^3$ -27
51. $(-2)^4$ 16
52. $(-1)^5$ -1

The rules for the signs in multiplication also are true when you're working with variables.

EXAMPLE 1 $4(6a) = 24a$

EXAMPLE 2 $-6(4a) = -24a$

EXAMPLE 3
$$\begin{aligned} -6a(-4b) &= (-6)(-4)(a)(b) \\ &= 24ab \end{aligned}$$

You know that we can rewrite $2(x + 5)$ in the following way.

$$\begin{aligned} 2(x + 5) &= (2 \cdot x) + (2 \cdot 5) \\ &= 2x + 10 \end{aligned}$$

You can work in this way when some numbers are negative, too.

EXAMPLE 4
$$\begin{aligned} -2(x + 5) &= (-2 \cdot x) + (-2 \cdot 5) \\ &= -2x - 10 \end{aligned}$$

EXAMPLE 5
$$\begin{aligned} -3(a - 4) &= (-3 \cdot a) + (-3 \cdot -4) \\ &= -3a + 12 \end{aligned}$$

Recall from Chapter 1 that x means $1x$. You can then see that some other statements are also true.

$$x = 1x \text{ and } \begin{cases} -x = -1x \\ -(x - 2) = -1(x - 2) \end{cases}$$

These are useful facts in simplifying some expressions.

EXAMPLE 6
$$\begin{aligned} -(a - 4) &= -1(a - 4) \\ &= (-1 \cdot a) + (-1 \cdot -4) \\ &= -a + 4 \end{aligned}$$

EXAMPLE 7
$$\begin{aligned} -(-x + 3) &= -1(-x + 3) \\ &= (-1 \cdot -x) + (-1 \cdot 3) \\ &= x - 3 \end{aligned}$$

Classroom Practice

Multiply.

1. $2(-2a)$ $-4a$
2. $-2(-2a)$ $4a$
3. $6a \cdot 0$ 0
4. $-1 \cdot x$ $-x$
5. $8(-9x)$ $-72x$
6. $(-8x)(8)$ $-64x$
7. $-3b \cdot 0$ 0
8. $(2a)(-1)$ $-2a$
9. $a(-a + b)$ $-a^2 + ab$
10. $4(-x - 2)$ $-4x - 8$
11. $-c(c - 5)$ $-c^2 + 5c$
12. $-x(x - 2)$ $-x^2 + 2x$

Written Exercises

Multiply.

A

1. $-1(-2x)$ $2x$
2. $-4(-3x)$ $12x$
3. $2(-3x)$ $-6x$
4. $-1(7y)$ $-7y$
5. $-6a \cdot 0$ 0
6. $-4a(2)$ $-8a$
7. $-4(3x)$ $-12x$
8. $-5(2x)$ $-10x$
9. $3a(-2b)$ $-6ab$
10. $-a(4b)$ $-4ab$
11. $-3x(-2y)$ $6xy$
12. $-7y(-6z)$ $42yz$
13. $4x(-2y)$ $-8xy$
14. $-5x(-3y)$ $15xy$
15. $7a(-2b)$ $-14ab$
16. $-4r(-5s)$ $20rs$
17. $4(9 - x)$ $36 - 4x$
18. $-4(9 - x)$ $-36 + 4x$
19. $2(a - b)$ $2a - 2b$
20. $-2(a - b)$ $-2a + 2b$
21. $-x(6 - x)$ $-6x + x^2$
22. $-y(y - 8)$ $-y^2 + 8y$
23. $(a + b)(-3)$ $-3a - 3b$
24. $(c + d)(-2)$ $-2c - 2d$

Simplify.

25. $-(a + 4)$ $-a - 4$
26. $-(x + 6)$ $-x - 6$
27. $-(y - 1)$ $-y + 1$
28. $-(x - 3)$ $-x + 3$
29. $-(-x + 5)$ $x - 5$
30. $-(4 - y)$ $-4 + y$
31. $-(7 - a)$ $-7 + a$
32. $-(x - y)$ $-x + y$
33. $x - (x + 3)$ -3
34. $y - (y - 1)$ 1
35. $2x - (x + 4)$ $x - 4$
36. $3a - (a - 5)$ $2a + 5$
37. $8a - (a - 4)$ $7a + 4$
38. $2a - (a + 4)$ $a - 4$
39. $6x - (3 - x)$ $7x - 3$
40. $5x - (7 - 2x)$ $7x - 7$

Solve for the variable.

B

41. $x + 11 = 2(x - 1)$ 13
42. $3(n - 1) = 2n + 8$ 11
43. $-(n - 1) = -2(n + 3)$ -7
44. $p - (p - 2) = p$ 2

There is a mistake in this work. Can you correct it?

C

45.
$$4(7x - 1) - 3(8x - 2) = 4 - (-3x + 2)$$
$$28x - 4 - 24x - 6 = 4 + 3x - 2$$
$$4x - 10 = 3x + 2$$
$$x = 12$$

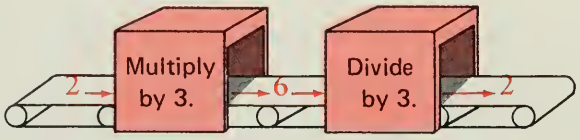
$28x - 4 - 24x + 6 = 4 + 3x - 2$
$4x + 2 = 3x + 2$
$x = 0$

For more practice, see page 413.

8 • Division

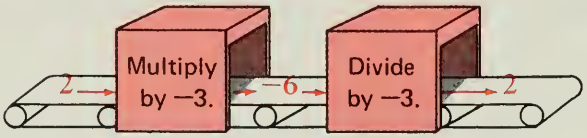
Once you know the rules for signs in multiplication, it's easy to learn the rules for division.

$2 \cdot 3 = 6$



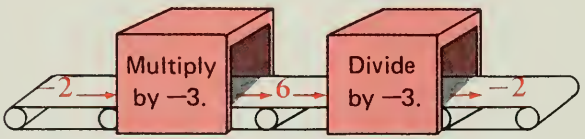
$6 \div 3 = 2$

$2 \cdot -3 = -6$



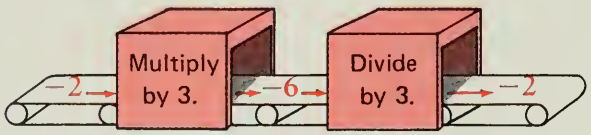
$-6 \div (-3) = 2$

$-2 \cdot -3 = 6$



$6 \div (-3) = -2$

$-2 \cdot 3 = -6$



$-6 \div 3 = -2$

If you study the division equations closely, you'll notice that the rules for the signs in division are similar to those in multiplication.

When two numbers have *like* signs, their quotient is a *positive* number.

positive ÷ positive = positive ⟶ $14 \div 2 = 7$
negative ÷ negative = positive ⟶ $-14 \div -2 = 7$

When two numbers have *unlike* signs, their quotient is a *negative* number.

positive ÷ negative = negative ⟶ $14 \div -2 = 7$
negative ÷ positive = negative ⟶ $-14 \div 2 = -7$

Classroom Practice

Divide.

1. $8 \div -8$ -1
2. $-10 \div -2$ 5
3. $-4 \div 2$ -2
4. $-12 \div 6$ -2
5. $-15 \div -5$ 3
6. $\frac{16}{-2}$ -8
7. $\frac{-45}{9}$ -5
8. $\frac{22}{2}$ 11
9. $\frac{-49}{-7}$ 7
10. $\frac{28}{-7}$ -4

Written Exercises

Divide.

A

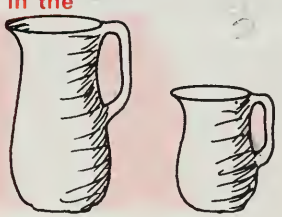
1. $3 \div -3$ -1
2. $-3 \div -3$ 1
3. $-56 \div 7$ -8
4. $56 \div -7$ -8
5. $4 \div -2$ -2
6. $-6 \div 2$ -3
7. $-10 \div -5$ 2
8. $-15 \div -3$ 5
9. $42 \div -6$ -7
10. $-16 \div 4$ -4
11. $-20 \div -4$ 5
12. $25 \div -5$ -5
13. $49 \div -7$ -7
14. $-18 \div 6$ -3
15. $-21 \div -7$ 3
16. $24 \div -4$ -6
17. $\frac{12}{-2}$ -6
18. $\frac{14}{7}$ 2
19. $\frac{-10}{5}$ -2
20. $\frac{-15}{-3}$ 5
21. $\frac{-21}{-3}$ 7
22. $\frac{18}{-2}$ -9
23. $\frac{-9}{-3}$ 3
24. $\frac{-20}{5}$ -4
25. $\frac{35}{7}$ 5
26. $\frac{56}{-8}$ -7
27. $\frac{-24}{6}$ -4
28. $\frac{-25}{-5}$ 5
29. $\frac{16}{2}$ 8
30. $\frac{32}{-8}$ -4
31. $\frac{48}{-6}$ -8

For more practice, see page 413.

Puzzles & Things

See page 4 of Answers to the Odd-Numbered Exercises in the back of the text.

A liter is a little more than a quart. I have 2 pitchers which hold exactly 3 liters and 5 liters. Using just these two pitchers, how can I give you exactly 4 liters of water?



9 • Solving Equations

Now that you can work with both negative and positive numbers, you can put the skills to work in solving equations.

EXAMPLE 1

$$\begin{aligned} n - 7 &= -16 \\ n - 7 + 7 &= -16 + 7 \\ n &= -9 \end{aligned}$$

Check:

$$\begin{array}{r|l} n - 7 = -16 & \\ \hline -9 - 7 & -16 \\ -16 & \end{array}$$

EXAMPLE 2

$$\begin{aligned} 3a &= -12 \\ \frac{3a}{3} &= \frac{-12}{3} \\ a &= -4 \end{aligned}$$

Check:

$$\begin{array}{r|l} 3a = -12 & \\ \hline 3 \cdot -4 & -12 \\ -12 & \end{array}$$

EXAMPLE 3

$$\begin{aligned} 4c - 5c &= 8 \\ -c &= 8 \\ -1 \cdot -c &= -1 \cdot 8 \\ c &= -8 \end{aligned}$$

If you know $-c$, multiply both sides by -1 to find c.

Check:

$$\begin{array}{r|l} 4c - 5c = 8 & \\ \hline (4 \cdot -8) - (5 \cdot -8) & 8 \\ -32 - (-40) & \\ -32 + 40 & \\ 8 & \end{array}$$

EXAMPLE 4

$$\begin{aligned} r - 8 &= -3r + 4 \\ r - 8 + 3r &= -3r + 4 + 3r \\ 4r - 8 &= 4 \\ 4r - 8 + 8 &= 4 + 8 \\ 4r &= 12 \\ r &= 3 \end{aligned}$$

Check:

$$\begin{array}{r|l} r - 8 = -3r + 4 & \\ \hline 3 - 8 & -3(3) + 4 \\ -5 & -9 + 4 \\ & -5 \end{array}$$

Classroom Practice

Solve for the variable. Check.

1. $x + 1 = -3$ (−4)
2. $y + 6 = 4$ (−2)
3. $x + 2 = 0$ (−2)
4. $-x + 4x = 9$ (3)
5. $-2x + 3x = 7$ (7)
6. $3x - 2x = -11$ (−11)
7. $4a = 3a - 3$ (−3)
8. $4x = -20$ (−5)
9. $-5x = -40$ (8)
10. $-3y = 24$ (−8)
11. $2x = 4 - 6$ (−1)
12. $2y - 5y = -9$ (3)
13. $-x = 11$ (−11)
14. $-a = -6$ (6)
15. $5n - 7n = 10$ (−5)
16. $2y - 7y = -15$ (3)

Written Exercises

Solve for the variable. Check.

A

1. $x + 2 = -4$ -6
2. $x + 6 = 3$ -3
3. $y + 12 = -12$ -24
4. $y - 12 = 12$ 24
5. $a - 3 = -2$ 1
6. $3x = -15$ -5
7. $-4x = 12$ -3
8. $-6x = 18$ -3
9. $5y - 3y = -10$ -5
10. $5x - 2x = -18$ -6
11. $15x = -45 + 30$ -1
12. $3x - 2x = -2$ -2

Solve for the variable.

13. $-x + 2x = 9$ 9
14. $-n + 4n = 15$ 5
15. $-2n + 5n = 12$ 4
16. $-x + 5x = 16$ 4
17. $-2a + 6a = 12$ 3
18. $-2n + 3n = -12$ -12
19. $4c - 3c = -6$ -6
20. $4x + 7 = 3x$ -7
21. $2n + 5 = n$ -5
22. $3y + 4 = 2y$ -4
23. $7y - 2 - 5y = 0$ 1
24. $-6m + 5 = -7m$ -5
25. $-3x - 8 = -5x$ 4
26. $3x + 1 = 2x$ -1
27. $6y + 4 = 4y$ -2
28. $-14 = -6x - 2$ 2
29. $2n + 16 = n + 2$ -14
30. $-3x + 2 = -x + 4$ -1
31. $2x - (3x + 2) = -7$ 5
32. $5y - (2y - 1) = -2$ -1
33. $-x - (5x - 7) = -5$ 2
34. $7 - (2 - x) = -4$ -9
35. $6 - (4 - x) = 3x$ 1
36. $2x - (3 - x) = x - 7$ -2

For more practice, see page 413.

SELF-TEST

Multiply.

1. $-9(3x)$ $-27x$
2. $-3a(-2b)$ $6ab$
3. $-4(y - 5)$ $-4y + 20$
4. $5(-c - d)$ $-5c - 5d$

Simplify.

5. $-(x + 6)$ $-x - 6$
6. $-(a + 5)$ $-a - 5$
7. $y - (-y - 7)$ $2y + 7$
8. $2x - 3(x - 1)$ $-x + 3$

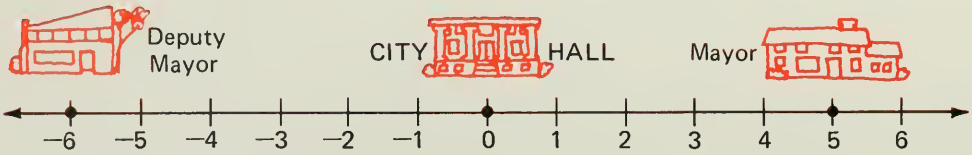
Divide.

9. $12 \div 3$ 4
10. $-15 \div -5$ 3
11. $21 \div -7$ -3
12. $-6 \div -2$ 3

Solve for x.

13. $3x - 4x = 7$ -7
14. $9x - 3x = -12$ -2
15. $5x + 3 = 4x$ -3

10 • Absolute Value (Optional)



The Deputy Mayor lives 6 blocks to the left of City Hall.

The Mayor lives 5 blocks to the right of City Hall.

Who lives farther from City Hall? Of course it's the Deputy Mayor. Sometimes we care only about the number of units and not whether the number is positive or negative. In cases like this we use the following notation.

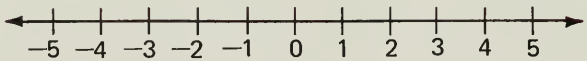
$|-6| = 6$ — Read: the absolute value of negative six

$|5| = 5$ — Read: the absolute value of five

$|-6| > |5|$

EXAMPLE Compare -5 and -4. Compare $|-5|$ and $|-4|$.

$-5 < -4$ $\qquad$ $|-5| > |-4|$



Can you guess what $|0|$ is? (Yes, it is 0.)

Written Exercises

True or false?

1. $|-3| < |-2|$ false
2. $|-6| > |2|$ true
3. $|-4| > |-3|$ true
4. $|5| > |-1|$ true
5. $|0| < |6|$ true
6. $|-3| < |0|$ false
7. $|1| > |-7|$ false
8. $|-1| > |-7|$ false
9. $|2| < |-2|$ false
10. $|-6| < |4|$ false
11. $|-7| > |7|$ false
12. $|3| > |-2|$ true

Compare. Write sentences with $>$, $<$, or $=$.

Sample Compare $|-4|$ and $|2|$. $|-4| > |2|$

13. $|5|$? $|6|$ $<$

14. $|2|$? $|-3|$ $<$

15. $|-7|$? $|-3|$ $>$

16. $|0|$? $|-4|$ $<$

17. $|9|$? $|-10|$ $<$

18. $|-6|$? $|-2|$ $>$

19. $|-11|$? $|7|$ $>$

20. $|-1|$? $|1|$ $=$

21. $|-5|$? $|2|$ $>$

Find the value of the expression.

22. $-|3|$ -3

23. $-|-2|$ -2

24. $-|3| + |-2|$ -1

25. $7 + |-4|$ 11

26. $-|5| + |-5|$ 0

27. $|-2| - |3|$ -1

Find two solutions to the equation.

Sample $|x| + 2 = 5$ x could be -3 or 3.

28. $|x| + 4 = 9$ 5 or -5

29. $|y| + 3 = 7$ 4 or -4

30. $|x| - 1 = 9$ 10 or -10

31. $|a| + 5 = 11$ 6 or -6

32. $|n| + 2 = 5$ 3 or -3

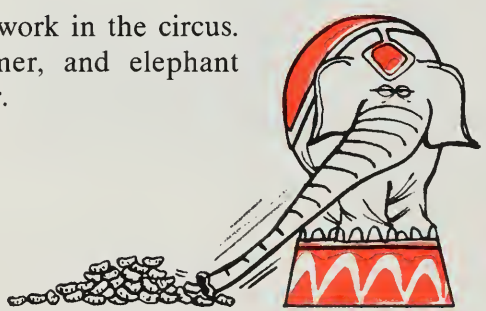
33. $|x| - 7 = 7$ 14 or -14

Puzzles & Things

A Logic Problem

Amos, Christopher, and Geraldine work in the circus. They are the ringmaster, lion tamer, and elephant trainer, not necessarily in that order.

1. Geraldine has red hair.
2. Amos has curly hair.
3. The ringmaster is taller than Amos.
4. The lion tamer is bald.

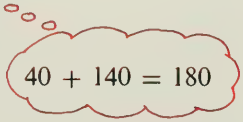


Who is the elephant trainer? Amos

Reviewing Arithmetic Skills

Use your estimating skills. Estimate in your head.

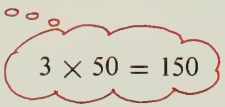
Sample $39 + 142$ *Answer:* about 180



1. $22 + 59$ about 80	**2.** $47 + 11$ about 60	**3.** $86 + 21$ about 110	**4.** $79 + 32$ about 110
5. $41 + 51$ about 90	**6.** $99 + 16$ about 120	**7.** $81 + 39$ about 120	**8.** $64 + 89$ about 150
9. $76 + 120$ about 200	**10.** $51 + 135$ about 190	**11.** $106 + 84$ about 190	**12.** $92 + 188$ about 280

Estimate in your head.

Sample 3×47 *Answer:* about 150



13. 6×18 about 120	**14.** 7×62 about 420	**15.** 8×12 about 80	**16.** 3×98 about 300
17. $4 \times \$.39$ about \$1.60	**18.** $5 \times \$.95$ about \$5.00	**19.** $2 \times \$.65$ about \$1.40	**20.** $5 \times \$.75$ about \$4.00
21. $4 \times \$.89$ about \$3.60	**22.** $6 \times \$.35$ about \$2.40	**23.** $3 \times \$.85$ about \$2.70	**24.** $7 \times \$.29$ about \$2.10

Suppose you have a calculator. Do these answers look reasonable, or have you touched a wrong button by mistake? (Don't calculate. Just check to see if the answer is *reasonable*.)

25. $16 + 47 = 99$ not reasonable	**26.** $86 + 92 = 178$ reasonable	**27.** $153 + 89 = 532$ not reasonable
28. $235 + 41 = 286$ reasonable	**29.** $723 + 19 = 714$ not reasonable	**30.** $811 + 49 = 890$ not reasonable
31. $4 \times 48 = 216$ not reasonable	**32.** $5 \times 39 = 1950$ not reasonable	**33.** $66 \times 23 = 158$ not reasonable
34. $70 \times 84 = 2048$ not reasonable	**35.** $49 \times 73 = 3577$ reasonable	**36.** $58 \times 91 = 3048$ not reasonable

Find the average.

Sample 10, 15, 26 $10 + 15 + 26 = 51$

Divide 51 by the number of items. → $51 \div 3 = 17$ ← This is the average.

37. 12, 8, 19 13	**38.** 3, 14, 16 11	**39.** 98, 20, 32 50	**40.** 10, 12, 20 14
41. 2, 4, 10, 20 9	**42.** 16, 5, 7, 12 10	**43.** 5, 10, 20, 21 14	**44.** 10, 12, 16, 20, 27 17

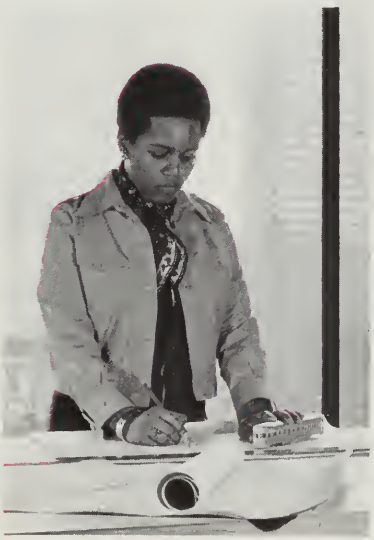
CAREER NOTEBOOK

Transportation

Ours is a very mobile society. The field of transportation offers a wide variety of opportunities. Among these are careers in research, design, construction, testing, operations, and maintenance.



City planners who plan transit systems study traffic patterns, predicted population figures, and so on.



Engineers design and develop safe and economical rapid transportation equipment.



Automotive technicians thoroughly test new cars to be sure that they meet all standards.

CONSUMER CORNER



Writing a Check

It isn't safe to carry much cash with you. It may be lost or stolen. Many people put their money in a checking account. Then they can pay their bills by check or they can cash a check for themselves when they need money.

Can you write a check correctly? This Consumer Corner is devoted to showing you how easy it is. On page 128 we'll show you how to keep track of your money in your checking account.

Here is a sample check.

Maria Lentini

No. 27

PAY TO THE ORDER OF Midtown Hardware $ 21 95/100

Oct. 20 19 ??

Twenty-one and 95/100 DOLLARS

National Bank of Seattle

Maria Lentini

⑆2117⑈6006⑆56800427

Write the amount of the check as a mixed numeral in terms of the number of dollars.

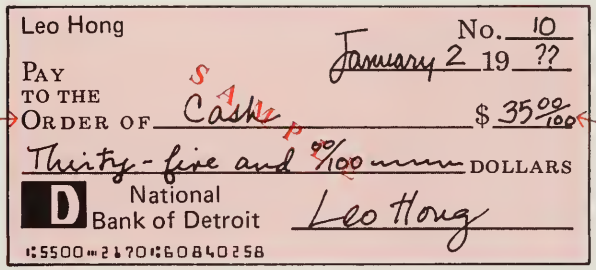
Sign your name. The check is not worth anything until you do.

Only Midtown Hardware can cash this check.

Write the amount of the check in words. The word *and* is written to show the decimal point.

The number of cents is written as a part of a dollar.

Suppose you wish to get some cash for yourself. You can just go to your bank and make out a check for *Cash*.



Write $\frac{00}{100}$ when there are no cents.

Be careful! It might be possible for somebody else to cash the check if you lose it.

Now why not try writing out a few checks, just for practice.

1. Write a check for $12.83 to Hub Grocery Store.
2. Write a check for $10 to pay Mr. Jackson for a window you just broke.
3. Write a check for $90.95 to Federal Bicycle Company for a new bicycle.
4. Write a check for $75 to Manny's Used Cars as a down payment for a car.
5. Write a check for $18.95 to Jones Department Store for a sweater.
6. Write a check for $16.00 to the Granada Theater for tickets to the rock concert.
7. Write a check for $23.75 to DiCenso Shoe Store for a pair of shoes.
8. Write a check for $11.50 to India Imports for a birthday present.

Reviewing the Chapter

Compare the numbers. Write $>$ or $<$. *(See pp. 72–73.)*

1. $10 \;?\; 12$ $<$
2. $15 \;?\; 4$ $>$
3. $-10 \;?\; -12$ $>$
4. $-6 \;?\; -20$ $>$
5. $2 \;?\; -2$ $>$
6. $-6 \;?\; 10$ $<$
7. $4 \;?\; -9$ $>$
8. $0 \;?\; 6$ $<$
9. $-7 \;?\; 0$ $<$
10. $-8 \;?\; -14$ $>$
11. $6 \;?\; -7$ $>$
12. $-10 \;?\; 12$ $<$

Graph the solution of the equation. *(See pp. 74–75.)* Check students' graphs.

13. $x + 2 = 4$ $x = 2$
14. $x + 6 = 7$ $x = 1$
15. $x + 9 = 3$ $x = -6$

Graph the solutions of the inequality. *(See pp. 74–75.)* Check students' graphs.

16. $x > 0$
17. $x < 1$
18. $x < -3$

Add. *(See pp. 76–77.)*

19. $(-6) + (-7)$ -13
20. $(-3) + (-8)$ -11
21. $(-4) + 1$ -3
22. $6 + (-6)$ 0
23. $3 + (-2)$ 1
24. $(-8) + 5$ -3
25. $(-5) + (-6)$ -11
26. $(-9) + 5$ -4
27. $4 + (-10)$ -6
28. $(-5) + 9$ 4
29. $(-7) + (-4)$ -11
30. $7 + (-11)$ -4

Subtract. *(See pp. 78–81.)*

31. $8 - (-10)$ 18
32. $-6 - (-2)$ -4
33. $3 - (-1)$ 4
34. $6 - 7$ -1
35. $-8 - 5$ -13
36. $13 - (-10)$ 23
37. $-9 - 3$ -12
38. $8 - (-5)$ 13
39. $6 - 11$ -5
40. $-9 - (-5)$ -4
41. $-5 - (-6)$ 1
42. $3 - (-12)$ 15

Simplify. *(See pp. 82–83.)*

43. $x - 4x$ $-3x$
44. $x - 10 - 4x$ $-3x - 10$
45. $5a - 15 - 5a$ -15
46. $h - 2h + 3$ $-h + 3$
47. $-x - y + 3x$ $2x - y$
48. $6b - 9b + 3$ $-3b + 3$
49. $-b^2 + 9b^2$ $8b^2$
50. $-a - 10 + 8a$ $7a - 10$
51. $-4x + 3y - 2x$ $-6x + 3y$

Solve for x. *(See pp. 90–91.)*

52. $-6 + x = -9$ -3

53. $7x - 6x = -4$ -4

54. $-8 + x = 15$ 23

55. $5x + 9 = 4x$ -9

56. $11x - 4 = 10x$ 4

57. $4x - 3x = -7$ -7

58. $-3 + x = -11$ -8

59. $5x - 4x = -17$ -17

60. $9x + 4 = 8x$ -4

Multiply. *(See pp. 84–87.)*

61. $-8(3)$ -24

62. $6(-9)$ -54

63. $(-3)(-8)$ 24

64. $(-9)(-3)$ 27

65. $-9(8)$ -72

66. $18(-2)$ -36

67. $(y - 5)y$ $y^2 - 5y$

68. $x(x - 4)$ $x^2 - 4x$

69. $-2(x + 3)$ $-2x - 6$

70. $(x - 4)(-5)$ $-5x + 20$

71. $-3(a - b)$ $-3a + 3b$

72. $6(-m + n)$ $-6m + 6n$

Simplify. *(See pp. 84–87.)*

73. $-(x)^2$ $-x^2$

74. $-(-b)^2$ $-b^2$

75. $-(y - 7)$ $-y + 7$

76. $-a - (2a + b)$ $-3a - b$

77. $-(-x - 9)$ $x + 9$

78. $-x - (-3 + y)$ $-x + 3 - y$

79. $(-y)^2$ y^2

80. $-(-a - 4)$ $a + 4$

81. $-y - (6x - 9)$ $-y - 6x + 9$

Divide. *(See pp. 88–89.)*

82. $-81 \div 3$ -27

83. $-72 \div 8$ -9

84. $-12 \div -12$ 1

85. $-16 \div 4$ -4

86. $27 \div 9$ 3

87. $30 \div -3$ -10

88. $-16 \div -4$ 4

89. $24 \div -6$ -4

90. $\frac{48}{-6}$ -8

91. $\frac{-25}{5}$ -5

92. $\frac{-24}{-2}$ 12

93. $\frac{18}{-9}$ -2

Solve for x. *(See pp. 90–91.)*

94. $6x = -30$ -5

95. $8x = -64$ -8

96. $5x - 6x = -8$ 8

97. $5x + 9 = 6x$ 9

98. $-9x = 36$ -4

99. $x + 4 = 3x$ 2

100. $8x - 9x = -2$ 2

101. $4x + 10 = 6x$ 5

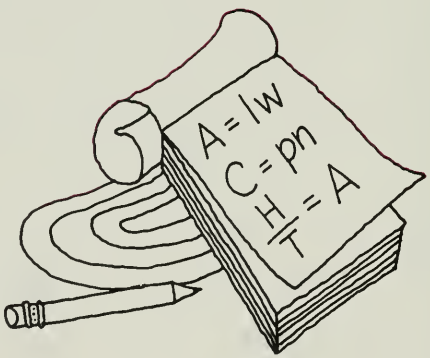
102. $9x - 4x = -15$ -3

A permission-to-reproduce chapter test can be found on page T13.

A cumulative review of Chapters 1–3 can be found on page 438.

Here's what you'll learn in this chapter:

1. To use formulas to find perimeters, areas, and volumes.
2. To make your own formulas for problems.
3. To solve motion and cost problems.
4. To solve a formula for a given variable.



Chapter 4

Formulas

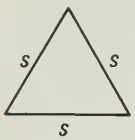


1 • Perimeter Formulas

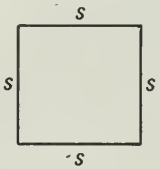


One lap around the inside lane of an Olympic track is 400 meters. We call the distance around a figure its **perimeter.**

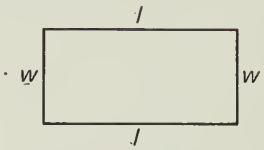
Many figures have special **formulas** for their perimeters.



$P = 3s$



$P = 4s$

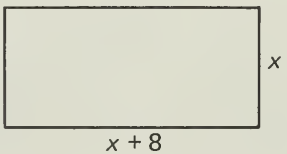


$P = 2l + 2w$

Knowing some formulas, you can solve many problems involving perimeters.

EXAMPLE 1 The length of a rectangle is 8 cm longer than the width. The perimeter is 44 cm. Find the length and width.

Let x = width.
Then $x + 8$ = length.



$P = 2l + 2w$

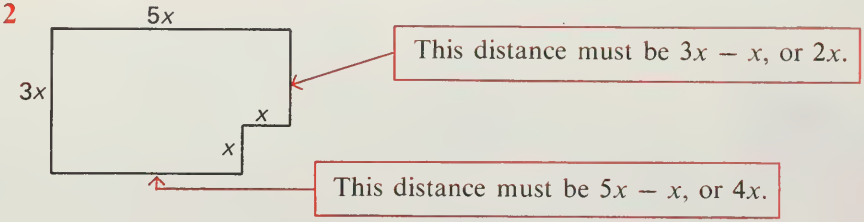
$44 = 2(x + 8) + 2x$
$44 = 2x + 16 + 2x$
$44 = 4x + 16$
$44 - 16 = 4x + 16 - 16$
$28 = 4x$
$7 = x$

Answer: width $= x = 7 \longrightarrow 7$ cm
length $= x + 8 = 15 \longrightarrow 15$ cm

Check: $15 - 7 = 8$
$2 \cdot 15 + 2 \cdot 7 = 30 + 14 = 44$ ✓

For some figures you can make up your own perimeter formula.

EXAMPLE 2

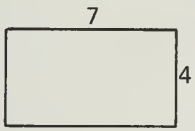


$P = 3x + 5x + 2x + x + x + 4x$
$P = 16x$

Written Exercises

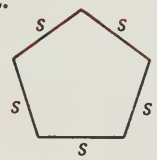
Find the perimeter.

A **1.**



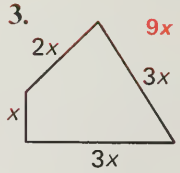
22

2.



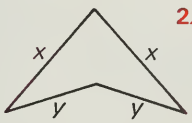
5s

3.



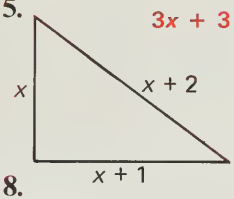
9x

4.



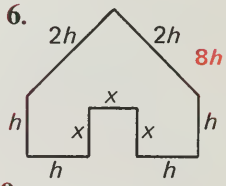
2x + 2y

5.



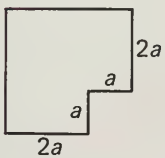
3x + 3

6.



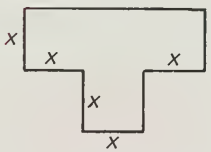
8h + 3x

7.



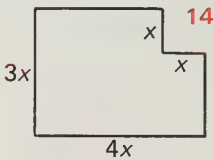
12a

8.



10x

9.

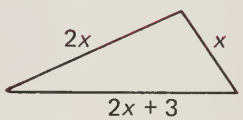


14x

10. The length of a rectangle is 2 cm more than the width. The perimeter is 20 cm. Find the length and width. width: 4 cm; length: 6 cm

11. The length of a rectangle is 5 cm more than the width. The perimeter is 50 cm. Find the length and width. width: 10 cm; length: 15 cm

12. The perimeter of the triangle shown is 48. Find the value of x. $x = 9$



For more practice, see page 414.

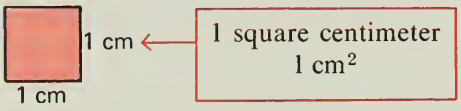
2 • Area Formulas



How much paint do I need to paint the walls of this room?

How much carpeting is needed to cover the floor?

Questions like those above involve area. The **area** of a figure is the amount of its surface. Small areas are often measured in square centimeters.

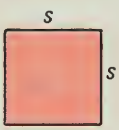


Of course larger regions are measured in square meters or square kilometers.



688,000 km²

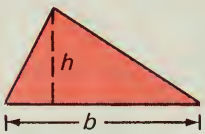
The areas of some figures can be given by formulas.



$A = s^2$

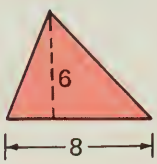


$A = lw$



$A = \frac{1}{2}bh$

EXAMPLE 1

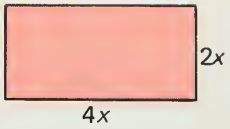


$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2} \cdot 8 \cdot 6$$

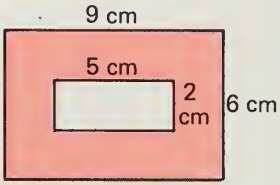
$$A = 24$$

EXAMPLE 2



$A = lw$
$A = (4x)(2x)$
$A = 8x^2$

EXAMPLE 3 Find the area of the shaded region.

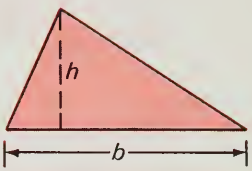


Area = Area large rectangle − Area small rectangle

$A = 9 \cdot 6 - 5 \cdot 2$
$A = 54 - 10$
$A = 44$
Answer: $44\ \text{cm}^2$

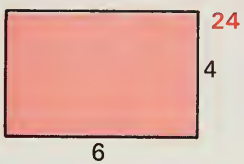
Classroom Practice

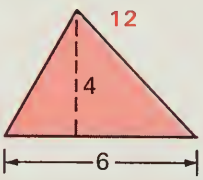
The formula for the area of a triangle is $A = \frac{1}{2}bh$.

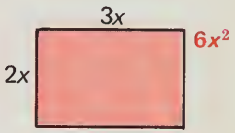


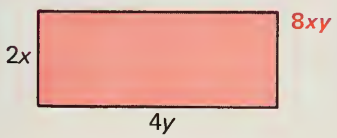
1. Can you guess what the letter b stands for? length of the base
2. Can you guess what the letter h stands for? height
3. In the formula $A = lw$, what do the letters l and w stand for? length and width

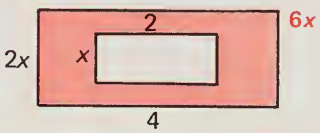
Find the area of the shaded figure.

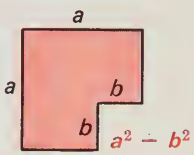
4. 

5. 

6. 

7. 

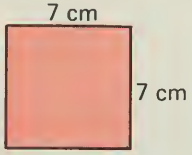
8. 

9. 

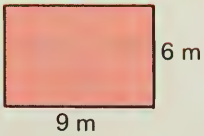
Written Exercises

Find the area.

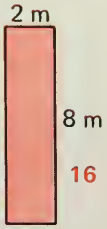
A

1. 

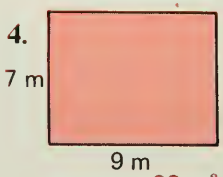
49 cm²

2. 

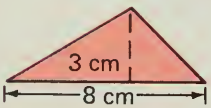
54 m²

3. 

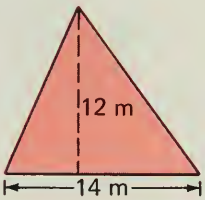
16 m²

4. 

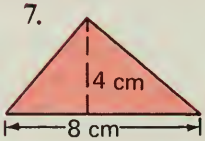
63 m²

5. 

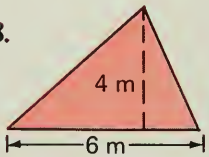
12 cm²

6. 

84 m²

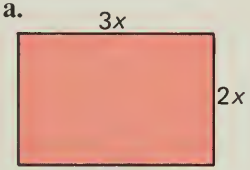
7. 

16 cm²

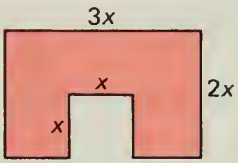
8. 

12 m²

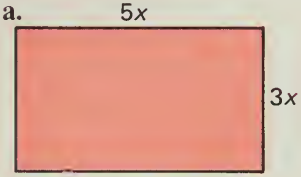
Find the area of the shaded figure. Give your answer in terms of the variables.

9.a. 

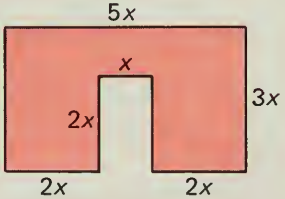
$6x^2$

b. 

$5x^2$

10.a. 

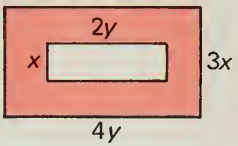
$15x^2$

b. 

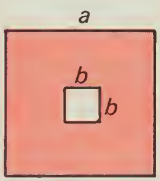
$13x^2$

11.a. 

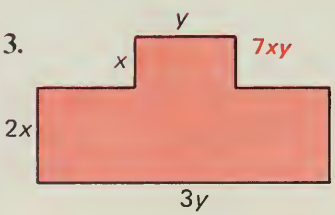
$12xy$

b. 

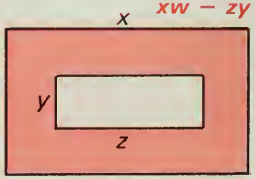
$10xy$

12. 

$a^2 - b^2$

13. 

$7xy$

14. 

$xw - zy$

15.



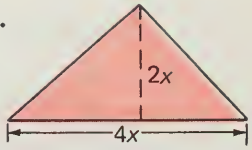
$25y^2$

16.



$20b^2$

17.



$4x^2$

18. Would the area of a rectangle be given in *centimeters* or in *square centimeters?* square centimeters

19. Would the perimeter of a rectangle be given in *centimeters* or in *square centimeters?* centimeters

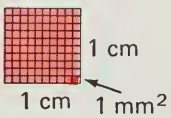
20. A nylon rug is priced at \$12 per square meter.
How much would it cost to buy a rug 5 meters long and 3 meters wide? \$180

21. It costs \$6 per square meter to pave a driveway.
How much would it cost to pave a driveway 20 meters long and 4 meters wide? \$480

22. You know that 1 cm = 10 mm.

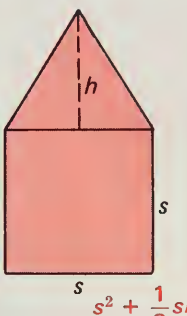
The following is then true.

$1\text{ cm}^2 = \underline{?}\ \text{mm}^2$ 100



Find the area.

B **23.**



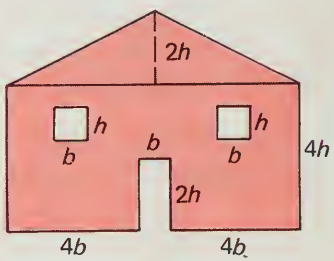
$s^2 + \frac{1}{2}sh$

24.



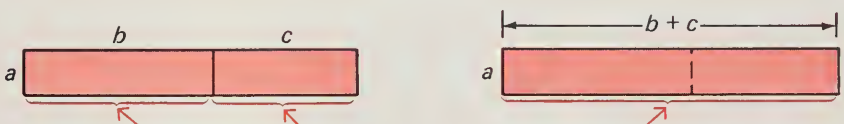
$wh + lw$

25.



$41bh$

26. What property is shown by the diagram and equations below?



This area + this area = this area

$ab + ac = a(b + c)$

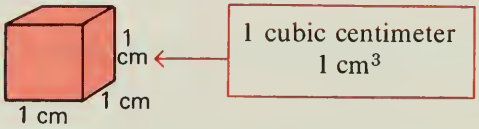
distributive property

For more practice, see page 414.

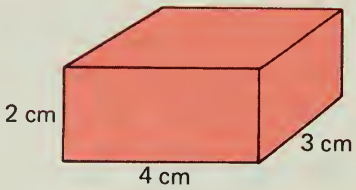
3 • Volume Formulas



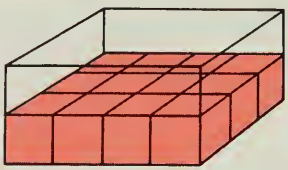
The **volume** of a telephone booth is the amount of space inside it. Volume can be measured in many units. One common unit is the cubic centimeter.



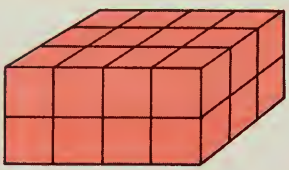
Suppose you want to find the volume of a box like this.



Think of centimeter cubes filling the base of the box. Twelve cubes will fit in the bottom layer.



Two layers of cubes will fit in the box. This means 2 · 12, or 24 cubes will fill the box. The volume is 24 cubic centimeters.

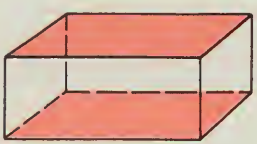


The box above is called a **prism.** In a prism, the bases are parallel and are the same size and shape.

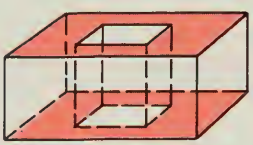
Volume of a Prism

Volume = Base Area × height

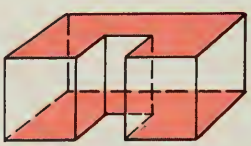
$V = Bh$



$V = Bh$

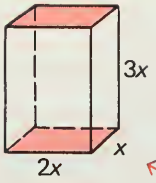


$V = Bh$



$V = Bh$

EXAMPLE 1

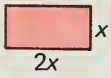


$V = Bh$

$V = (2x)(x)(3x)$

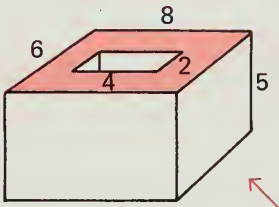
$V = 6x^3$

The bases are rectangles.



Base Area: $(2x)(x)$

EXAMPLE 2

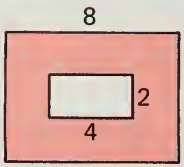


$V = Bh$

$V = 40 \cdot 5$

$V = 200$

The two bases look like this.

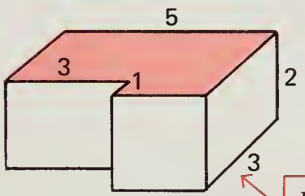


Base Area:

$(6 \cdot 8) - (4 \cdot 2) = 48 - 8$

$= 40$

EXAMPLE 3



$V = Bh$

$V = 12 \cdot 2$

$V = 24$

Base Area:

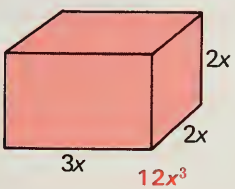
$(5 \cdot 3) - (3 \cdot 1) = 15 - 3$

$= 12$

Classroom Practice

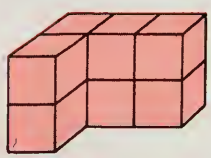
Find the volume.

1.



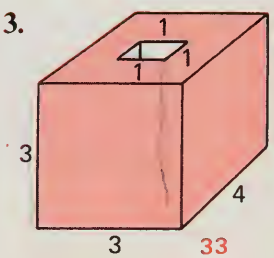
$12x^3$

2.



8

3.



33

Another unit of volume often used is the liter. (The symbol is the letter ℓ.) A liter is a little more than a quart.

1 liter = 1000 cubic centimeters

Complete.

4. $3\ell =$ ___?___ cm^3 3000

5. $\frac{1}{2}\ell =$ ___?___ cm^3 500

6. ___?___ $\ell = 4000\ cm^3$ 4

7. ___?___ $\ell = 4500\ cm^3$ $4\frac{1}{2}$

Written Exercises

Draw a sketch to help you.

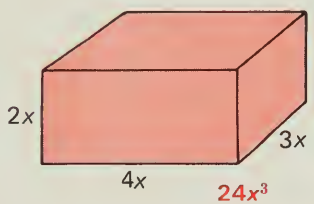
A 1. A box has a rectangular base which measures 9 cm by 5 cm. The height of the box is 4 cm. Find the volume. 180 cm^3

2. Find the volume of a cube 4 cm on a side. 64 cm^3

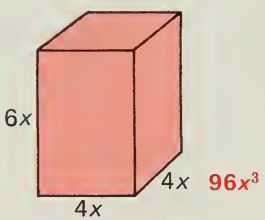
3. A box measures 20 cm by 15 cm by 10 cm. Its volume is ___?___ cm^3, or ___?___ liters.
3000 3

4. A box measures 40 cm by 30 cm by 50 cm. Its volume is ___?___ cm^3, or ___?___ liters.
60,000 60

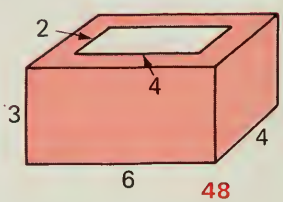
Find the volume.

5. 

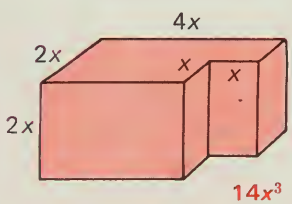
$24x^3$

6. 

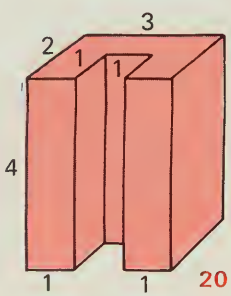
$96x^3$

7. 

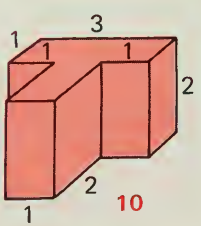
48

8. 

$14x^3$

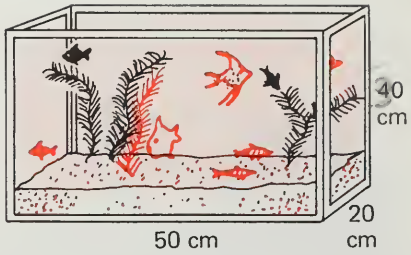
9. 

20

10. 

10

Exercises 11–14 relate to the aquarium shown.



B **11.** Find the volume in cubic centimeters. 40,000 cm^3

12. How many liters of water will it hold? (1 liter = 1000 cubic centimeters.) 40 liters

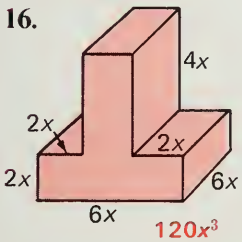
13. How many grams of water is this? (1 cubic centimeter holds 1 gram.) 40,000 grams

14. (How many kilograms will the water be? (1 kilogram = 1000 grams.) 40 kilograms

15. Will an aquarium measuring 45 cm by 30 cm by 30 cm hold more or less than the aquarium in the drawing? 40,500 cm^3; more

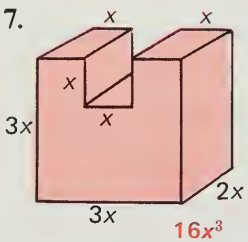
Find the volume.

16.



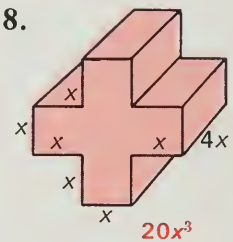
$120x^3$

17.



$16x^3$

18.



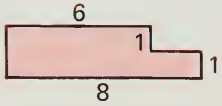
$20x^3$

For more practice, see page 415.

SELF-TEST

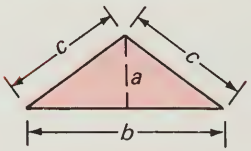
1. What is the perimeter of the figure? 20

2. What is the area of the figure? 14



3. What is the perimeter of the figure? $b + 2c$

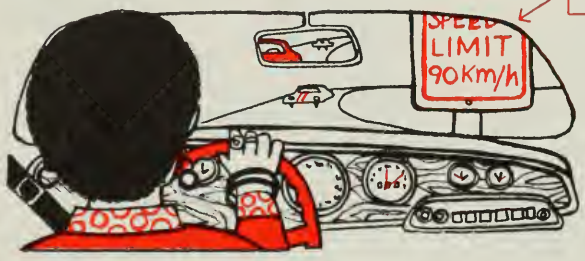
4. What is the area of the figure? $\frac{1}{2}ab$



5. A box has rectangular bases which measure 12 cm by 9 cm. The height of the box is 6 cm. Find the volume. 648 cm^3

4 • Writing Formulas

A formula is a general rule for solving a problem. It tells you *how* to solve the problem, even if you do not know all the numbers.



90 km/h means 90 kilometers per hour.

Suppose you drive 90 km/h for 3 hours. How would you find the distance you traveled? You would *multiply*.

$$90 \times 3 = 270$$

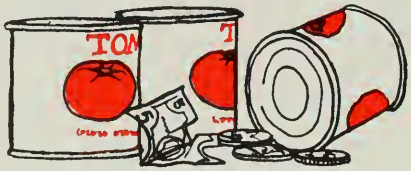
Answer: 270 km

Here's a formula to find distance.

Distance = rate × time

$D = rt$

Now think of some cost problems. How would you figure the cost of 3 cans of tomatoes at 40¢ each? Multiply!



$$40 \times 3 = 120$$

Answer: \$1.20

Here's a formula to find cost.

Cost = price per item × number of items

$C = pn$

Classroom Practice

Complete.

1. You travel at 60 km/h for n hours. Your distance $D =$ __?__. 60n

2. You travel at x km/h for n hours. Your distance $D =$ __?__. nx

3. You buy n colas at 35¢ each. The cost $C =$ __?__. 35n

4. You buy s sodas at y cents each. The cost $C =$ __?__. sy

Written Exercises

Complete.

A **1.** You bike at x km/h for t hours.
Your distance $D =$ __?__. xt

2. You walk for y hours at w km/h.
Your distance $D =$ __?__. wy

3. You buy 6 donuts at x cents each.
The cost $C =$ __?__. $6x$

4. You buy m records at n dollars each.
The cost $C =$ __?__. mn

5. Making a basket in a basketball game scores 2 points.
Making b baskets scores __?__ points. $2b$



6. Making a free throw scores 1 point.
Making f free throws scores __?__ points. f

7. A player makes b baskets and f free throws.
The total $T =$ __?__ points. $2b + f$

8. A telephone operator earns d dollars per hour.
The pay for 35 hours is __?__. $35d$

9. You make \$3 per hour.
Your pay for x hours is __?__. $3x$

10. You earn d dollars per hour.
You work for n hours.
Your pay $P =$ __?__. dn

11. You buy something for b dollars.
You sell it for s dollars.
You make a profit.
Your profit $P =$ __?__. $s - b$

12. You buy something for b dollars.
You sell it for s dollars.
You take a loss.
Your loss $L =$ __?__. $b - s$

B **13.** Buy 3 hotdogs at h cents each.
Buy 2 colas at c cents each.
Total cost $C =$ __?__. $3h + 2c$

14. Salary: \$2 per hour for x hours.
Overtime: \$3 per hour for y hours.
Total pay $P =$ __?__. $2x + 3y$

15. A can contains 3 tennis balls.
Each ball is b grams.
The can alone is c grams.
The total is __?__ grams. $3b + c$

16. A phone call costs x cents for the first 3 minutes and y cents for each additional minute.
A 10-minute call costs __?__. $x + 7y$

5 • Motion Formulas

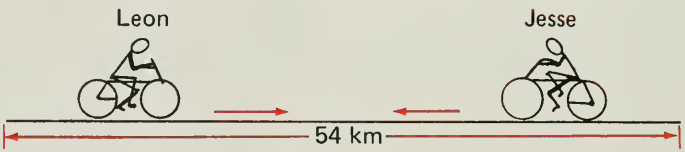
In the next two sections you will be using formulas to solve motion problems and cost problems. In all of these, you will use basically the same problem-solving methods you used in Chapter 2.

A Guide for Problem Solving

1. Read the problem. Read it more than once, if you wish.
2. Let a variable stand for one of the unknowns. Show the other unknowns in terms of that variable.
3. Write an equation and solve it.
4. Answer the question.
5. Check your answer.

EXAMPLE 1 Leon and Jesse live 54 km apart. At noon each boy rides his bike toward the other. Leon travels at 15 km/h. Jesse travels at 12 km/h. At what time will they meet?

Drawing a sketch for yourself often helps.



Let t = number of hours until they meet.

Making a table helps a lot.

	rate	× time	= Distance
Leon	15	t	$15t$
Jesse	12	t	$12t$
			54 ← total distance

$$\underbrace{\text{Leon's distance}} + \underbrace{\text{Jesse's distance}} = 54$$

$$15t + 12t = 54$$
$$27t = 54$$
$$t = 2$$

Answer: The boys will meet 2 hours after they start, at 2:00 P.M.

Check: Leon goes $15 \cdot 2 = 30$ km
Jesse goes $12 \cdot 2 = 24$ km
54 km ✓

EXAMPLE 2 Terry can ride her bike 2 km/h faster than Gina.
They travel from school in opposite directions for 2 hours.
They are then 84 km apart.
How fast does each girl travel?



Let r = Gina's speed.
Then $r + 2$ = Terry's speed.

	rate	× time	= Distance
Gina	r	2	$2r$
Terry	$r + 2$	2	$2(r + 2)$
			84 ← total distance

Gina's distance + Terry's distance = 84

$$\begin{aligned} 2r + 2(r + 2) &= 84 \\ 2r + 2r + 4 &= 84 \\ 4r + 4 &= 84 \\ 4r &= 80 \\ r &= 20 \end{aligned}$$

Answer: Gina's speed: $r = 20 \longrightarrow$ 20 km/h
Terry's speed: $r + 2 = 22 \longrightarrow$ 22 km/h

Check: Gina goes $2 \cdot 20 = 40$ km
Terry goes $2 \cdot 22 = 44$ km
84 km ✓

Classroom Practice

1. Two cars are 360 km apart.
They travel toward each other.
One travels at 80 km/h.
The other travels at 100 km/h.
How long will it take them to meet?

2 hours

	rate	× time	= Distance
1st car	80	t	$80t$
2nd car	100?	t ?	$100t$?
			360

2. Gloria walks 1 km/h faster than Mary Lou.
They walk in opposite directions for 2 hours.
They are then 30 km apart.
How fast do they walk? Mary Lou: 7 km/h; Gloria: 8 km/h

	rate × time = Distance		
Gloria	? $x + 1$	? 2	? $2(x + 1)$
Mary Lou	x	2	? $2x$
			? 30

Written Exercises

A 1. Bill and Mike are 56 km apart.
Each boy travels toward the other.
Bill travels at 20 km/h.
Mike travels at 8 km/h.
After how long do they meet? 2 hours

	rate × time = Distance		
Bill	20	t	$20t$
Mike	? 8	? t	? $8t$
			56

2. At noon, a train leaves Omaha to go to Mobile at 100 km/h.
At the same time a train leaves Mobile for Omaha at 120 km/h.
The distance between the cities is 1760 km.
At what time do the trains pass each other? 8:00 P.M.



	rate × time = Distance		
Omaha—Mobile	100	t	? $100t$
Mobile—Omaha	? 120	? t	? $120t$
			? 1760

3. Kate bikes 4 km/h faster than Barb.
Kate travels east for 2 hours.
Barb travels west for 2 hours.
They are then 80 km apart.
How fast is each traveling?
Barb: 18 km/h; Kate: 22 km/h

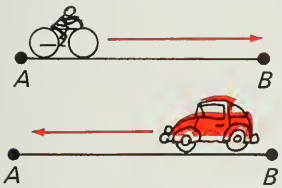
	rate × time = Distance		
Barb	r	? 2	? $2r$
Kate	$r + 4$	? 2	? $2(r + 4)$
			? 80

4. Paula and Tony start at their office.
They drive in opposite directions for 3 hours.
They are then 450 km apart.
Paula's speed is 80 km/h.
What is Tony's speed? 70 km/h



	rate × time = Distance		
Paula	80	? 3	? 240
Tony	r	? 3	? 3r
			? 450

5. A man bikes from A to B at 20 km/h.
He returns by car at 60 km/h.
The bike trip is 2 hours longer than the car trip.
How far is it from A to B? 60 km



	rate × time = Distance		
Bike	? 20	t + ? 2	? 20(t + 2)
Car	60	t	60t

Hint: These two distances are equal.

6. Cindy bikes from A to B at 20 km/h.
She returns by car at 80 km/h.
The bike trip is 3 hours longer than the car trip.
How far is it from A to B? 80 km

	rate × time = Distance		
Bike	20	? + 3	? 20(t + 3)
Car	? 80	t	? 80t

B **7.** Ralph bikes twice as fast as Sam jogs.
In $\frac{1}{2}$ hour Ralph can go 6 km farther than Sam.
How fast can Sam jog? 12 km/h

	rate × time = Distance		
Sam	x	$\frac{1}{2}$	? $\frac{1}{2}x$
Ralph	$2x$	$\frac{1}{2}$	? x

6 • Cost and Money Problems

In section 4 of this chapter you worked with a formula to find total cost.

$$\text{Cost} = \text{price per item} \times \text{number of items}$$
$$C = pn$$

There are many problems which can make use of this formula.



Gail bought some donuts at 10¢ each.
She ate 2 of them and sold the rest at 15¢ each.
Her profit was $1.50.
How many donuts did she buy?

Let n = number of donuts Gail bought.
Then $n - 2$ = number of donuts Gail sold.

	$p \times n = C$			
Sell	15	$n - 2$	$15(n - 2)$ ←	Selling price
Buy	10	n	$10n$ ←	Buying price
			150 ←	Profit

$$\underbrace{\text{Selling price}} - \underbrace{\text{Buying price}} = \text{Profit}$$

$$15(n - 2) - 10n = 150 \quad \text{(Since the prices are given in cents, you write \$1.50 as 150¢.)}$$
$$15n - 30 - 10n = 150$$
$$5n - 30 = 150$$
$$5n = 180$$
$$n = 36$$

Answer: Gail bought 36 donuts.

Check: Sold 34 at 15¢ each → $34 \times 15¢ = 510¢$
Bought 36 at 10¢ each → $36 \times 10¢ = 360¢$ ← Subtract.
Profit = 150¢ ✓

Classroom Practice

1. Marcia bought some apples at 5¢ each.
She sold all but 4 of them at 10¢ each.
Her profit was $1.60.
How many apples did she buy? 40 apples

	price × number = Cost		
Sell	10	$n - 4$?	? ← Selling price $10n - 4$
Buy	5	n	? ← Buying price $5n$
			? ← Profit 160

2. I bought several notebooks at $.80 each.
I kept 2 and sold the rest at $1.00 each.
I didn't make any money, but I didn't lose any either.
How many notebooks did I buy? 10 notebooks

	price × number = Cost		
Sell	?100	$?x - 2$	? ← Selling price $100(x - 2)$
Buy	80	x	? ← Buying price $80x$
			? ← Profit 0

3. A dress shop bought 15 dresses at $20 each.
They sold some of them at $35 each.
They sold the rest at a sale price of $28.
Their profit was $190.
How many dresses were sold at the sale price? 5 dresses

Written Exercises

A **1.** Jake bought some donuts at 10¢ each.
He ate 4, and sold the rest at 15¢ each.
His profit was $2.40.
How many donuts did he buy?
60 donuts

	price × number = Cost		
Sell	15	$n - 4$	$?15(n - 4)$
Buy	10	n	$?10n$
			240

2. Midtown TV Center bought several Model LB370 TV sets.
Each set cost them \$400.
They gave away one set, and sold the others for \$500 each.
Their profit was \$1100.
How many sets did they sell? 15 sets

	price	× number	= Cost
Sell	500	$n - 1$	? $500(n - 1)$
Buy	400	n	? $400n$
			? 1100



3. Hank bought several ball-point pens at 40¢ each.
He kept 2 and sold the rest at 60¢ each.
He neither gained nor lost any money.
How many pens did he buy? 6 pens

	price	× number	= Cost
Sell	? 60	$?_{x - 2}$	? $60(x - 2)$
Buy	40	x	? $40x$
			0

4. Mr. and Mrs. Haley both make \$3 an hour at work.
One week, Mr. Haley worked 4 hours more than Mrs. Haley.
Their combined pay for the week came to \$228.
How many hours did each work? Mr. Haley: 40 hours; Mrs. Haley: 36 hours

	pay per hour	× number of hours	= Weekly pay
Mrs. Haley	3	x	$?_{3x}$
Mr. Haley	? 3	? $x + 4$	$?_{3(x + 4)}$
			$?_{228}$

Combined pay

5. Lana earns \$1 more per hour than her husband Tom.
After working a 40-hour work week, their combined pay totaled \$280.
How much did each earn per hour?
(Make a chart like the one in Exercise 4.) Lana: \$4/hr; Tom: \$3/hr

6. Tickets to the class play were \$1 for students and \$2 for adults.
There were 80 more student tickets sold than adult tickets.
The ticket sales totaled \$1280.
How many student tickets were sold? 480 tickets

	price	×	number	=	Cost	
Student tickets	? 1		x?+ 80		?	$x + 80$
Adult tickets	? 2		x		?	$2x$
					1280	

B 7. Ticket sales for the Glee Club concert totaled \$1200.
The adult tickets were \$2 each.
Student tickets were \$1 each.
There were twice as many adult tickets sold as student tickets.
How many of each kind of ticket were sold? 480 adult tickets, 240 student tickets

8. For a local charity, people pledged \$15 or \$25 donations.
Twice as many people pledged \$15 as those who pledged \$25.
Altogether, \$5,500 was pledged.
How many people pledged \$15? 200 people

For more practice, see page 415.

SELF-TEST

1. You walk for k hours at m km/h.
Your distance $D =$ ___?___. km

2. You buy x hotdogs at y cents each.
The cost $C =$ ___?___. xy

3. You earn s dollars per hour.
You work for t hours.
Your pay $P =$ ___?___. st

4. You buy something for d dollars.
You sell it for c dollars.
Your profit $P =$ ___?___. $c - d$

5. Steven and Len start at the same place.
They drive in opposite directions for 2 hours.
They are then 310 km apart.
Len's speed is 75 km/h.
What is Steven's speed? 80 km/h

6. Lou and Bill Peters each earn \$4 an hour at work.
One week, Bill worked 6 hours more than Lou.
Their combined pay that week was \$304.
How many hours did each person work? Lou: 35 hours; Bill: 41 hours



Suppose a race car in the Indianapolis 500-mile race finished the course in 3 hours. You could figure the average rate, or speed, in two ways.

Method 1

$$D = rt$$

Substitute. → $500 = r \cdot 3$

Solve for r. → $\frac{500}{3} = r$

$$166\frac{2}{3} = r$$

Method 2

$$D = rt$$

$\frac{D}{t} = r$ ← Solve for r.

$\frac{500}{3} = r$ ← Substitute.

$$166\frac{2}{3} = r$$

The car went $166\frac{2}{3}$ miles per hour!

In the first step of Method 2, we solved the formula $D = rt$ for r. Here are some other examples that show how to solve a formula.

EXAMPLE 1 Solve $A = lw$ for w.

$$A = lw$$

$$\frac{A}{l} = \frac{lw}{l}$$

$$\frac{A}{l} = w$$

Of course, you can also write $w = \frac{A}{l}$.

EXAMPLE 2 Solve $P = S - B$ for S.

$$\begin{aligned} P &= S - B \\ P + B &= S - B + B \\ P + B &= S \end{aligned}$$

← Or, $S = P + B$.

EXAMPLE 3 Solve $4y + x = 11$ for x.

$$\begin{aligned} 4y + x &= 11 \\ 4y + x - 4y &= 11 - 4y \\ x &= 11 - 4y \end{aligned}$$

EXAMPLE 4 Solve $2x + y = 9$ for y.

$$\begin{aligned} 2x + y &= 9 \\ 2x + y - 2x &= 9 - 2x \\ y &= 9 - 2x \end{aligned}$$

EXAMPLE 5 Solve $3x - y = 7$ for y.

$$\begin{aligned} 3x - y &= 7 \\ 3x - y - 3x &= 7 - 3x \\ -y &= 7 - 3x \\ -1(-y) &= -1(7 - 3x) \\ y &= -7 + 3x \end{aligned}$$

← Multiply both sides by -1 so that you have solved the equation for y, not $-y$.

Classroom Practice

Solve for x.

1. a. $5x = 15$ 3 **b.** $5x = y$ $\frac{y}{5}$ **c.** $ax = y$ $\frac{y}{a}$

Solve for y.

2. a. $y + 2 = 9$ 7 **b.** $y + 2 = a$ $a - 2$ **c.** $y + x = a$ $a - x$

Solve for s.

3. a. $8 + s = 11$ 3 **b.** $8 + s = t$ $t - 8$ **c.** $r + s = t$ $t - r$

4. Solve $A = lw$ for l. $l = \frac{A}{w}$

5. Solve $F = ma$ for m. $m = \frac{F}{a}$

6. Solve $F = ma$ for a. $a = \frac{F}{m}$

Written Exercises

Solve for y.

A **1.** a. $y - 5 = 7$ 12 b. $y - x = 7$ $x + 7$ c. $y - x = t$ $x + t$

2. a. $y + 5 = 7$ 2 b. $y + x = 7$ $7 - x$ c. $y + x = t$ $t - x$

3. $2 + y = 5$ 3 **4.** $2x + y = 4$ $4 - 2x$ **5.** $3 + y = 11$ 8 **6.** $4s + y = 9$ $9 - 4s$

7. $y - 2x = 5$ $2x + 5$ **8.** $y + 3x = 7$ $7 - 3x$ **9.** $7 - y = 5$ 2 **10.** $3x - y = 2$ $3x - 2$

Solve for x.

11. a. $8x = 24$ 3 b. $8x = y$ $\frac{y}{8}$ c. $ax = y$ $\frac{y}{a}$

12. a. $3x = 45$ 15 b. $3x = y$ $\frac{y}{3}$ c. $kx = y$ $\frac{y}{k}$

13. $3 + x = 11$ 8 **14.** $y + x = 9$ $9 - y$ **15.** $2y + x = 4$ $4 - 2y$ **16.** $x + 3y = 9$ $9 - 3y$

17. $4y + x = 6$ $6 - 4y$ **18.** $x - 3y = 5$ $3y + 5$ **19.** $y - x = 5$ $y - 5$ **20.** $5y - x = 10$ $5y - 10$

21. Solve $A = 8w$ for w. $w = \frac{A}{8}$ **22.** Solve $A = Pr$ for r. $r = \frac{A}{P}$

23. Solve $D = rt$ for t. $t = \frac{D}{r}$ **24.** Solve $E = 360n$ for n. $n = \frac{E}{360}$

25. Solve $N = a + b$ for b. $b = N - a$ **26.** Solve $P = a + b + c$ for c. $c = P - a - b$

Some common formulas are given in Exercises 27–29. You don't have to know how they are used, but their meanings are given, just for interest.

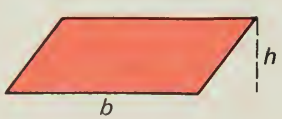
27. Solve for d: $W = fd$ ← work = force × distance $d = \frac{W}{f}$

28. Solve for V: $W = AV$ ← watts = amperes × volts $V = \frac{W}{A}$

29. Solve for H: $\frac{H}{T} = A$ ← hits ÷ times at bat = batting average $H = AT$

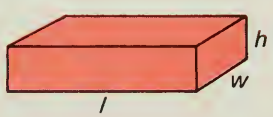
Complete.

30.



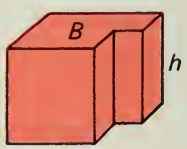
$A = bh$
$A = 84, b = 14$
$h =$? 6

31.



$V = lwh$
$V = 100, l = 10$
$w = 5, h =$? 2

32.

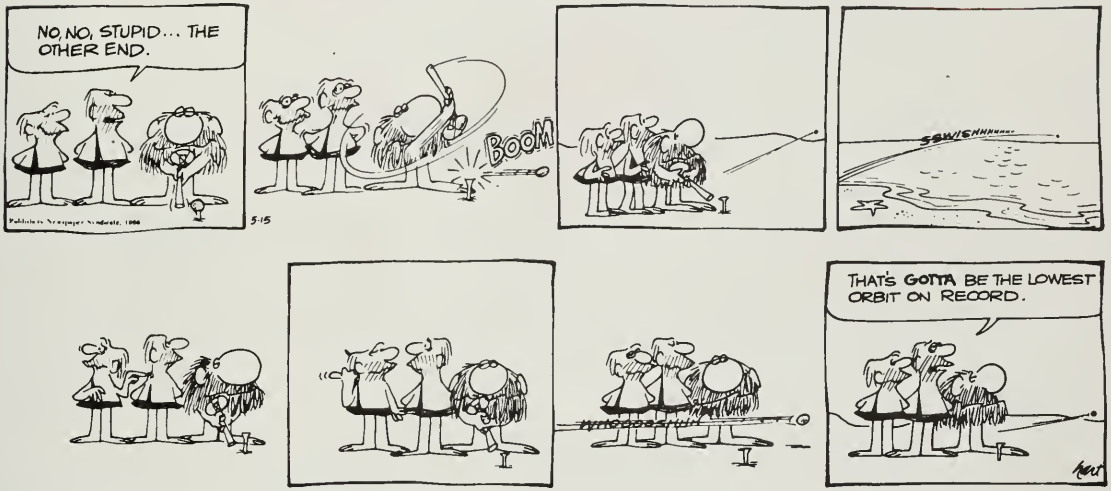


$V = Bh$
$V = 450, B = 50$
$h =$? 9

For Exercises 33–34 you'll want to use the formula $D = rt$.

33. The Grand Prix of Monaco is a 256 kilometer race. Suppose the winning car one year took exactly 2 hours. What was the average speed? 128 km/h

34. The famous Le Mans race is won by the automobile which goes the greatest distance in 24 hours. The winning distance for the first Le Mans race in 1923 was 2208 km. What was the average rate? 92 km/h



B.C. by permission of John Hart and Field Enterprises, Inc.

B **35.** Suppose a golf ball is in orbit around Earth. Suppose the orbital distance in the cartoon above is 48,000 km. Suppose it took the golf ball 4 hours to orbit. What was its rate? 12,000 km/h

$$\text{orbital distance} = \text{rate} \cdot \text{time}$$

For more practice, see page 415.

SELF-TEST

Solve for x.

1. a. $9x = 45$ 5 b. $9x = y$ $\frac{y}{9}$ c. $ax = y$ $\frac{y}{a}$

2. a. $x + 2 = 20$ 18 b. $x + 2 = y$ $y - 2$ c. $x + k = y$ $y - k$

Solve for y.

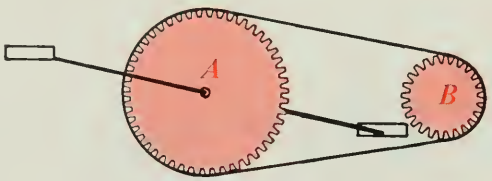
3. $y - x = 10$ $x + 10$ **4.** $2x + y = 8$ $8 - 2x$ **5.** $2x - y = 8$ $2x - 8$

6. $y - 8 = 2x$ $2x + 8$ **7.** $y + 3x = 9$ $9 - 3x$ **8.** $xy = 12$ $\frac{12}{x}$

8 • Formulas from Mechanics (Optional)

Did you know that there is a formula relating the speeds of bicycle gears? Example 1 explains this formula.

EXAMPLE 1 The pedal gear A on a bicycle has 48 teeth. The wheel gear B has 24 teeth. A chain links both gears so that when you turn gear A, gear B will turn. If A turns at 1 turn per minute, how fast will B turn?



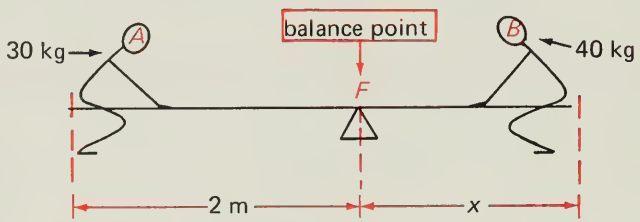
Gear Formula → $\underbrace{\text{speed of } A}_{1} \times \underbrace{\text{number of teeth in } A}_{48} = \underbrace{\text{speed of } B}_{s} \times \underbrace{\text{number of teeth in } B}_{24}$

$$\begin{aligned} 1 \times 48 &= s \times 24 \\ 48 &= 24s \\ 2 &= s \end{aligned}$$

Answer: 2 turns per minute

The gear formula is like the lever formula. The following ideas will not seem surprising to you if you have ever been on a seesaw.

EXAMPLE 2 Study the lever (seesaw) below. Then find x.

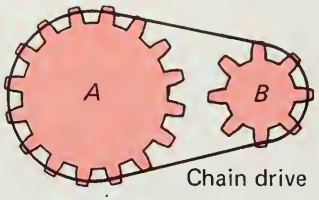


Lever Formula → $\underbrace{A}_{30} \times \underbrace{\text{distance of } A \text{ from } F}_{2} = \underbrace{B}_{40} \times \underbrace{\text{distance of } B \text{ from } F}_{x}$

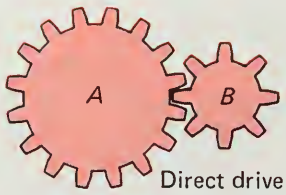
$$\begin{aligned} 30 \times 2 &= 40 \times x \\ 60 &= 40x \\ 1\tfrac{1}{2} &= x \end{aligned}$$

Written Exercises

The diagrams show gear *A* driving gear *B*.



Chain drive



Direct drive

A **1.** In the first diagram, when *A* turns clockwise, gear *B* turns ___?___ (clockwise or counterclockwise). clockwise

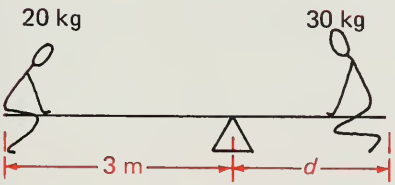
2. In the second diagram, when *A* turns clockwise, then *B* turns ___?___ (clockwise or counterclockwise). counterclockwise

3. In the second diagram, count the number of teeth for both gears. When *A* turns at a speed of 2 turns per second, then how fast does *B* turn? 4 turns/second

4. On a certain five-speed bike, the pedal gear has 52 teeth. The wheel gear used for going up hills has 13 teeth. Suppose you turn the pedal gear at a speed of 1 turn per second. How fast will the first gear turn? 4 turns/second

5. Study the lever shown and find the value of *d*. 12 m

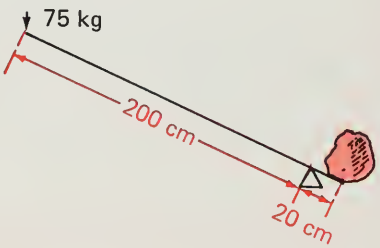




6. Two children, 20 kilograms and 30 kilograms, are balanced on a seesaw as shown. What is the distance from the balancing point to the bigger child? 2 m

7. A person pushes on the end of the crowbar shown at the right, and is just able to move the large rock. About how many kilograms is the rock? 750 kilograms

(*Hint:* Pretend the person is putting a 75-kilogram object on the end of the crowbar.)



CONSUMER CORNER



Balancing Your Checkbook

When you have a checking account you have to be careful to keep track of how much money you have spent and how much money you have left in your account. Here are some tips on how to balance your account.

Be sure to record all checks *when* you write them. It's just too easy to forget about the check in only a few hours.

This is a check stub for the check you have just written.

No. 5	$ 10		Alison Rc
Nov. 20	19 33		
To Ames Theater			
Bal.	45	25	Pay to
Deposit			
Total	45	25	
This check	10	00	
Bal. For'd.	35	25	

After writing the check, be sure to figure how much money you have left in your account.

When you deposit money into your account, remember to save the deposit slips. Record your deposit on a check stub too.

First City Bank
Checking Account Deposit

Name Alison Rogers
Date Nov. 14 19 33
Account No. 881019564

CASH	380	00
CHECKS		

When you get a bank statement, you will also get all those checks which have been processed and cancelled by the bank.

First City Bank

ACCOUNT NUMBER
881019564

ALISON ROGERS
18 OAK BLUFFS RD.
OCEANSIDE, NJ

PREVIOUS BALANCE
$87.21

DATE	CHECKS		DEPOSITS		BALANCE
10/20	20.00				67.21
10/22	3.00				64.21
10/30	17.25				46.96
11/4	6.47				40.49
11/14	19.30		80.00		101.19
11/14	14.66	4.62			81.91

The final bank balance

Here's what you can do to make sure your checkbook balances.

1. Did you make any deposits which are not on the statement? Add that amount to the balance on the statement.

2. Arrange the returned cancelled checks by number. Mark on your check stubs those checks that have been processed.

3. Have you written some checks which have not been processed? Subtract the total of these checks from the balance you found in step 1.

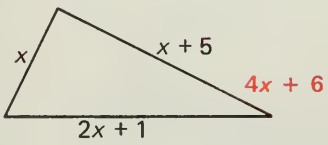
4. Does the final figure check with the balance you have on the last stub in your checkbook? It should!

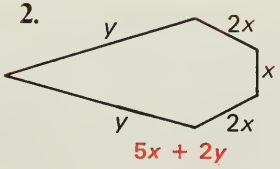
Does Alison Rogers' checkbook balance? Here's her work so far.

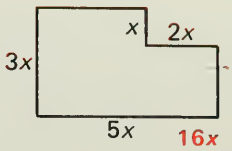
Deposits not on Statement	Checks not Processed	Balance in Checkbook
$45.00	$10.00 13.88 24.75	$78.28

Reviewing the Chapter

Find the perimeter. *(See pp. 104–105.)*

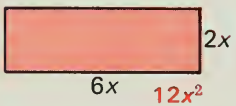
1. 

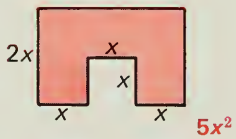
2. 

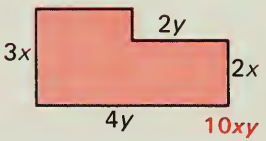
3. 

4. The length of a rectangle is 7 cm more than the width. The perimeter is 46 cm. Find the length and width. length: 15 cm; width: 8 cm

Find the area. *(See pp. 106–109.)*

5. 

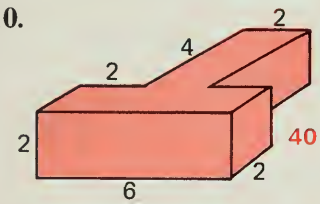
6. 

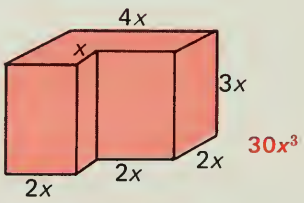
7. 

8. A rug is priced at \$11 per square meter. How much would it cost to buy a rug 4 meters long and 3 meters wide? \$132

9. One can of paint will cover 5 square meters of area. How many cans are needed to paint the side of a barn that is 10 meters long and 8 meters high? 16 cans

Find the volume. *(See pp. 110–113.)*

10. 

11. 

12. A box measures 50 cm by 30 cm by 20 cm. Its volume is __?__ cm^3, 30,000 or __?__ liters. 30

13. Find the volume of a box that measures $5x$ units by $3x$ units by $2x$ units. $30x^3$

Complete. *(See pp. 114–115.)*

14. You travel at 90 km/h for h hours. Your distance D = __?__. $90h$

15. You buy 4 pizzas at \$$x$ each. The cost C = __?__. $4x$

Solve. *(See pp. 116–123.)*

16. You have a bat and m balls.
The bat is x grams.
Each ball is y grams.
Together the bat and balls are ___?___. $x + my$

17. Vince bikes for 2 hours at 18 km/h.
Then he rides by car for 3 hours.
His total distance is 246 km.
How fast does the car travel? 70 km/h

	rate × time = Distance		
Bike	? 18	? 2	? 36
Car	? r	? 3	? $3r$
			246

18. Judy can bike 2 km/h faster than Linda.
They travel in opposite directions for 2 hours.
They are then 68 km apart.
How fast is each girl traveling? Judy: 18 km/h; Linda: 16 km/h

	rate × time = Distance			
Judy	r	? 2	?	$2r$
Linda	?	? 2	?	$2(r - 2)$
	$r - 2$		?	68

19. Bailey's Bargain Barn bought several electronic calculators.
Each calculator cost them \$9.
They gave 2 calculators away and sold the rest for \$15 each.
Their profit was \$570.
How many calculators did they sell? 98 calculators

	rate × number = Cost			
Sell	? 15	? $n - 2$	?	$15(n - 2)$
Buy	9	n	?	$9n$
			?	570

Solve for x. *(See pp. 124–127.)*

20. **a.** $7x = 21$ 3 **b.** $kx = 21$ $\frac{21}{k}$ **c.** $kx = a$ $\frac{a}{k}$

21. **a.** $x + y = 7$ $7 - y$ **b.** $x - y = 7$ $y + 7$ **c.** $y - x = 7$ $y - 7$

22. Solve $\frac{t}{3} = 8$ for t. $t = 24$

23. Solve $F = ma$ for a. $a = \frac{F}{m}$

A permission-to-reproduce chapter test can be found on page T14.

Cumulative Review

Simplify.

1. $7b - 4b + 3$ $3b + 3$
2. $x(8x)(-3y)$ $-24x^2y$
3. $5(a + 6) - 11$ $5a + 19$
4. $9(3n^2 + 2m) - 18m$ $27n^2$
5. $6(s - 4t) - 2s$ $4s - 24t$
6. $7n - 11n$ $-4n$
7. $-3a^2 + 5a^2$ $2a^2$
8. $x - (7 - 3x)$ $4x - 7$
9. $-6a - 2(4 + 3a)$ $-12a - 8$
10. $\frac{12t}{t}$ 12
11. $17 \cdot \frac{0}{b}$ 0
12. $(-m)^2 + m$ $m^2 + m$

Tell which of the numbers shown in color are solutions.

13. $x - 13 < 10$ 21 25, 23, 21
14. $a^2 + 2 > 10$ 3, 5 1, 3, 5
15. $5b - 6 > 11$ 5 1, 3, 5
16. $3(t + 1) < 15$ 2 2, 4, 6

Solve for x.

17. $x + 15 = 22$ 7
18. $x - 9 = 11$ 20
19. $9x = 117$ 13
20. $7x + 5 = 61$ 8
21. $3(x - 6) = 12$ 10
22. $x + 14 = 3x$ 7
23. $5x - 7 = 4x$ 7
24. $6x = 48 - 6x$ 4
25. $8x = -48$ −6
26. $-11x = 132$ −12
27. $15x - 8x = -14$ −2
28. $x + 9 = 4x$ 3
29. $3x - 5x = 4$ −2
30. $5x - 2 = 7x$ −1
31. $13x - 8 = 9x$ 2
32. $9x - 3 = 2x + 11$ 2
33. $x - s = t$ $t + s$
34. $ax = 32$ $\frac{32}{a}$

Solve.

35. Jackie is 8 kilograms more than Laura.
Together they are 104 kilograms.
How many kilograms is Jackie? 56 kilograms

36. Student tickets for the game Saturday were $1.25 each.
There were 700 student tickets and 375 adult tickets sold.
The ticket sales totaled $1625.
How much did each adult ticket cost? $2.00

37. Eugene and Dale drive in opposite directions for 3 hours.
They are then 480 km apart.
Dale's speed is 85 km/h.
What is Eugene's speed? 75 km/h

UNIT C

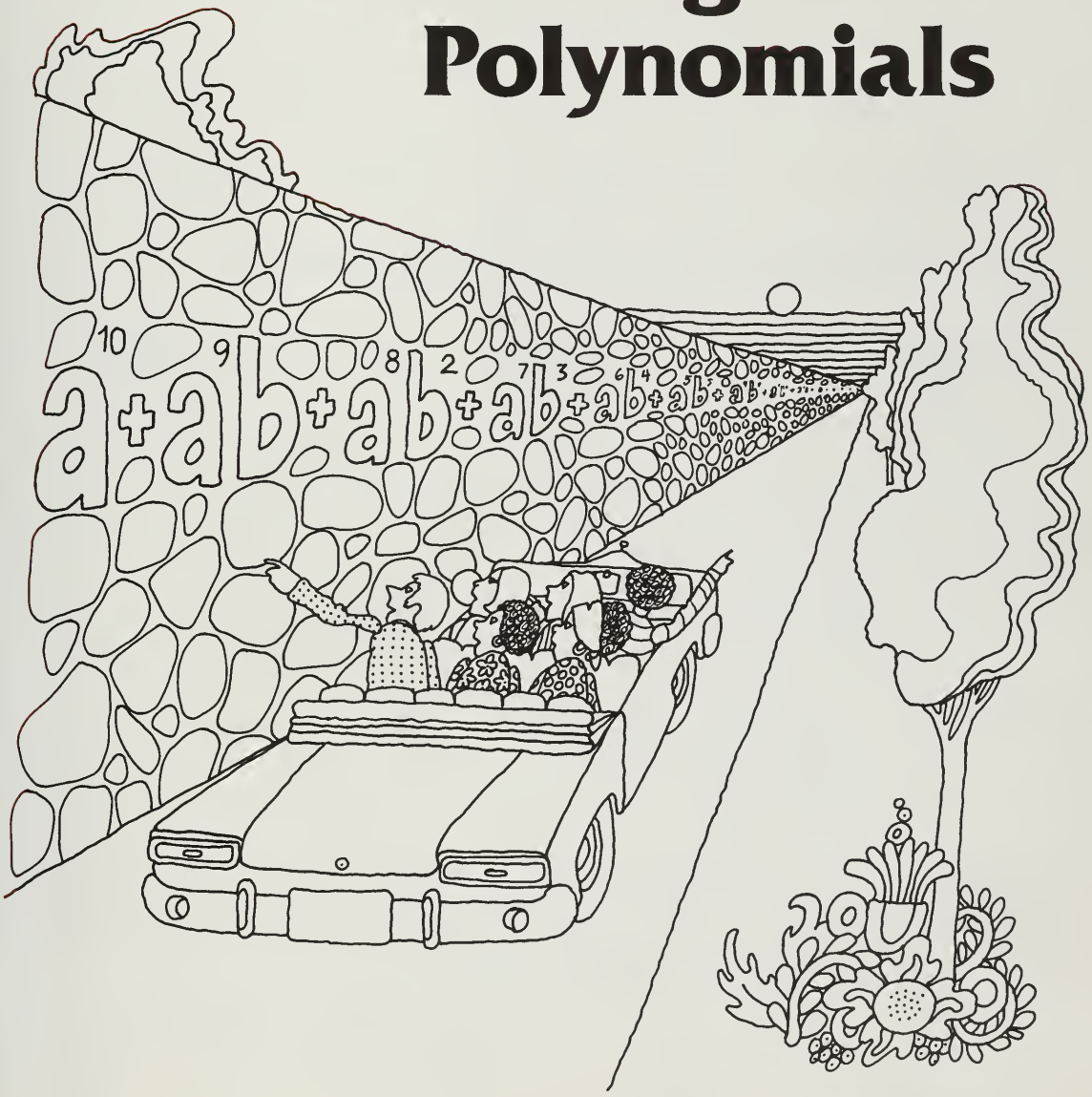
Here's what you'll learn in this chapter:

1. To add and subtract polynomials.
2. To multiply polynomials.
3. To multiply binomials at sight.
4. To square a binomial.
5. To divide by a monomial.



Chapter 5

Working With Polynomials



1 • Addition

The three expressions below are **polynomials.** A polynomial may have any number of terms. Notice the terms are connected by + or − signs.

$$3xy$$
$$4a + 2b$$
$$5x^2 - 12xy + y^2$$

Special names are given to polynomials with one, two, or three terms.

one term $3xy$ → **monomial**

two terms $4a + 2b$ → **binomial**

three terms $5x^2 - 12xy + y^2$ → **trinomial**

It's easy to remember these words if you compare them with words you know.



monocycle



bicycle



tricycle

You can add polynomials in a vertical form or in a horizontal form.

EXAMPLE 1

$$\begin{array}{r} 2a^2 + 3b^2 + ab - 3 \\ 6a^2 - b^2 \qquad + 1 \\ \hline 8a^2 + 2b^2 + ab - 2 \end{array}$$

EXAMPLE 2

$$\begin{aligned} &(2xy + 2x + 3y) + (4xy - 2x + 5y) \\ &= 2xy + 4xy + 2x - 2x + 3y + 5y \\ &= 6xy + 8y \end{aligned}$$

Classroom Practice

Is the expression a monomial, a binomial, or a trinomial?

1. $a + 2$ — binomial
2. x — monomial
3. $4ab$ — monomial
4. $w + 4$ — binomial
5. $x^2 + 2xy + y^2$ — trinomial
6. -1 — monomial
7. $t^3 - 3$ — binomial
8. $a^2 + 4ab + 4$ — trinomial

Add.

9. $2x + 4$
$5x - 1$
$7x + 3$

10. $3a + 2b$
$a + 5b$
$4a + 7b$

11. $a^2 - 3ab - b^2$
$6a^2 + 2ab + 5b^2$
$7a^2 - ab + 4b^2$

12. $(3a + 4ab) + (6a + ab)$
$9a + 5ab$

13. $(x^2 + 3xy) + (4x^2 - xy)$
$5x^2 + 2xy$

Written Exercises

Add.

A **1.** $2y + 4$
$y + 7$
$3y + 11$

2. $2x^2 + 8$
$7x^2 + 7$
$9x^2 + 15$

3. $m^2 - n^2$
$m^2 + n^2$
$2m^2$

4. $-3x + 4y$
$-8x + 9y$
$-11x + 13y$

5. $12ab + b^2$
$-7ab + b^2$
$5ab + 2b^2$

6. $a^2b - 3ab^2$
$2a^2b + 5ab^2$
$3a^2b + 2ab^2$

7. $x^2 + xy + y^2$
$x^2 + y^2$
$2x^2 + xy + 2y^2$

8. $-n^2 + 2n - 1$
$n^2 + 8n - 12$
$10n - 13$

9. $4y^4 - 4y^2 + 1$
$-4y^4 - y^2$
$-5y^2 + 1$

10. $(2x - y) + (7x + y)$
$9x$

11. $(n + 1) + (11n + 1)$
$12n + 2$

12. $(3y + z + 1) + (18y + 9z + 16)$
$21y + 10z + 17$

13. $(m + 3n + 8) + (9m + 9n + 7)$
$10m + 12n + 15$

14. $(n^2 + 2n + 1) + (3n^2 + 4n - 3)$
$4n^2 + 6n - 2$

15. $(-m^2 + 4m) + (m^2 - 8m + 8)$
$-4m + 8$

16. $(a^2 - 2ab + b^2) + (-2a^2 + b^2)$
$-a^2 - 2ab + 2b^2$

17. $(x^2 - 10x + 5) + (9x^2 - 10x - 3)$
$10x^2 - 20x + 2$

18. $(3a^2 + 3a - 6) + (a^2 - 1)$
$4a^2 + 3a - 7$

19. $(x^2 + 2x + 4) + (4x^2 - 5)$
$5x^2 + 2x - 1$

20. $(n^3 - 4) + (n^3 - n^2 + 10)$
$2n^3 - n^2 + 6$

21. $(k^3 - 1) + (k^2 - 2k + 7)$
$k^3 + k^2 - 2k + 6$

22. $(n^2 - n) + (n^3 - 2n)$
$n^3 + n^2 - 3n$

23. $(2x^2 + 3) + (x^3 + 4x^2 - 6)$
$x^3 + 6x^2 - 3$

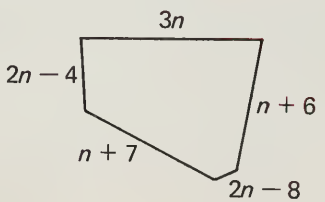
24. $(2a^3 - ab + 1) + (a^3 - 2ab)$
$3a^3 - 3ab + 1$

25. $(4a^2 - 2ab) + (-3a^2 + ab - b^2)$
$a^2 - ab - b^2$

26. A city park has the dimensions shown.

a. Write an expression for its perimeter.
$9n + 1$

b. Evaluate if $n = 20$.
181



2 • Subtraction

You know that when you want to subtract, you add the opposite.

$$\begin{aligned} 4 - 7 &= 4 + (-7) \\ &= -3 \end{aligned}$$

The same is true when you're subtracting binomials or trinomials. The vertical form is sometimes easier to use.

EXAMPLE 1 $(3x - 1) - (2x + 2)$

$$\begin{array}{r} 3x - 1 \\ \underline{-(2x + 2)} \end{array} \longrightarrow \boxed{\text{Rewrite and add.}} \longrightarrow \begin{array}{r} 3x - 1 \\ \underline{-2x - 2} \\ x - 3 \end{array}$$

EXAMPLE 2 $(5x^2 + x + 1) - (4x^2 - x + 1)$

$$\begin{array}{r} 5x^2 + x + 1 \\ \underline{-(4x^2 - x + 1)} \end{array} \longrightarrow \boxed{\text{Rewrite and add.}} \longrightarrow \begin{array}{r} 5x^2 + x + 1 \\ \underline{-4x^2 + x - 1} \\ x^2 + 2x \end{array}$$

Of course, the idea of changing signs and adding can be used in the horizontal form too.

EXAMPLE 3 $(2a + 4) - (a - 6) = (2a + 4) + (-a + 6)$
$= 2a - a + 4 + 6$
$= a + 10$

EXAMPLE 4 $(x^2 + 4x + 2) - (2x^2 - x + 1) = (x^2 + 4x + 2) + (-2x^2 + x - 1)$
$= x^2 - 2x^2 + 4x + x + 2 - 1$
$= -x^2 + 5x + 1$

Classroom Practice

Name the opposite.

1. $(4x^2 - 1)$ $-4x^2 + 1$
2. $(-x + 4)$ $x - 4$
3. $(x^2 + 2x - 1)$ $-x^2 - 2x + 1$
4. $(x^2 - 2xy + y^2)$ $-x^2 + 2xy - y^2$

Subtract.

5. $\begin{array}{r} 2a - 4 \\ \underline{-(3a + 2)} \end{array}$ $-a - 6$
6. $\begin{array}{r} x^2 - 3y \\ \underline{-(x^2 - 6y)} \end{array}$ $3y$
7. $\begin{array}{r} a^2 - 2b + 5b^2 \\ \underline{-(3a^2 - b + 4b^2)} \end{array}$ $-2a^2 - b + b^2$
8. $\begin{array}{r} 2x^2 + 3x - y^2 \\ \underline{-(x^2 - 3x + y^2)} \end{array}$ $x^2 + 6x - 2y^2$

Written Exercises

Subtract.

A

1. $2x + y$ minus $(x - 3y)$ — $x + 4y$
2. $3a - 4b$ minus $(a + 5b)$ — $2a - 9b$
3. $-x^2 + y^2$ minus $(x^2 - y^2)$ — $-2x^2 + 2y^2$
4. $x^2 - 2x - 1$ minus $(x^2 + 2x + 1)$ — $-4x - 2$
5. $3x^2 - 4x + 5$ minus $(x^2 - 4x + 1)$ — $2x^2 + 4$
6. $n^2 - 4n + 5$ minus $(3n^2 + 7n + 3)$ — $-2n^2 - 11n + 2$
7. $x^2 - 2xy + y^2$ minus $(x^2 + 2xy + y^2)$ — $-4xy$
8. $y^3 + y^2 + 6$ minus $(y^3 + 4)$ — $y^2 + 2$
9. $x^2y + xy^2 + 4$ minus $(-3xy^2 + 2)$ — $x^2y + 4xy^2 + 2$

10. $(a + 1) - (2a - 4)$ — $-a + 5$
11. $(2b - 2) - (b - 4)$ — $b + 2$
12. $(x^2 + 1) - (x^2 + 1)$ — 0
13. $(5m - 16) - (m + 2)$ — $4m - 18$
14. $(3n - 2) - (-n - 2)$ — $4n$
15. $(y + 6) - (-3y - 8)$ — $4y + 14$
16. $(x^2 + 3x + 2) - (x^2 - 4x + 1)$ — $7x + 1$
17. $(x^2 + 2x + 1) - (4x^2 - 3x + 7)$ — $-3x^2 + 5x - 6$
18. $(a^2 + 5ab - 2c) - (3a^2 + ab - 4c)$ — $-2a^2 + 4ab + 2c$
19. $(2ab - 4c) - (a^2 - 3ab + 6c)$ — $-a^2 + 5ab - 10c$
20. $(x^2 - 8x + 7) - (5x^2 + 9)$ — $-4x^2 - 8x - 2$
21. $(4y^2 - 8) - (2y^2 - 5y + 10)$ — $2y^2 + 5y - 18$

Solve for the variable.

22. $3x - (-24 - x) = 60$ — $x = 9$
23. $(13n + 5) - (3n + 6) = 99$ — $n = 10$
24. $(2x - 4) - (x + 8) = 24$ — $x = 36$
25. $(12n - 40) - (10n + 30) = n + 10$ — $n = 80$

B

26. You have two pencils, $3n + 1$ centimeters and $4n - 7$ centimeters. What is the difference in their lengths? — $n - 8$ cm



For more practice, see page 416.

SELF-TEST

Add or subtract.

1. $(3x - 4) + (4x + 9)$ — $7x + 5$
2. $(5m + n + 1) + (-3m - n + 3)$ — $2m + 4$
3. $(x + 4) - (3x - 2)$ — $-2x + 6$
4. $(-a^2 + 3b) + (2a^2 - 4b + 1)$ — $a^2 - b + 1$
5. $(m^2 + 2m + 1) - (m^2 - 3m + 2)$ — $5m - 1$
6. $(3b - 4) - (3b^2 + 2b + 5)$ — $-3b^2 + b - 9$

3 • Multiplying Monomials

You will recall from your work with exponents that $x^2 = x \cdot x$, $x^3 = x \cdot x \cdot x$, and so on. Therefore, you can do the following.

$$\begin{aligned} x^2 \cdot x^3 &= (x \cdot x)(x \cdot x \cdot x) \\ &= x^5 \end{aligned}$$

You can use this idea to multiply any two monomials.

EXAMPLE 1 $\begin{aligned} x^4 \cdot x^3 &= (x \cdot x \cdot x \cdot x)(x \cdot x \cdot x) \\ &= x^7 \end{aligned}$

EXAMPLE 2 $\begin{aligned} (3a^2)(a^3) &= (3 \cdot a \cdot a)(a \cdot a \cdot a) \\ &= 3(a \cdot a \cdot a \cdot a \cdot a) \\ &= 3a^5 \end{aligned}$

Once you have done a few problems like these, you'll probably see a short cut.



When you multiply two powers of the same number, add the exponents.

$$x^a \cdot x^b = x^{a+b}$$

EXAMPLE 3 $x^5 \cdot x^7 = x^{5+7} = x^{12}$

EXAMPLE 4 $n^6 \cdot n = n^{6+1} = n^7$

Remember, $n = n^1$.

EXAMPLE 5 $\begin{aligned} (2x^2)(3x^3) &= (2 \cdot 3)(x^2 \cdot x^3) \\ &= 6x^{2+3} \\ &= 6x^5 \end{aligned}$

EXAMPLE 6 $\begin{aligned} (4a^2b)(-3ab^2) &= (4 \cdot -3)(a^2 \cdot a)(b \cdot b^2) \\ &= -12a^{2+1}b^{1+2} \\ &= -12a^3b^3 \end{aligned}$

Classroom Practice

Multiply.

1. $x^4 \cdot x^2$ — x^6
2. $n \cdot n^2$ — n^3
3. $y^2 \cdot y^3$ — y^5
4. $-2x \cdot x^5$ — $-2x^6$
5. $(x^3)(4x^2)$ — $4x^5$
6. $(2a)(4a^3)$ — $8a^4$
7. $(-9y^2)(y^5)$ — $-9y^7$
8. $(6n^4)(2n^4)$ — $12n^8$
9. $(5ab)(a^2b)$ — $5a^3b^2$
10. $(2x^2)(4x^2y)$ — $8x^4y$
11. $(-5x^2y)(-2y^2)$ — $10x^2y^3$
12. $(8a^2b^5)(-4ab^2)$ — $-32a^3b^7$

Written Exercises

Multiply.

A

1. $a \cdot a^2$ — a^3
2. $b^2 \cdot b^2$ — b^4
3. $x^2 \cdot 3x$ — $3x^3$
4. $x^4 \cdot x$ — x^5
5. $n^3 \cdot n^4$ — n^7
6. $a^6 \cdot a$ — a^7
7. $(2x)(2x^2)$ — $4x^3$
8. $(3x)(-2x^4)$ — $-6x^5$
9. $(c^2)(-5c^3)$ — $-5c^5$
10. $(ab)(a^2b)$ — a^3b^2
11. $(3x^2)(-2x^5)$ — $-6x^7$
12. $(-y^2)(-y^7)$ — y^9
13. $(3x^2)(4x^4)$ — $12x^6$
14. $(-x^2)(-4x)$ — $4x^3$
15. $(5a)(-ab^2)$ — $-5a^2b^2$
16. $(xy)(-2x)$ — $-2x^2y$
17. $(cd)(-3d^3)$ — $-3cd^4$
18. $(2mn)(-8m^2)$ — $-16m^3n$
19. $(5x^2y)(4xy^2)$ — $20x^3y^3$
20. $(-5xy)(2xy^2)$ — $-10x^2y^3$
21. $(-r^2s)(-10r^2s^2)$ — $10r^4s^3$
22. $(-6a^2)(4ab^5)$ — $-24a^3b^5$
23. $(-x^4)(-3xyz^2)$ — $3x^5yz^2$
24. $(-6a^2b^5)(abc^3)$ — $-6a^3b^6c^3$
25. $(a^2b)(-5a^2b^2)$ — $-5a^4b^3$
26. $(-x^3)(-5x^2y)$ — $5x^5y$
27. $(xy^3)(-2x^3y^2)$ — $-2x^4y^5$
28. $(-a^4b^3)(a^2bc^5)$ — $-a^6b^4c^5$

Puzzles & Things

See page 5 of Answers to Odd-Numbered Exercises in the back of the text.

The 27¢ Special

How many different combinations of coins have a sum of exactly 27¢? Complete this table to find out.

Coin	25¢	10¢	5¢	1¢
Combinations	1	0	0	2
	0	2	1	2
	and so on			



4 • Powers of Monomials

Study the examples below and see if you can discover another important rule of exponents.

$$(x^3)^2 = x^3 \cdot x^3 = x^6$$
$$(a^4)^2 = a^4 \cdot a^4 = a^8$$
$$(y^5)^3 = y^5 \cdot y^5 \cdot y^5 = y^{15}$$



To find the power of a power of a number, multiply the exponents.

$$(x^a)^b = x^{ab}$$

EXAMPLE 1 $(x^4)^3 = x^{4 \cdot 3} = x^{12}$

EXAMPLE 2 $(y^8)^2 = y^{8 \cdot 2} = y^{16}$

Suppose you have an expression like $(xy)^2$. You can rewrite it in the following way.

$$(xy)^2 = (xy)(xy) = x^2y^2$$



Make a note of this rule of exponents.

$$(xy)^n = x^n y^n$$

EXAMPLE 3 $(ab)^3 = a^3b^3$

EXAMPLE 4 $(2a)^2 = 2^2 \cdot a^2 = 4a^2$

WARNING! $2a^2$ and $(2a)^2$ are not the same.

$2a^2 \stackrel{?}{=} (2a)^2$ ← ~~$2a^2 = (2a)^2$~~

$2a^2$	$2^2 \cdot a^2$
	$4a^2$

EXAMPLE 5 $(-4y)^2 = (-4)^2 \cdot y^2 = 16y^2$

Classroom Practice

1. In 5^3, 5 is used as a factor ___?___ times. 3

2. In $(x^3)^2$, x^3 is used as a factor ___?___ times. 2

3. Does $(3x)^2$ equal $3x^2$, or does it equal $9x^2$? $9x^2$

4. Does $(2x)^3$ equal $2x^3$, or does it equal $8x^3$? $8x^3$

Simplify.

5. $(a^4)^2$ a^8
6. $(c^2)^5$ c^{10}
7. $(ab)^4$ a^4b^4
8. $(5a)^2$ $25a^2$
9. $(x^2)^3$ x^6
10. $(2a)^2$ $4a^2$
11. $(4n^2)^2$ $16n^4$
12. $(-3xy)^2$ $9x^2y^2$

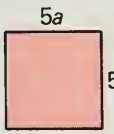
Written Exercises

Simplify.

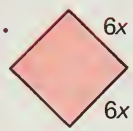
A

1. $(x^2)^3$ x^6
2. $(a^3)^4$ a^{12}
3. $(b^6)^2$ b^{12}
4. $(x^2)^5$ x^{10}
5. $(c^3)^5$ c^{15}
6. $(n^4)^{10}$ n^{40}
7. $(2x)^2$ $4x^2$
8. $(4a)^2$ $16a^2$
9. $(ab)^4$ a^4b^4
10. $(xy)^6$ x^6y^6
11. $(6ax)^2$ $36a^2x^2$
12. $(-2xy)^3$ $-8x^3y^3$

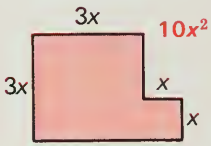
Find the area. Use $A = s^2$.

13. 

$25a^2$

14. 

$36x^2$

15. 

$10x^2$

Use both of the exponent rules on the opposite page. Simplify.

Sample $(4x^3)^2 = 4^2(x^3)^2 = 16x^6$

16. $(5x^3)^2$ $25x^6$
17. $(3a^2)^2$ $9a^4$
18. $(2b^3)^2$ $4b^6$
19. $(5x^5)^2$ $25x^{10}$
20. $(-2a^5)^2$ $4a^{10}$
21. $(2a^2b)^2$ $4a^4b^2$
22. $(3xy^3)^2$ $9x^2y^6$
23. $(-x^3y^3)^2$ x^6y^6
24. $2(ab^4)^2$ $2a^2b^8$
25. $3(x^2y)^2$ $3x^4y^2$
26. $-(2n^2)^3$ $-8n^6$
27. $-(4n^2)^2$ $-16n^4$

B

28. $(xy)^2(-xy)$ $-x^3y^3$
29. $(-xy^3)(-xy)^2$ $-x^3y^5$
30. $(3xy^4)^2(-4x^2)^2$ $144x^6y^8$
31. $(2mn)^3(3n)^2$ $72m^3n^5$
32. $-(x^2y^2)(x^2)$ $-x^4y^2$
33. $-(r^4s^3)^2(rs^2)^3$ $-r^{11}s^{12}$

For more practice, see page 416.

5 • Polynomials Times Monomials

Now that you can multiply monomials, you can put the distributive property to work.

You know that $5(3 + 4) = (5 \cdot 3) + (5 \cdot 4)$

In the same way $5(3x + 4y) = (5 \cdot 3x) + (5 \cdot 4y)$
$= 15x + 20y$

EXAMPLE 1 $3(a + 3b) = (3 \cdot a) + (3 \cdot 3b)$
$= 3a + 9b$

EXAMPLE 2 $-1(2x - 3y) = (-1 \cdot 2x) - (-1 \cdot 3y)$
$= -2x + 3y$

EXAMPLE 3 $a(a^2 + 2ab + b^2) = (a \cdot a^2) + (a \cdot 2ab) + (a \cdot b^2)$
$= a^3 + 2a^2b + ab^2$

EXAMPLE 4 $-2x(9x^3 + 3x^2 + x) = (-2x \cdot 9x^3) + (-2x \cdot 3x^2) + (-2x \cdot x)$
$= -18x^4 - 6x^3 - 2x^2$

Classroom Practice

Multiply.

1. $2(a + 4)$ — $2a + 8$
2. $3(x + y)$ — $3x + 3y$
3. $-1(2x + 3y)$ — $-2x - 3y$
4. $x(x - 2)$ — $x^2 - 2x$
5. $a(a + 2b)$ — $a^2 + 2ab$
6. $2c(a + b)$ — $2ac + 2bc$
7. $-4(x^2 + 1)$ — $-4x^2 - 4$
8. $-5(a^2 - 2c)$ — $-5a^2 + 10c$
9. $x(x^2 + 2x + 4)$ — $x^3 + 2x^2 + 4x$
10. $-x(x^2 - 2x + 4)$ — $-x^3 + 2x^2 - 4x$
11. $x^2(x^2 - 3x + 1)$ — $x^4 - 3x^3 + x^2$
12. $a^2b^2(-a - b)$ — $-a^3b^2 - a^2b^3$

Written Exercises

Multiply.

A

1. $2(x + 4)$ — $2x + 8$
2. $3(a - b)$ — $3a - 3b$
3. $4(x + y)$ — $4x + 4y$
4. $5(a^2 + b)$ — $5a^2 + 5b$
5. $-6(n + 2m)$ — $-6n - 12m$
6. $-1(5a + b^2)$ — $-5a - b^2$
7. $a(a - b)$ — $a^2 - ab$
8. $x(x + 3y)$ — $x^2 + 3xy$
9. $-c(a + b)$ — $-ca - cb$
10. $-ab(2a - 4b)$ — $-2a^2b + 4ab^2$
11. $-5x(3x + 2y)$ — $-15x^2 - 10xy$
12. $2x(3x - 1)$ — $6x^2 - 2x$
13. $4a(a + 2b + 3)$ — $4a^2 + 8ab + 12a$
14. $-4(1 + 5x + x^2)$ — $-4 - 20x - 4x^2$
15. $-1(2x + y + z)$ — $-2x - y - z$
16. $2x(x^2 - 2x - 4)$ — $2x^3 - 4x^2 - 8x$
17. $-4y(y^3 - 2y + 1)$ — $-4y^4 + 8y^2 - 4y$
18. $ab(a^2 + 2ab - 1)$ — $a^3b + 2a^2b^2 - ab$
19. $-x^2(x + 2x^2)$ — $-x^3 - 2x^4$
20. $-3c(2c^2 + 4c - 5)$ — $-6c^3 - 12c^2 + 15c$
21. $-y^2(y^3 - 2y^2 + 4y)$ — $-y^5 + 2y^4 - 4y^3$
22. A rectangle is $10n$ centimeters long by $(n + 6)$ centimeters wide. Write its area as a polynomial. $(10n^2 + 60n)$ cm^2

B

23. You have collected $(3n + 1)$ dimes. What is their value in cents? $(30n + 10)$ cents

Solve.

24. $4(2n + 3) - 3(n - 1) = 0$ -3
25. $-(n + 3) + 2(n + 7) = 0$ -11
26. $5x + 2 - 2(2x + 6) = 0$ 10
27. $(2y - 3) - (y + 6) = 63$ 72
28. $2(5x - 6) - 3(2x - 4) = 0$ 0
29. $3(1 - 2a) - (6 - 2a) = -7$ 1
30. $3(x - 4) + 2(2x + 1) = 4$ 2
31. $2(n - 6) + 5(2n + 4) = 32$ 2
32. $6(1 - 3x) - 2(2x + 5) = 40$ -2
33. $4(2a - 3) - 2(a - 8) = 22$ 3

SELF-TEST

Simplify.

1. $(4x)(7x^3)$ — $28x^4$
2. $(-m^2n^2)(2mn^2)$ — $-2m^3n^4$
3. $(ab)^3$ — a^3b^3
4. $(-7x^2y)^2$ — $49x^4y^2$

Multiply.

5. $x(y + 7)$ — $xy + 7x$
6. $m(m - n)$ — $m^2 - mn$
7. $-a(a^2 - b)$ — $-a^3 + ab$
8. $-2(3x - 4y)$ — $-6x + 8y$
9. $5x(4x + x^2)$ — $20x^2 + 5x^3$
10. $m^2(3m^3 - m^2 + 2m)$ — $3m^5 - m^4 + 2m^3$

6 • Multiplying Polynomials

To multiply by a binomial you also use the distributive property. Let's use a vertical form. Your work is a lot like multiplying in arithmetic.

$$\begin{array}{r} n + 1 \\ n + 3 \\ \hline n^2 + n \\ \quad + 3n + 3 \\ \hline n^2 + 4n + 3 \end{array}$$

$n(n + 1)$

$3(n + 1)$

Be sure that like terms are lined up.

Here are a few more examples to study.

EXAMPLE 1

$$\begin{array}{r} x + 7 \\ x - 2 \\ \hline x^2 + 7x \\ \quad - 2x - 14 \\ \hline x^2 + 5x - 14 \end{array}$$

$x(x + 7)$

$-2(x + 7)$

EXAMPLE 2

$$\begin{array}{r} 2y - 5 \\ y - 4 \\ \hline 2y^2 - 5y \\ \quad - 8y + 20 \\ \hline 2y^2 - 13y + 20 \end{array}$$

$y(2y - 5)$

$-4(2y - 5)$

EXAMPLE 3

$$\begin{array}{r} a^2 + 2a - 1 \\ a - 3 \\ \hline a^3 + 2a^2 - a \\ \quad - 3a^2 - 6a + 3 \\ \hline a^3 - a^2 - 7a + 3 \end{array}$$

$a(a^2 + 2a - 1)$

$-3(a^2 + 2a - 1)$

Classroom Practice

Multiply.

1. $(x + 2)(x + 4)$ — $x^2 + 6x + 8$
2. $(n + 2)(n + 7)$ — $n^2 + 9n + 14$
3. $(a + 4)(a - 1)$ — $a^2 + 3a - 4$
4. $(x + 3)(x - 5)$ — $x^2 - 2x - 15$
5. $(2x + 1)(x + 5)$ — $2x^2 + 11x + 5$
6. $(a + 6)(2a - 4)$ — $2a^2 + 8a - 24$
7. $(2a + b)(a + 3b)$ — $2a^2 + 7ab + 3b^2$
8. $(2x - 4)(5x + 1)$ — $10x^2 - 18x - 4$
9. $(3y - 6)(2y - 5)$ — $6y^2 - 27y + 30$
10. $(2x - 5)(x + 2)$ — $2x^2 - x - 10$
11. $(3n + 6)(n - 3)$ — $3n^2 - 3n - 18$
12. $(4x - 1)(2x + 3)$ — $8x^2 - 10x - 3$

Written Exercises

Multiply.

A

1. $(x + 1)(x + 2)$ — $x^2 + 3x + 2$
2. $(y + 3)(y + 4)$ — $y^2 + 7y + 12$
3. $(n + 4)(n + 6)$ — $n^2 + 10n + 24$
4. $(a + 3)(a + 5)$ — $a^2 + 8a + 15$
5. $(x + 1)(x + 7)$ — $x^2 + 8x + 7$
6. $(y + 9)(y + 3)$ — $y^2 + 12y + 27$
7. $(x + y)(x + y)$ — $x^2 + 2xy + y^2$
8. $(a + b)(a + c)$ — $a^2 + ab + ac + bc$
9. $(x + y)(x + 4)$ — $x^2 + xy + 4x + 4y$
10. $(x - 1)(x - 2)$ — $x^2 - 3x + 2$
11. $(y - 3)(y - 4)$ — $y^2 - 7y + 12$
12. $(n - 3)(n - 1)$ — $n^2 - 4n + 3$
13. $(x - 1)(x + 2)$ — $x^2 + x - 2$
14. $(y + 3)(y - 4)$ — $y^2 - y - 12$
15. $(y - 5)(y + 4)$ — $y^2 - y - 20$
16. $(2x + y)(x + 2y)$ — $2x^2 + 5xy + 2y^2$
17. $(4x + 1)(4x + 1)$ — $16x^2 + 8x + 1$
18. $(4x + 1)(4x - 1)$ — $16x^2 - 1$
19. $(a + b)(a - b)$ — $a^2 - b^2$
20. $(a + 2b)(a - 2b)$ — $a^2 - 4b^2$
21. $(2a - 1)(a + b)$ — $2a^2 - a + 2ab - b$
22. $(6 - x)(6 - x)$ — $36 - 12x + x^2$
23. $(6x - 5)(6x - 5)$ — $36x^2 - 60x + 25$
24. $(3y - 2)(2y + 1)$ — $6y^2 - y - 2$

B

25. $(n + 1)(n^2 + 2n + 1)$ — $n^3 + 3n^2 + 3n + 1$
26. $(x + 2)(x^2 + 2x + 1)$ — $x^3 + 4x^2 + 5x + 2$
27. $(x - 2)(x^2 - 4x + 4)$ — $x^3 - 6x^2 + 12x - 8$
28. $(y + 3)(y^2 + 6y + 9)$ — $y^3 + 9y^2 + 27y + 27$
29. $(m - 2)(m^2 - 2m + 4)$ — $m^3 - 4m^2 + 8m - 8$
30. $(n - 4)(n^2 - 8n + 16)$ — $n^3 - 12n^2 + 48n - 64$
31. $(a + b)(a^2 + 2ab + b^2)$ — $a^3 + 3a^2b + 3ab^2 + b^3$
32. $(a - b)(a^2 - b^2)$ — $a^3 - ab^2 - a^2b + b^3$

C

33. $(n^2 + 2n + 1)(n^2 + 2n + 1)$ — $n^4 + 4n^3 + 6n^2 + 4n + 1$
34. $(x^2 - 7x + 12)(x^2 - 3x - 1)$ — $x^4 - 10x^3 + 32x^2 - 29x - 12$
35. $(2a^2 - ab + 4b^2)(5a^2 + ab + b^2)$ — $10a^4 - 3a^3b + 21a^2b^2 + 3ab^3 + 4b^4$
36. $(x - 4)(2x^4 - 5x^3 - 7x^2 + 10)$ — $2x^5 - 13x^4 + 13x^3 + 28x^2 + 10x - 40$
37. $(n - 1)(n^4 - 3n^3 + n^2 + 1)$ — $n^5 - 4n^4 + 4n^3 - n^2 + n - 1$
38. $(a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$ — $a^5 - b^5$

For more practice, see page 417.

7 • Multiplying at Sight

When you multiplied binomials by using the vertical form, you may have noticed a pattern. The pattern is pointed out at the right.

$$\begin{array}{r} x + 2 \\ x + 3 \\ \hline x^2 + 2x \\ \quad + 3x + 6 \\ \hline x^2 + 5x + 6 \end{array}$$

product of the first terms

product of the last terms

sum of $2 \cdot x$ and $3 \cdot x$

You don't have to write out each step every time you multiply two binomials. You can multiply at sight if you follow three steps.

Step 1: $(x + 3)(x + 2) = x^2 \ldots$ ← Multiply the *first* terms.

Step 2: $(x + 3)(x + 2) = x^2 + 5x \ldots$ ← Multiply the *outer* terms. Multiply the *inner* terms. Add the two products.

Step 3: $(x + 3)(x + 2) = x^2 + 5x + 6$ ← Multiply the *last* terms.

An easy way to remember this is to think of it as the **FOIL method.**



FOIL stands for **F**irst, **O**uter, **I**nner, **L**ast!

Here's how the FOIL method works.

EXAMPLE 1 Multiply $(x + 4)(x + 5)$.

Step 1: $(x + 4)(x + 5) = x^2 \ldots$ ← F

Step 2: $(x + 4)(x + 5) = x^2 + 9x \ldots$ ← O + I

Step 3: $(x + 4)(x + 5) = x^2 + 9x + 20$ ← L

Be sure to watch the signs!

EXAMPLE 2 Multiply $(x - 4)(x - 2)$.

Step 1: $(x - 4)(x - 2) = x^2 \ldots$ ← F

Step 2: $(x - 4)(x - 2) = x^2 - 6x \ldots$ ← O + I

Step 3: $(x - 4)(x - 2) = x^2 - 6x + 8$ ← L

EXAMPLE 3 Multiply $(x - 4)(x + 2)$.

$$(x - 4)(x + 2) = x^2 - 2x - 8$$

F | O + I | L

EXAMPLE 4 Multiply $(2x + 3y)(x + y)$.

$$(2x + 3y)(x + y) = 2x^2 + 5xy + 3y^2$$

F | O + I | L

Classroom Practice

Complete.

1. $(x + 1)(x + 4) = _?_ + 5x + 4$ x^2
2. $(a + 2)(a + 5) = a^2 + 7a + _?_$ 10
3. $(y + 3)(y + 2) = y^2 + 5y + _?_$ 6
4. $(x + 4)(x + 5) = x^2 + _?_ + 20$ $9x$
5. $(n + 1)(n + 7) = n^2 + _?_ + 7$ $8n$
6. $(x + 5)(x - 2) = x^2 + 3x - _?_$ 10
7. $(a + 8)(a - 2) = a^2 + 6a - _?_$ 16
8. $(c - 6)(c + 1) = c^2 - 5c - _?_$ 6
9. $(x - 3)(x - 4) = x^2 - 7x + _?_$ 12
10. $(a - 5)(a - 9) = a^2 - _?_ + 45$ $14a$

Multiply.

11. $(x + 2)(x + 3)$ — $x^2 + 5x + 6$
12. $(y + 5)(y + 1)$ — $y^2 + 6y + 5$
13. $(a + 6)(a + 2)$ — $a^2 + 8a + 12$
14. $(x - 4)(x - 3)$ — $x^2 - 7x + 12$
15. $(y - 6)(y - 1)$ — $y^2 - 7y + 6$
16. $(x + 3)(x - 7)$ — $x^2 - 4x - 21$
17. $(c - 9)(c + 6)$ — $c^2 - 3c - 54$
18. $(2x + 1)(x + 3)$ — $2x^2 + 7x + 3$
19. $(4y + 1)(2y - 6)$ — $8y^2 - 22y - 6$
20. $(5x + 2)(3x - 1)$ — $15x^2 + x - 2$
21. $(2a - b)(3a + 4b)$ — $6a^2 + 5ab - 4b^2$
22. $(7r - s)(r + 4s)$ — $7r^2 + 27rs - 4s^2$

Written Exercises

Multiply.

A

1. $(x + 4)(x + 3)$ — $x^2 + 7x + 12$
2. $(x + 3)(x + 5)$ — $x^2 + 8x + 15$
3. $(x + 6)(x + 2)$ — $x^2 + 8x + 12$
4. $(x + 2)(x + 1)$ — $x^2 + 3x + 2$
5. $(x - 1)(x - 2)$ — $x^2 - 3x + 2$
6. $(x - 6)(x - 3)$ — $x^2 - 9x + 18$
7. $(x - 4)(x - 3)$ — $x^2 - 7x + 12$
8. $(a - 3)(a - 5)$ — $a^2 - 8a + 15$
9. $(y - 4)(y + 1)$ — $y^2 - 3y - 4$
10. $(a - 1)(a + 3)$ — $a^2 + 2a - 3$
11. $(n + 2)(n - 1)$ — $n^2 + n - 2$
12. $(x + 5)(x - 2)$ — $x^2 + 3x - 10$
13. $(y + 6)(y - 2)$ — $y^2 + 4y - 12$
14. $(a + 5)(a - 3)$ — $a^2 + 2a - 15$
15. $(2x + 1)(x + 4)$ — $2x^2 + 9x + 4$
16. $(x + 2)(x - 2)$ — $x^2 - 4$
17. $(a + b)(a - b)$ — $a^2 - b^2$
18. $(2x + 5)(2x - 5)$ — $4x^2 - 25$
19. $(6a + 7)(6a - 7)$ — $36a^2 - 49$
20. $(2y - 3)(2y + 3)$ — $4y^2 - 9$
21. $(3x + 4)(3x - 4)$ — $9x^2 - 16$
22. $(5n - 1)(2n + 3)$ — $10n^2 + 13n - 3$
23. $(3x - 1)(2x + 5)$ — $6x^2 + 13x - 5$
24. $(3x + 2y)(2x + 3y)$ — $6x^2 + 13xy + 6y^2$
25. $(3x + y)(7x + 2y)$ — $21x^2 + 13xy + 2y^2$
26. $(3a - b)(a - 3b)$ — $3a^2 - 10ab + 3b^2$
27. $(4x - 3y)(x - 5y)$ — $4x^2 - 23xy + 15y^2$
28. $(3a - b)(7a + 4b)$ — $21a^2 + 5ab - 4b^2$
29. $(6x - y)(4x + 2y)$ — $24x^2 + 8xy - 2y^2$
30. $(3n + 2p)(7n - p)$ — $21n^2 + 11np - 2p^2$

Find the area. Use $A = lw$.

31. 

$x^2 + 10x + 24$

32. 

$x^2 + 6x + 9$

33. 

$2a^2 + 5a - 12$

34. A rectangle measures $(17 - x)$ centimeters by $(x - 4)$ centimeters. Find its area. $21x - 68 - x^2$

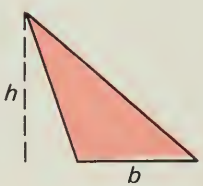
35. A rectangle measures $(2y + 3)$ centimeters by $(3y + 1)$ centimeters. Find its area. $6y^2 + 11y + 3$

Find the area. Use $A = \frac{1}{2}bh$.

B

36. 

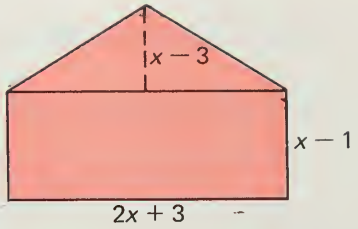
$b = 3n - 1$
$h = 4n$
$6n^2 - 2n$

37. 

$b = 2n - 1$
$h = n + 3$
$\frac{1}{2}(2n^2 + 5n - 3)$

38. Find the area of the side of the house shown at the right.

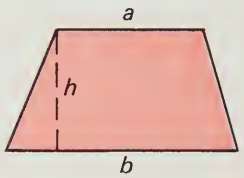
$\frac{1}{2}(2x^2 - 3x - 9) + 2x^2 + x - 3$



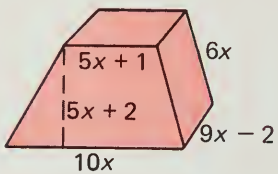
C **39.** A trapezoid is shown at the right. Its area may be found by using the formula $A = \frac{1}{2}h(a + b)$.

$h = 2x$
$a = 3x - 1$
$b = 3x + 1$

Find the area. $6x^2$



40. The box shown at the right is designed to hold a cup and saucer. The top, the bottom, and two sides are rectangles. Two sides are trapezoids. Find the area of the whole surface of the box. $318x^2 - 10x$



Puzzles & Things

There are 3 red hats and 2 black hats in a drawer. Melba, Fran, and Brent line up in a single file, and a hat is placed on each one's head. They are asked to figure out what color hat they are wearing.

Brent, who could look ahead and see Fran and Melba, says, "I don't know." Fran, who could see only Melba, says, "I don't know." But Melba, who could see nobody, says, "I know."

What color is Melba's hat and how does she know?
Melba's hat is red.



8 · Square of a Binomial

You know that when you square a number you multiply it by itself.

$$6^2 = 6 \cdot 6 \qquad x^2 = x \cdot x$$

The same is true when you square a binomial. All you do is to write the two factors, then multiply as you did in the last section.

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b) \\ &= a^2 + 2ab + b^2\end{aligned}$$

The square of a binomial is called a **trinomial square.**

Study some more examples.

EXAMPLE 1

$$\begin{aligned}(x + 3)^2 &= (x + 3)(x + 3) \\ &= x^2 + 6x + 9\end{aligned} \qquad \begin{aligned}(x - 4)^2 &= (x - 4)(x - 4) \\ &= x^2 - 8x + 16\end{aligned}$$

Once you have done a few multiplications like those above, you might notice a pattern in squaring a binomial.



To square a binomial:

1. Square the first term.
2. Combine twice the product of the two terms.
3. Combine the square of the second term.

$$\begin{aligned}(a + b)^2 &= a^2 + 2ab + b^2 \\ (a - b)^2 &= a^2 - 2ab + b^2\end{aligned}$$

Here are a couple of examples. Be sure to watch the signs too.

EXAMPLE 2

$(x + 5)^2 = x^2 + 10x + 25$

square of x — twice $5 \cdot x$ — square of 5

$(n - 3)^2 = n^2 - 6n + 9$

square of n — twice $3 \cdot n$ — square of 3

Of course, if you find this short cut difficult to remember, you can multiply as in Example 1.

Written Exercises

Express as a trinomial.

A

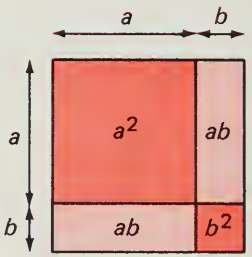
1. $(x + 4)^2$ — $x^2 + 8x + 16$
2. $(m + 1)^2$ — $m^2 + 2m + 1$
3. $(r + 2)^2$ — $r^2 + 4r + 4$
4. $(y - 3)^2$ — $y^2 - 6y + 9$
5. $(x + 3)^2$ — $x^2 + 6x + 9$
6. $(n + 6)^2$ — $n^2 + 12n + 36$
7. $(x - 2)^2$ — $x^2 - 4x + 4$
8. $(m - 6)^2$ — $m^2 - 12m + 36$
9. $(x - 4)^2$ — $x^2 - 8x + 16$
10. $(m + 5)^2$ — $m^2 + 10m + 25$
11. $(y - 7)^2$ — $y^2 - 14y + 49$
12. $(a + 10)^2$ — $a^2 + 20a + 100$
13. $(x + 9)^2$ — $x^2 + 18x + 81$
14. $(y - 8)^2$ — $y^2 - 16y + 64$
15. $(m + 7)^2$ — $m^2 + 14m + 49$
16. $(n + 8)^2$ — $n^2 + 16n + 64$
17. $(n - 5)^2$ — $n^2 - 10n + 25$
18. $(y - 1)^2$ — $y^2 - 2y + 1$
19. $(3a + 2)^2$ — $9a^2 + 12a + 4$
20. $(2x + 3)^2$ — $4x^2 + 12x + 9$
21. $(4x - 2)^2$ — $16x^2 - 16x + 4$
22. $(2a + b)^2$ — $4a^2 + 4ab + b^2$
23. $(3x + y)^2$ — $9x^2 + 6xy + y^2$
24. $(6n - 1)^2$ — $36n^2 - 12n + 1$
25. $(10a - b)^2$ — $100a^2 - 20ab + b^2$
26. $(2m + 3n)^2$ — $4m^2 + 12mn + 9n^2$
27. $(4x - 3y)^2$ — $16x^2 - 24xy + 9y^2$
28. $(3x - 4y)^2$ — $9x^2 - 24xy + 16y^2$
29. $2(a + 1)^2$ — $2a^2 + 4a + 2$
30. $2(a + 5)^2$ — $2a^2 + 20a + 50$
31. $3(x - 2)^2$ — $3x^2 - 12x + 12$
32. $-1(x + 3)^2$ — $-x^2 - 6x - 9$
33. $-2(x - 2)^2$ — $-2x^2 + 8x - 8$
34. $-1(2n + 3)^2$ — $-4n^2 - 12n - 9$
35. $2(2x + y)^2$ — $8x^2 + 8xy + 2y^2$
36. $-1(a - 2b)^2$ — $-a^2 + 4ab - 4b^2$

37. Study the figure. Then complete the equation below.

Area of big square

Sum of the areas of the four regions

$$(\underset{a}{?} + \underset{b}{?})^2 = a^2 + \underset{ab}{?} + \underset{ab}{?} + \underset{b^2}{?}$$



For more practice, see page 417.

SELF-TEST

Multiply.

1. $(x + 2)(x + 4)$ — $x^2 + 6x + 8$
2. $(a - 1)(a - 5)$ — $a^2 - 6a + 5$
3. $(y + 4)(y - 3)$ — $y^2 + y - 12$
4. $(m - 5)(m + 1)$ — $m^2 - 4m - 5$
5. $(2a - 7b)(2a + 3b)$ — $4a^2 - 8ab - 21b^2$
6. $(3m + 1)(2m + 5)$ — $6m^2 + 17m + 5$

Express as a trinomial.

7. $(a + 4)^2$ — $a^2 + 8a + 16$
8. $(n - 3)^2$ — $n^2 - 6n + 9$
9. $(6x + y)^2$ — $36x^2 + 12xy + y^2$
10. $(4a - 7b)^2$ — $16a^2 - 56ab + 49b^2$

9 • Division with Monomials

You know that a division is often shown as a fraction. Here is one way you can simplify.

In Arithmetic

$$\frac{10}{5} = \frac{\overset{2}{\cancel{10}}}{\underset{1}{\cancel{5}}} = 2$$

Divide both 10 and 5 by the same number, 5.

In Algebra

$$\frac{2a}{a} = \frac{2 \cdot \overset{1}{\cancel{a}}}{\underset{1}{\cancel{a}}} = 2$$

Divide both $2a$ and a by the same number, a.

This method works for all divisions.

EXAMPLE 1 $\dfrac{x^4}{x} = \dfrac{\overset{1}{\cancel{x} \cdot x \cdot x \cdot x}}{\underset{1}{\cancel{x}}} = x^3$

EXAMPLE 2 $\dfrac{2x^3}{x^2} = \dfrac{2 \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot x}{\underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}}} = 2x$

EXAMPLE 3 $\dfrac{-4ab^4}{2ab} = \dfrac{\overset{-2}{\cancel{-4}} \cdot \overset{1}{\cancel{a}} \cdot \overset{1}{\cancel{b}} \cdot b \cdot b \cdot b}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{a}} \cdot \underset{1}{\cancel{b}}} = -2b^3$

Classroom Practice

Divide.

1. $\dfrac{x^6}{x}$ x^5
2. $\dfrac{x^4}{x^3}$ x
3. $\dfrac{a^7}{a^4}$ a^3
4. $\dfrac{2a}{2}$ a
5. $\dfrac{5x}{x}$ 5
6. $\dfrac{3c^2}{c}$ $3c$
7. $\dfrac{4ab}{a}$ $4b$
8. $\dfrac{6a^3}{a^2}$ $6a$
9. $\dfrac{8a^2b}{ab}$ $8a$
10. $\dfrac{-7x^2}{x^2}$ -7
11. $\dfrac{-9m^2r^3}{3mr^3}$ $-3m$
12. $\dfrac{24rs^4}{-8rs^3}$ $-3s$

Written Exercises

Divide.

A

1. $\frac{2a}{2a}$ 1
2. $\frac{-6a}{a}$ -6
3. $\frac{a^2}{a}$ a
4. $\frac{6c}{2}$ $3c$
5. $\frac{-5ax}{a}$ $-5x$
6. $\frac{x^4}{x^3}$ x
7. $\frac{c^9}{c^4}$ c^5
8. $\frac{2x^2}{x}$ $2x$
9. $\frac{7n^3}{n^2}$ $7n$
10. $\frac{10a^5}{2a^3}$ $5a^2$
11. $\frac{-6mn}{mn}$ -6
12. $\frac{2xy}{2y}$ x
13. $\frac{12rs}{-3}$ $-4rs$
14. $\frac{4ab}{-2b}$ $-2a$
15. $\frac{-9x^2y}{-3}$ $3x^2y$
16. $\frac{-8ab^2}{-2a}$ $4b^2$
17. $\frac{-56x^4}{8x^2}$ $-7x^2$
18. $\frac{30m^3}{-6m}$ $-5m^2$
19. $\frac{-63b^2}{9b}$ $-7b$
20. $\frac{36n^6}{6n^4}$ $6n^2$
21. $\frac{-10cd^4}{-5d^2}$ $2cd^2$
22. $\frac{48bc^2}{-8c^2}$ $-6b$
23. $\frac{42a^2b^2}{-6ab}$ $-7ab$
24. $\frac{-72x^4y^3}{9xy^2}$ $-8x^3y$

Simplify. Be sure to divide first.

B

25. $\frac{4x^2}{x} - \frac{2x^3}{x^2}$ $2x$
26. $\frac{2x^4}{x^3} - \frac{3x}{3}$ x
27. $\frac{5a^2}{5a} + \frac{6a^5}{2a^4}$ $4a$
28. $\frac{10xy}{5x} + \frac{4x^2y^3}{4x^2y^2}$ $3y$
29. $\frac{18ab^3}{6b^2} + \frac{5a^2b}{5a}$ $4ab$
30. $\frac{-4x^3y^4}{-2x^2y^2} - \frac{3x^2y^5}{xy^3}$ $-xy^2$

Puzzles & Things

A Sticky Problem

Here are five squares made from sticks. It is possible to make four squares by moving just two of the sticks. Can you do it?



See page 6 of Answers to Odd-Numbered Exercises in the back of the text.

10 • Exponent Rules (Optional)

You may have already guessed that you can work with exponents in division in the following way.

$$\frac{x^5}{x^2} = x^{5-2} = x^3$$

Sometimes you may find that the exponent in the denominator is greater than the exponent in the numerator.

$$\frac{x^2}{x^5} = \frac{1}{x^{5-2}} = \frac{1}{x^3}$$



Make a note of the following rules.

$\frac{x^a}{x^b} = x^{a-b}$ when a is greater than b.

$\frac{x^a}{x^b} = \frac{1}{x^{b-a}}$ when b is greater than a.

$\frac{x^a}{x^a} = x^{a-a} = x^0 = 1$

You can use these rules when a and b are positive integers.

EXAMPLE 1 $\frac{3n^6}{n^2} = 3n^{6-2} = 3n^4$

EXAMPLE 2 $\frac{5x^4}{x^6} = \frac{5}{x^{6-4}} = \frac{5}{x^2}$

EXAMPLE 3 $\frac{a^3b^4}{a^3b} = a^{3-3}b^{4-1} = 1 \cdot b^3 = b^3$

Classroom Practice

Divide.

1. $\frac{x^5}{x^3}$ x^2
2. $\frac{a^7}{a^4}$ a^3
3. $\frac{c^9}{c^3}$ c^6
4. $\frac{x}{x^6}$ $\frac{1}{x^5}$
5. $\frac{2a^4}{a^3}$ $2a$
6. $\frac{5x^4}{5x^2}$ x^2
7. $\frac{10n^5}{2n^3}$ $5n^2$
8. $\frac{-8y}{2y^2}$ $\frac{-4}{y}$

Written Exercises

Divide.

A

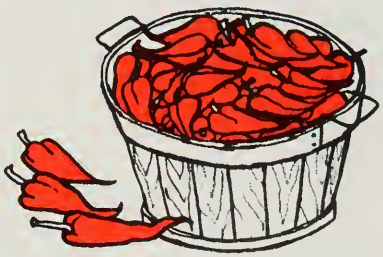
1. $\frac{a^5}{a^2}$ a^3
2. $\frac{x^9}{x^4}$ x^5
3. $\frac{x^3}{x^7}$ $\frac{1}{x^4}$
4. $\frac{5a^7}{a^3}$ $5a^4$
5. $\frac{9x^6}{x^3}$ $9x^3$
6. $\frac{12x^3}{3x^5}$ $\frac{4}{x^2}$
7. $\frac{-8n^2}{4n^5}$ $\frac{-2}{n^3}$
8. $\frac{14b^{12}}{2b^2}$ $7b^{10}$
9. $\frac{-3a^2b^2}{ab^2}$ $-3a$
10. $\frac{-18a^3b}{-3a^2}$ $6ab$
11. $\frac{16x^4y^3}{-4xy}$ $-4x^3y^2$
12. $\frac{25x^4y^2}{5xy}$ $5x^3y$
13. $\frac{-12x^5y^2}{-6x^2y^2}$ $2x^3$
14. $\frac{14a^3b^2}{-7a^2b^5}$ $\frac{-2a}{b^3}$
15. $\frac{-42m^6n^3}{-7m^4n^4}$ $\frac{6m^2}{n}$
16. $\frac{56x^5y^5}{7xy^4}$ $8x^4y$
17. $\frac{-20x^6y^7}{5xy^6}$ $-4x^5y$
18. $\frac{-12a^3b^8}{-3a^2b^5}$ $4ab^3$
19. $\frac{9x^9y^4z}{3xy}$ $3x^8y^3z$
20. $\frac{15a^2b^4c^3}{-3ac^2}$ $-5ab^4c$
21. $\frac{-7a^9bc}{a^5bc^3}$ $\frac{-7a^4}{c^2}$
22. $\frac{49c^2d^{10}x}{-7c^2d^5}$ $-7d^5x$
23. $\frac{-3r^4st^6}{-rst^4}$ $3r^3t^2$
24. $\frac{36a^6b^2c^7}{-12cbc^6}$ $-3a^5bc$
25. $\frac{12x^2y}{-2xy}$ $-6x$
26. $\frac{21a^2b^3c}{-3a^2b}$ $-7b^2c$
27. $\frac{-40a^4b^2c^5}{-10a^3bc^3}$ $4abc^2$
28. $\frac{30x^2y^3z}{-5xy^2}$ $-6xyz$
29. $\frac{-42a^5b^2}{6a^3b}$ $-7a^2b$
30. $\frac{36x^4y^2z}{-6x^4}$ $-6y^2z$
31. $\frac{-81a^4b^2c^9}{-3ab^2c^5}$ $27a^3c^4$
32. $\frac{54r^2s^4t^{10}}{-9s^3t^4}$ $-6r^2st^6$

Puzzles & Things

The Pickled Pepper Problem

Paula and Peter Piper are picking pickled peppers. They're putting the pickled peppers in a basket. The number of peppers they pick doubles every minute. In one hour the basket is full. When was the basket half full?

the 59th minute



11 • Polynomials Divided by Monomials

You probably remember that in arithmetic you add fractions in the following way.

$$\frac{20}{4} + \frac{3}{4} = \frac{20 + 3}{4} = \frac{23}{4}$$

If this process is turned around, you have the following.

$$\frac{23}{4} = \frac{20 + 3}{4} = \frac{20}{4} + \frac{3}{4}$$

Now just use this idea in working with division problems.

EXAMPLE 1 $\dfrac{3a + 3b}{3} = \dfrac{3a}{3} + \dfrac{3b}{3}$

Divide each term of $3a + 3b$ by 3.

$= a + b$

EXAMPLE 2 $\dfrac{2rs - 6r^2s^2}{2r} = \dfrac{2rs}{2r} - \dfrac{6r^2s^2}{2r}$

$= s - 3rs^2$

EXAMPLE 3 $\dfrac{2x^2 + 4x + 8xy}{2x} = \dfrac{2x^2}{2x} + \dfrac{4x}{2x} + \dfrac{8xy}{2x}$

$= x + 2 + 4y$

Classroom Practice

Divide.

1. $\dfrac{5x + 5y}{5}$ $x + y$

2. $\dfrac{2x + 3x}{x}$ 5

3. $\dfrac{4m + 2m}{2m}$ 3

4. $\dfrac{8ab - 4a}{2a}$ $4b - 2$

5. $\dfrac{5a^2b - 3ab}{ab}$ $5a - 3$

6. $\dfrac{6mn^2 + 9m^2n}{3mn}$ $2n + 3m$

7. $\dfrac{2x^2 + 3x - 4xy}{x}$ $2x + 3 - 4y$

8. $\dfrac{3a^2b - 6a + 9a^2}{3a}$ $ab - 2 + 3a$

9. $\dfrac{4xy^2 + 2xy - 6x^2y}{2xy}$ $2y + 1 - 3x$

Written Exercises

Divide.

A

1. $\frac{5a + 15b}{5}$ $a + 3b$
2. $\frac{7a - 7b}{7}$ $a - b$
3. $\frac{8x - 4y}{4}$ $2x - y$
4. $\frac{3a + 12c}{3}$ $a + 4c$
5. $\frac{8x - 4y}{2}$ $4x - 2y$
6. $\frac{20x - 16}{4}$ $5x - 4$
7. $\frac{56m + 63n}{7}$ $8m + 9n$
8. $\frac{42x + 54y}{6}$ $7x + 9y$
9. $\frac{5a^2 + 45a}{5a}$ $a + 9$
10. $\frac{6x^2 - 48x}{6x}$ $x - 8$
11. $\frac{ab^2 - a^2b}{a}$ $b^2 - ab$
12. $\frac{xy - 4x^2y^2}{xy}$ $1 - 4xy$
13. $\frac{a^3 + a^2b + ab^2}{a}$ $a^2 + ab + b^2$
14. $\frac{x^3 + 3x^2 - x}{x}$ $x^2 + 3x - 1$
15. $\frac{6m^2n^2 - 2m + 4m^2}{2m}$ $3mn^2 - 1 + 2m$
16. $\frac{3n^4 + 3n^3 - 6n^2}{3n}$ $n^3 + n^2 - 2n$
17. $\frac{5x^2 - 15x^3y + 10x}{5x}$ $x - 3x^2y + 2$
18. $\frac{a^2b^3 - a^3b + 4a^2b^2}{a^2b}$ $b^2 - a^2 + 4b$
19. $\frac{6r^2s + 9rs^2 - 18r^3}{3r}$ $2rs + 3s^2 - 6r^2$
20. $\frac{n^2 - 3n^4 + 5n^5}{-n^2}$ $-1 + 3n^2 - 5n^3$
21. $\frac{24a^2b - 6a^2b^2 + 12a^3}{-6}$ $-4a^2b + a^2b^2 - 2a^3$

B

22. $\frac{-4x^4 + 12x^3 - 48x^2 + 8x}{4x}$ $-x^3 + 3x^2 - 12x + 2$
23. $\frac{32n - 16n + 16n^2 + 32n^2}{-8n}$ $-2 - 6n$
24. $\frac{6a^4b^4 + 2a^3b^3 + 4a^2b^2 + 2ab}{-2ab}$ $-3a^3b^3 - a^2b^2 - 2ab - 1$
25. $\frac{-18x^3y + 48x^2y^2 - 54xy^3}{-6xy}$ $3x^2 - 8xy + 9y^2$
26. $\frac{-24a^6b^3 + 12a^4b^2 - 9a^2b^3}{-3a^2b^2}$ $8a^4b - 4a^2 + 3b$
27. $\frac{21x^3b^4 - 28x^5b^3 + 42x^2b^5}{-7x^2b^3}$ $-3xb + 4x^3 - 6b^2$

For more practice, see page 418.

SELF-TEST

Divide.

1. $\frac{4x}{2x}$ 2
2. $\frac{2a^6}{a^3}$ $2a^3$
3. $\frac{-6mn^2}{-3mn}$ $2n$
4. $\frac{9x^2y^4}{-3x^2y^3}$ $-3y$
5. $\frac{9m - 12n}{3}$ $3m - 4n$
6. $\frac{4a^2 + 16a}{2a}$ $2a + 8$
7. $\frac{x^2y^2 - xy^2}{xy}$ $xy - y$
8. $\frac{3a^2b - 6ab^2}{3ab}$ $a - 2b^2$

12 • Dividing Polynomials (Optional)

Dividing by a polynomial is a little like doing a long division problem in arithmetic. Compare the steps.

$\frac{255}{15}$ $\qquad$ $\frac{x^2 + 7x + 12}{x + 3}$

$15\overline{)255}$ $\qquad$ $x + 3\overline{)x^2 + 7x + 12}$

$\begin{array}{r} 1 \\ 15\overline{)255} \end{array}$ $\qquad$ $\begin{array}{r} x \\ x + 3\overline{)x^2 + 7x + 12} \end{array}$

$\begin{array}{r} 1 \\ 15\overline{)255} \\ \underline{15} \end{array}$ $\qquad$ $\begin{array}{r} x \\ x + 3\overline{)x^2 + 7x + 12} \\ \underline{x^2 + 3x} \end{array}$

$\begin{array}{r} 1 \\ 15\overline{)255} \\ \underline{15} \\ 105 \end{array}$ $\qquad$ $\begin{array}{r} x \\ x + 3\overline{)x^2 + 7x + 12} \\ \underline{x^2 + 3x} \\ 4x + 12 \end{array}$

$\begin{array}{r} 17 \\ 15\overline{)255} \\ \underline{15} \\ 105 \end{array}$ $\qquad$ $\begin{array}{r} x + 4 \\ x + 3\overline{)x^2 + 7x + 12} \\ \underline{x^2 + 3x} \\ 4x + 12 \end{array}$

$\begin{array}{r} 17 \\ 15\overline{)255} \\ \underline{15} \\ 105 \\ \underline{105} \\ 0 \end{array}$ $\qquad$ $\begin{array}{r} x + 4 \\ x + 3\overline{)x^2 + 7x + 12} \\ \underline{x^2 + 3x} \\ 4x + 12 \\ \underline{4x + 12} \\ 0 \end{array}$

Check: $15 \times 17 = 255$ ✓ $\qquad$ $(x + 3)(x + 4) = x^2 + 7x + 12$ ✓

Written Exercises

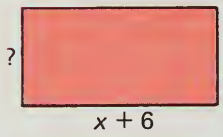
Divide. Check by multiplication.

A

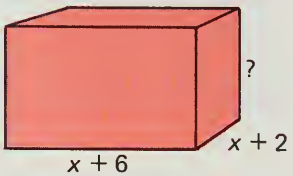
1. $\dfrac{x^2 + 4x + 4}{x + 2}$ $x + 2$
2. $\dfrac{a^2 + 6a + 8}{a + 4}$ $a + 2$
3. $\dfrac{y^2 + 10y + 24}{y + 6}$ $y + 4$
4. $\dfrac{x^2 - 4x + 4}{x - 2}$ $x - 2$
5. $\dfrac{x^2 - 4x - 12}{x - 6}$ $x + 2$
6. $\dfrac{x^2 - x - 2}{x - 2}$ $x + 1$
7. $\dfrac{x^2 - x - 12}{x + 3}$ $x - 4$
8. $\dfrac{n^2 - 3n - 4}{n + 1}$ $n - 4$
9. $\dfrac{a^2 - 3a - 10}{a + 2}$ $a - 5$
10. $\dfrac{s^2 - 7s + 10}{s - 5}$ $s - 2$
11. $\dfrac{x^2 - 9x + 18}{x - 3}$ $x - 6$
12. $\dfrac{c^2 - 10c + 21}{c - 7}$ $c - 3$

B

13. The area of the rectangle at the right is $x^2 + 4x - 12$. The length is $x + 6$. What is the width? $x - 2$



14. The volume of this rectangular prism is $x^3 + 11x^2 + 36x + 36$. Find the height. $x + 3$



Puzzles & Things

Do You Agree?

If you live in Parry Sound, then you live in Ontario.
If you live in Ontario, then you live in Canada.
Therefore, if you live in Parry Sound, you live in Canada.

Crackers are better than nothing.
Nothing is better than a banana split.
Therefore, crackers are better than a banana split.



What happened? See page 6 of Answers to Odd-Numbered Exercises in the back of the text.

Reviewing Arithmetic Skills

Simplify.

Sample $\frac{4}{6} = \frac{2 \cdot 2}{3 \cdot 2} = \frac{2}{3}$ *or* $\frac{4}{6} = \frac{2}{3}$

Divide the numerator and the denominator by 2.

1. $\frac{2}{4}$ $\frac{1}{2}$
2. $\frac{3}{15}$ $\frac{1}{5}$
3. $\frac{4}{16}$ $\frac{1}{4}$
4. $\frac{6}{8}$ $\frac{3}{4}$
5. $\frac{5}{10}$ $\frac{1}{2}$
6. $\frac{3}{9}$ $\frac{1}{3}$
7. $\frac{8}{10}$ $\frac{4}{5}$
8. $\frac{12}{3}$ 4
9. $\frac{16}{12}$ $\frac{4}{3}$
10. $\frac{18}{9}$ 2

Add. Simplify if possible.

11. $\frac{1}{3} + \frac{1}{3}$ $\frac{2}{3}$
12. $\frac{3}{5} + \frac{1}{5}$ $\frac{4}{5}$
13. $\frac{2}{8} + \frac{4}{8}$ $\frac{3}{4}$
14. $\frac{2}{6} + \frac{3}{6}$ $\frac{5}{6}$
15. $\frac{3}{12} + \frac{5}{12}$ $\frac{2}{3}$
16. $\frac{2}{9} + \frac{4}{9}$ $\frac{2}{3}$
17. $\frac{3}{14} + \frac{5}{14}$ $\frac{4}{7}$
18. $\frac{3}{10} + \frac{4}{10}$ $\frac{7}{10}$
19. $\frac{2}{7} + \frac{3}{7}$ $\frac{5}{7}$
20. $\frac{1}{4} + \frac{2}{4}$ $\frac{3}{4}$
21. $\frac{5}{14} + \frac{2}{14}$ $\frac{1}{2}$
22. $\frac{1}{12} + \frac{3}{12}$ $\frac{1}{3}$
23. $\frac{3}{10} + \frac{2}{10}$ $\frac{1}{2}$
24. $\frac{3}{15} + \frac{7}{15}$ $\frac{2}{3}$
25. $\frac{4}{13} + \frac{6}{13}$ $\frac{10}{13}$
26. $\frac{3}{14} + \frac{6}{14}$ $\frac{9}{14}$

Subtract. Simplify if possible.

27. $\frac{4}{5} - \frac{2}{5}$ $\frac{2}{5}$
28. $\frac{7}{8} - \frac{3}{8}$ $\frac{1}{2}$
29. $\frac{7}{9} - \frac{2}{9}$ $\frac{5}{9}$
30. $\frac{9}{11} - \frac{5}{11}$ $\frac{4}{11}$
31. $\frac{3}{4} - \frac{1}{4}$ $\frac{1}{2}$
32. $\frac{9}{10} - \frac{3}{10}$ $\frac{3}{5}$
33. $\frac{11}{12} - \frac{4}{12}$ $\frac{7}{12}$
34. $\frac{9}{14} - \frac{7}{14}$ $\frac{1}{7}$
35. $\frac{8}{13} - \frac{5}{13}$ $\frac{3}{13}$
36. $\frac{7}{12} - \frac{4}{12}$ $\frac{1}{4}$
37. $\frac{5}{6} - \frac{2}{6}$ $\frac{1}{2}$
38. $\frac{6}{10} - \frac{3}{10}$ $\frac{3}{10}$
39. $\frac{5}{8} - \frac{3}{8}$ $\frac{1}{4}$
40. $\frac{10}{14} - \frac{5}{14}$ $\frac{5}{14}$
41. $\frac{5}{12} - \frac{1}{12}$ $\frac{1}{3}$
42. $\frac{11}{13} - \frac{7}{13}$ $\frac{4}{13}$

CAREER NOTEBOOK

Marketing

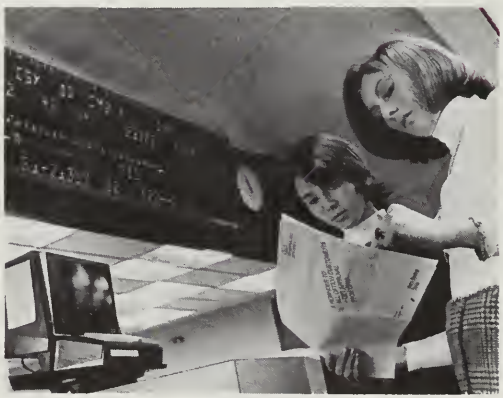
Many people aren't aware of all that is involved in putting a product on the market. Considerations must be given to consumer needs, competitive products, advertising, packaging, and profitability.



Packaging designers use artistic talent and marketing skills to design packages that protect and promote their contents.



Product designers work to improve the appearance and functional design of products.



Market researchers analyze data to forecast sales and recommend product design, packaging, and advertising.

CONSUMER CORNER



Which is a Better Buy?

At some time you have probably looked at two different brands of an item, and have thought about which one was the better buy. Of course, there are many things like quality and packaging to consider. As a consumer, however, you should be able to recognize which item is less expensive, and therefore be able to make good decisions.

Which cheese spread is less expensive?



8 oz.
$1.15



10 oz.
$1.50

Think:

$$8\overline{)\$1.15} \quad \$\ .14 +$$

about 14¢ per ounce

Think:

$$10\overline{)\$1.50} \quad \$\ .15$$

about 15¢ per ounce

Which strawberry jam is less expensive?



10 oz.
78¢

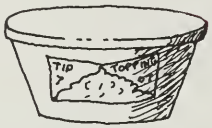


12 oz.
96¢

You don't always have to do this kind of arithmetic while you're shopping. Unit pricing labels on store shelves show you the price of the item per pound, per gram, per kilogram, per fluid ounce, or whatever unit seems easiest.

Most unit price labels look something like this.





UNIT PRICE	PRICE
$.08 Per oz.	$.72 Creamy Whip 9 oz.

UNIT PRICE	PRICE
$.09 Per oz.	$.63 Tip-Topping 7 oz.

Compare.

Round the unit price to the nearest cent. Complete the labels and compare the two brands. Which is less expensive?

1.

UNIT PRICE	PRICE
? Per fl. oz.	$.84 Green Mouthwash 7 fl. oz.

12¢

UNIT PRICE	PRICE
? Per fl. oz.	$.69 Fresh Mouthwash 6 fl. oz.

11.5¢ is about 12¢.
Fresh Mouthwash is slightly less expensive.

2.

UNIT PRICE	PRICE
? Per oz.	$.29 Spicy Mustard 8 oz.

about 4¢

UNIT PRICE	PRICE
? Per oz.	$.31 Amarillo Mustard 9 oz.

about 3¢
Amarillo Mustard is less expensive.

3.

UNIT PRICE	PRICE
? Per fl. oz.	$1.69 Dazzle Shampoo 8 fl. oz.

about 21¢

UNIT PRICE	PRICE
? Per fl. oz.	$1.39 Dew Drops Shampoo 7 fl. oz.

about 20¢
Dew Drops Shampoo is less expensive.

Reviewing the Chapter

Add. *(See pp. 136–137.)*

1. $x + 2$ and $x - 4$ → $2x - 2$
2. $x^2 - 1$ and $2x^2 - 5$ → $3x^2 - 6$
3. $6n - 14$ and $n + 16$ → $7n + 2$
4. $a^2 + 12$ and $4a^2 - 13$ → $5a^2 - 1$
5. $x^2 - 3y^2$ and $4x^2 + 10y^2$ → $5x^2 + 7y^2$
6. $a^2 + 7b^2$ and $-a^2 - 6b^2$ → b^2
7. $ax + by$ and $-3ax - by$ → $-2ax$
8. $9m + 4n$ and $-5m - n$ → $4m + 3n$

9. $(n + 3) + (2n + 5)$ → $3n + 8$
10. $(2x - y) + (9x - 7y)$ → $11x - 8y$
11. $(n^2 - 1) + (3n^2 + 1)$ → $4n^2$
12. $(3x - 7) + (x + 14)$ → $4x + 7$
13. $(a^2 + b^2) + (5a^2 + 7b^2)$ → $6a^2 + 8b^2$
14. $(1 - x^2) + (5 + 9x^2)$ → $6 + 8x^2$

Subtract. *(See pp. 138–139.)*

15. $x + 2$ and $-(x - 3)$ → 5
16. $3y - 1$ and $-(2y + 1)$ → $y - 2$
17. $12x - 7y$ and $-(10x + 5y)$ → $2x - 12y$
18. $15m^2 + n$ and $-(11m^2 - 2n)$ → $4m^2 + 3n$
19. $a^2 + 5$ and $-(2a^2 - 3)$ → $-a^2 + 8$
20. $5a^2b^2 + 6$ and $-(a^2b^2 - 6)$ → $4a^2b^2 + 12$
21. $3x^2 - y^2$ and $-(4x^2 + y^2)$ → $-x^2 - 2y^2$
22. $9x + 4y$ and $-(11x - 3y)$ → $-2x + 7y$

23. $(5m - n) - (m + 4n)$ → $4m - 5n$
24. $(6a - b) - (7a + b)$ → $-a - 2b$
25. $(9x + y) - (2x + 3y)$ → $7x - 2y$
26. $(6x - 3y) - (5x - 3y)$ → x
27. $(2x + 1) - (9x + 1)$ → $-7x$
28. $(8a + b) - (a - 5b)$ → $7a + 6b$

Multiply. *(See pp. 140–141.)*

29. $a^2 \cdot a^4$ → a^6
30. $2a^2 \cdot a$ → $2a^3$
31. $a \cdot a^3$ → a^4
32. $(5a)(-6a^2)$ → $-30a^3$
33. $(14x)(2x^3)$ → $28x^4$
34. $(3x^2)(8x)$ → $24x^3$
35. $(-7y^2)(8y^3)$ → $-56y^5$
36. $(-2n^3)(-9n^2)$ → $18n^5$

Simplify. *(See pp. 142–143.)*

37. $(a^2)^3$ → a^6
38. $-(a^2)^3$ → $-a^6$
39. $(-a^2)^3$ → $-a^6$
40. $(-3x^2)^3$ → $-27x^6$
41. $(mn^2)^4$ → m^4n^8
42. $(3mn^2)^3$ → $27m^3n^6$
43. $(-x^2y^3)^3$ → $-x^6y^9$
44. $(3xy^4)^2$ → $9x^2y^8$

Multiply. *(See pp. 144–147.)*

45. $5(6 + n)$
$30 + 5n$

46. $7(6x + 3)$
$42x + 21$

47. $a(a + 4)$
$a^2 + 4a$

48. $-3(a + 2b)$
$-3a - 6b$

49. $9(2x - 3y)$
$18x - 27y$

50. $-2n(6 - n)$
$-12n + 2n^2$

51. $(n - 3)(n - 2)$
$n^2 - 5n + 6$

52. $(3n - 1)(4n - 1)$
$12n^2 - 7n + 1$

53. $(6x + y)(x - y)$
$6x^2 - 5xy - y^2$

54. $(r + 4)(2r + 8)$
$2r^2 + 16r + 32$

55. $(2a - 7)(5a + 2)$
$10a^2 - 31a - 14$

56. $(3x + y)(x - y)$
$3x^2 - 2xy - y^2$

Multiply by the FOIL method. *(See pp. 148–151.)*

57. $(n - 6)(n - 3)$
$n^2 - 9n + 18$

58. $(2x - 1)(7x + 2)$
$14x^2 - 3x - 2$

59. $(3n + 1)(10n + 2)$
$30n^2 + 16n + 2$

60. $(1 + 4n)(1 - 2n)$
$1 + 2n - 8n^2$

61. $(y + 4)(3y + 1)$
$3y^2 + 13y + 4$

62. $(x - 5)(x - 7)$
$x^2 - 12x + 35$

Express as a trinomial square. *(See pp. 152–153.)*

63. $(n + 4)^2$
$n^2 + 8n + 16$

64. $(n - 4)^2$
$n^2 - 8n + 16$

65. $(x + 1)^2$
$x^2 + 2x + 1$

66. $(m - 6)^2$
$m^2 - 12m + 36$

67. $(2x + y)^2$
$4x^2 + 4xy + y^2$

68. $(x - y)^2$
$x^2 - 2xy + y^2$

69. $(3m - 2n)^2$
$9m^2 - 12mn + 4n^2$

70. $(4a + 3b)^2$
$16a^2 + 24ab + 9b^2$

Divide. *(See pp. 154–155, 158–159.)*

71. $\dfrac{6n^2}{2n}$ $3n$

72. $\dfrac{50x^3}{5x}$ $10x^2$

73. $\dfrac{-10x^3}{x^2}$ $-10x$

74. $\dfrac{24xy^2}{-8xy}$ $-3y$

75. $\dfrac{18a^2b^4}{-3ab}$ $-6ab^3$

76. $\dfrac{40a^4b^3}{8ab^2}$ $5a^3b$

77. $\dfrac{121mn^2}{11m}$ $11n^2$

78. $\dfrac{48x^2y^2}{-4x^2y}$ $-12y$

79. $\dfrac{6n^2 + n}{n}$ $6n + 1$

80. $\dfrac{4m^2 + 12m}{4m}$ $m + 3$

81. $\dfrac{6x^2 - 3x}{3x}$ $2x - 1$

82. $\dfrac{2x^2y^2 - 4xy}{2xy}$ $xy - 2$

83. $\dfrac{3a^3 + 15a^2 + 18a}{3a}$
$a^2 + 5a + 6$

84. $\dfrac{15x^3 - 10x^2 - 5x}{5x}$
$3x^2 - 2x - 1$

85. $\dfrac{12a^2b^2 - 9ab}{3ab}$ $4ab - 3$

86. $\dfrac{4a^3 - 2a^2 + 2a}{2a}$
$2a^2 - a + 1$

87. $\dfrac{6x^2y + 8xy^2 + 10xy}{2xy}$
$3x + 4y + 5$

A permission-to-reproduce chapter test can be found on page T15.

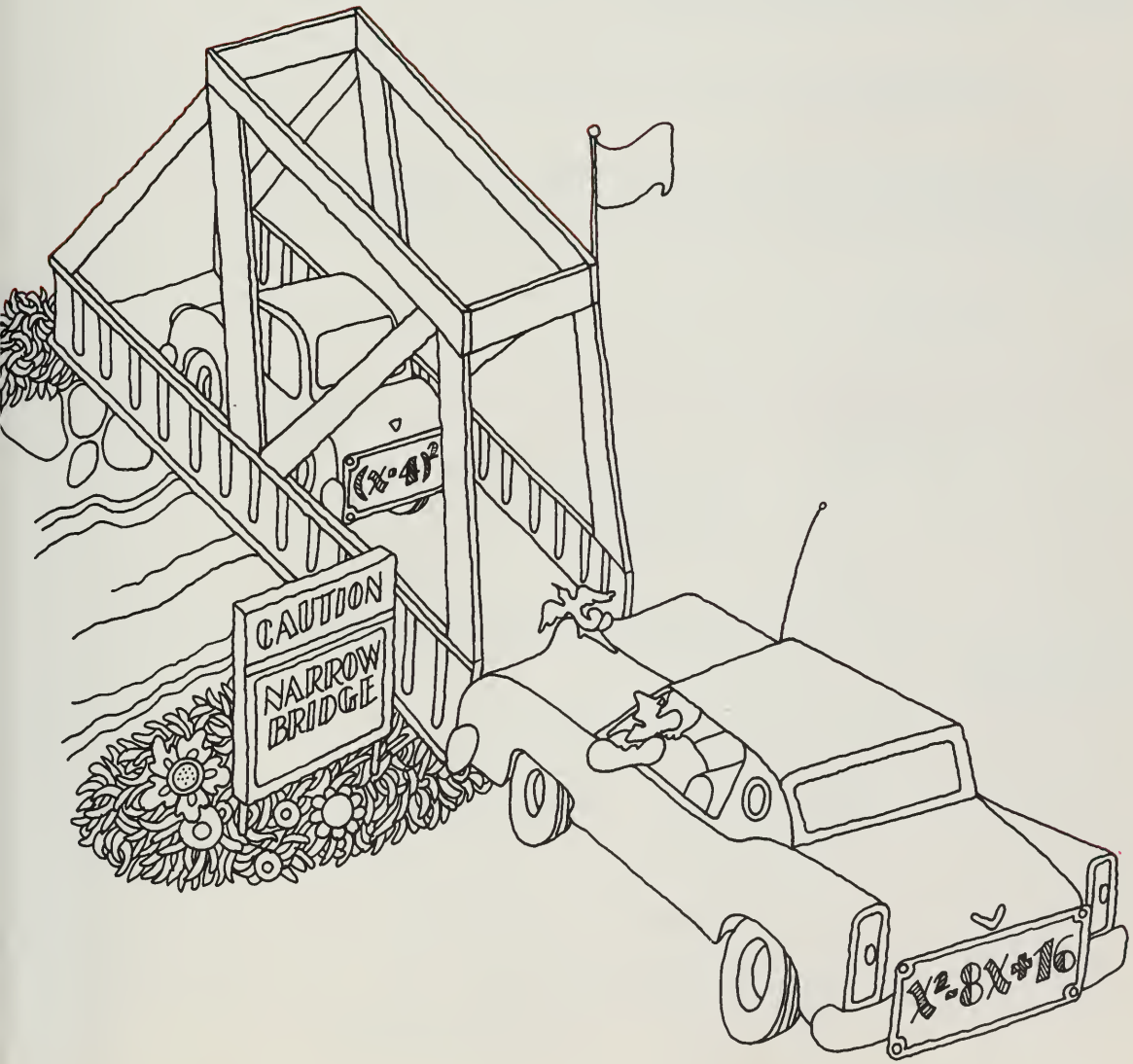
Here's what you'll learn in this chapter:

1. To find a common monomial factor in a polynomial.
2. To factor trinomials which are the product of two binomials.
3. To multiply binomials which are the sum and the difference of two monomials.
4. To factor the difference of two squares.



Chapter 6

Factoring



1 • Factoring in Arithmetic

The number 12 can be written as a product of its factors in several ways. The numbers 1, 2, 3, 4, 6, and 12 are all factors of 12.

$12 = 3 \cdot 4$ — factors of 12

$12 = 2 \cdot 6$ — factors of 12

$12 = 1 \cdot 12$ — factors of 12

If a number has no factors except itself and 1, it is called a **prime number.** Here are some prime numbers.

prime numbers → 2, 3, 5, 7, 11, 13, 19

Since 2 and 3 are prime numbers and also factors of 12, they are called **prime factors** of 12. Note how you can find *all* the prime factors of a number.

$12 = 3 \cdot 4$

$= 3 \cdot 2 \cdot 2$

$= 2 \cdot 2 \cdot 3$

The prime factors are usually arranged in order like this.

$140 = 10 \cdot 14$

$= 5 \cdot 2 \cdot 2 \cdot 7$

$= 2 \cdot 2 \cdot 5 \cdot 7$

Notice that 2 is a factor of both 12 and 140. We call 2 a **common factor** of 12 and 140.

EXAMPLE 1 Find a common factor of 35 and 28.

$35 = 5 \cdot 7$
$28 = 2 \cdot 2 \cdot 7$

7 is a common factor.

EXAMPLE 2 Find a common factor of 18 and 45.

$18 = 2 \cdot 3 \cdot 3$
$45 = 3 \cdot 3 \cdot 5$

3 is a common factor.

In the second example, 3 is a common factor, but $3 \cdot 3$, or 9, is also a common factor. The number 9 is the greatest number which is a common factor of 18 and 45. We call 9 the **greatest common factor** of 18 and 45.

EXAMPLE 3 Find the greatest common factor of 28 and 70.

$28 = 2 \cdot 2 \cdot 7$
$70 = 2 \cdot 5 \cdot 7$

$2 \cdot 7$, or 14, is the greatest common factor.

Here are a few clues for finding factors.

1. A number has 2 as a factor if its last digit is 0, 2, 4, 6, or 8.
2. A number has 5 as a factor if its last digit is 0 or 5.
3. A number has 3 as a factor if the sum of its digits can be divided evenly by 3.

Sample 726 is divisible by 3 because $7 + 2 + 6$, or 15, is divisible by 3.

Written Exercises

Find the prime factors.

A

1. 42 2, 3, 7	**2.** 100 2, 2, 5, 5	**3.** 36 2, 2, 3, 3	**4.** 54 2, 3, 3, 3	**5.** 310 2, 5, 31
6. 125 5, 5, 5	**7.** 120 2, 2, 2, 3, 5	**8.** 625 5, 5, 5, 5	**9.** 624 2, 2, 2, 2, 3, 13	**10.** 93 3, 31

Find the greatest common factor of the pair of numbers.

11. 12, 15 3	**12.** 16, 22 2	**13.** 14, 21 7	**14.** 27, 36 9
15. 11, 20 1	**16.** 12, 18 6	**17.** 22, 33 11	**18.** 15, 40 5
19. 16, 64 16	**20.** 21, 42 21	**21.** 36, 48 12	**22.** 14, 49 7
23. 42, 28 14	**24.** 18, 24 6	**25.** 100, 125 25	**26.** 30, 100 10
27. 16, 36 4	**28.** 15, 24 3	**29.** 24, 56 8	**30.** 19, 21 1
31. 12, 20 4	**32.** 28, 35 7	**33.** 40, 100 20	**34.** 15, 35 5
35. 42, 48 6	**36.** 50, 60 10	**37.** 100, 400 100	**38.** 36, 81 9

2 • Common Monomial Factors

Now you can find common factors of variable expressions.

$$\left.\begin{aligned} 2a &= 2 \cdot a \\ 6a^2 &= 2 \cdot 3 \cdot a \cdot a \end{aligned}\right\}$$ $2 \cdot a$, or $2a$, is the greatest common factor.

Use this idea to find a common monomial factor of the terms of a polynomial. When you do this, you **factor** the polynomial.

$$\begin{aligned} 2a + 6a^2 &= (2a \cdot 1) + (2a \cdot 3a) \\ &= 2a(1 + 3a) \end{aligned}$$

This is a common monomial factor of $2a$ and $6a^2$.

Check: Use the distributive property. → $2a(1 + 3a) = 2a + 6a^2$ ✓

EXAMPLE 1 Factor $3x^2 - ax^2$.

$$\begin{aligned} 3x^2 - ax^2 &= (3 \cdot x^2) - (a \cdot x^2) \\ &= x^2(3 - a) \end{aligned}$$

With practice you will be able to recognize the common monomial factors at sight.

EXAMPLE 2 $12mx - 6m^2x^2 = 6mx(2 - mx)$

Think: $6mx \cdot 2$ *Think:* $6mx \cdot mx$

EXAMPLE 3 $x^3 + x^2 + 2x = x(x^2 + x + 2)$

Think: $x \cdot x^2$ *Think:* $x \cdot x$ *Think:* $2 \cdot x$

EXAMPLE 4 $y^4 + y^3 + 4y^2 = y^2(y^2 + y + 4)$

Think: $y^2 \cdot y^2$ *Think:* $y^2 \cdot y$ *Think:* $4 \cdot y^2$

Classroom Practice

Complete.

1. $4xy = 4x(_?_)$ y
2. $25x^2 = 5x(_?_)$ $5x$
3. $3x^2y^2 = 3xy(_?_)$ xy
4. $36a^3 = 12a^2(_?_)$ $3a$
5. $72b^2 = 8b(_?_)$ $9b$
6. $40a^4b^4 = 4a^2b^2(_?_)$ $10a^2b^2$

Find the greatest common factor of the monomial pair.

7. $2a, 6a$ $2a$
8. $15n, 5n$ $5n$
9. $n^2, 3n$ n
10. $3x, 6x^2$ $3x$

Factor.

11. $3x + 2x^2$ $x(3 + 2x)$
12. $2a^2 + 4a$ $2a(a + 2)$
13. $7x^3 + 7x^2$ $7x^2(x + 1)$
14. $3x^2 + 6xy$ $3x(x + 2y)$

Written Exercises

Factor.

A

1. $9 + 3x$ $3(3 + x)$
2. $5 - 15n$ $5(1 - 3n)$
3. $2x - 10$ $2(x - 5)$
4. $4y - 16$ $4(y - 4)$
5. $3x^2 - x$ $x(3x - 1)$
6. $5n^2 - 2n$ $n(5n - 2)$
7. $2x^2 - 6x$ $2x(x - 3)$
8. $y^2 - 2y$ $y(y - 2)$
9. $28n^2 - 7n$ $7n(4n - 1)$
10. $2xy - y^2$ $y(2x - y)$
11. $4x^2 - 8x$ $4x(x - 2)$
12. $21y^2 - 7xy$ $7y(3y - x)$
13. $25mn - 5m^2n^2$ $5mn(5 - mn)$
14. $8a^2b - 24ab^2$ $8ab(a - 3b)$
15. $9x^2 - 27x^2y$ $9x^2(1 - 3y)$
16. $12a^2 + 36a^2b$ $12a^2(1 + 3b)$
17. $3x^2 - 6x + 21$ $3(x^2 - 2x + 7)$
18. $5n^3 + 15n^2 + 25n$ $5n(n^2 + 3n + 5)$
19. $55y^2 + 22y + 44$ $11(5y^2 + 2y + 4)$
20. $2n^2 + 4mn + 80m^2$ $2(n^2 + 2mn + 40m^2)$
21. $4a^2 + 12ab - 16b^2$ $4(a^2 + 3ab - 4b^2)$
22. $3x^2 - 12xy + 9y^2$ $3(x^2 - 4xy + 3y^2)$

B

23. $6x^2 + 6x + 24xy + 42$ $6(x^2 + x + 4xy + 7)$
24. $-13x + 26x^2 + 39x^3$ $13x(-1 + 2x + 3x^2)$
25. $-50a^2 + 25b^2 + 75ab$ $25(-2a^2 + b^2 + 3ab)$
26. $48mn + 72m^2n^2 + 60m^3n^3$ $12mn(4 + 6mn + 5m^2n^2)$
27. $56x^3y^3 - 72x^2y^2 - 64xy$ $8xy(7x^2y^2 - 9xy - 8)$
28. $32a^2b^4 - 16ab^3 + 48a^3b^5$ $16ab^3(2ab - 1 + 3a^2b^2)$

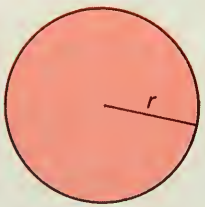
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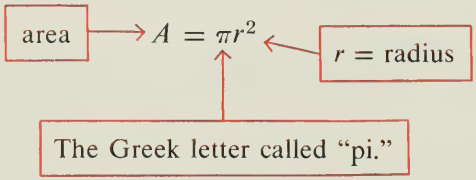
29. $x(a - b) + 2(a - b)$ $(x + 2)(a - b)$
30. $y(a + d) - 4(a + d)$ $(y - 4)(a + d)$
31. $mn - nx + my - xy$ $(n + y)(m - x)$
32. $ab - 3ad + b^2 - 3bd$ $(a + b)(b - 3d)$
33. $2a - a^2 + 2b - ab$ $(2 - a)(a + b)$
34. $3x^2 - 2xy - 3x + 2y$ $(3x - 2y)(x - 1)$
35. $2x^2 - 8 + x^2y - 4y$ $(x^2 - 4)(y + 2)$
36. $a^3 - ab^2 - a^2b + b^3$ $(a^2 - b^2)(a - b)$

For more practice, see page 419.

3 • Using Factoring

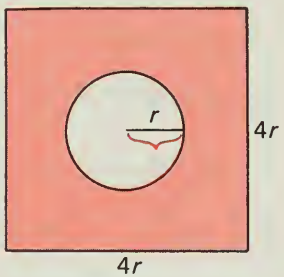
You are probably familiar with the formula $A = \pi r^2$. It is used to calculate the area of a circle. Take a look at this formula and what it means.





The symbol π in the formula has a value of about 3.14 or $\frac{22}{7}$. Sometimes it's difficult to work with these values, so we often just use the symbol π.

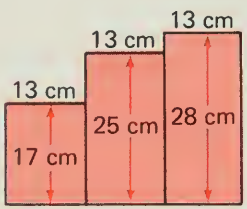
EXAMPLE 1 Find the area of the shaded part in the figure at the left.



Area of square: $(4r)^2 = 16r^2$
Area of circle: πr^2
Area of shaded part: $16r^2 - \pi r^2$
$= r^2(16 - \pi)$

In doing arithmetic problems with large numbers, be sure to look for common monomial factors as you work. By factoring, you may be able to save yourself some computation.

EXAMPLE 2 Find the area of the shaded part.



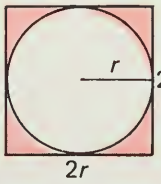
Area $= (13 \times 17) + (13 \times 25) + (13 \times 28)$
$= 13(17 + 25 + 28)$
$= 13 \times 70$
$= 910$

Answer: 910 cm^2

Written Exercises

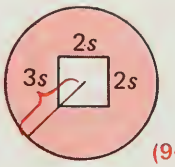
Find the area of the shaded part. Factor the expression. Leave your answer in terms of π.

A **1.**



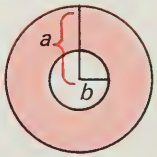
$(4 - \pi)r^2$

2.



$(9\pi - 4)s^2$

3.



$(a^2 - b^2)\pi$

Find the area of the shaded part.

4.



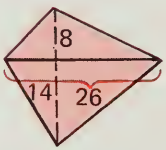
560 cm²

5.



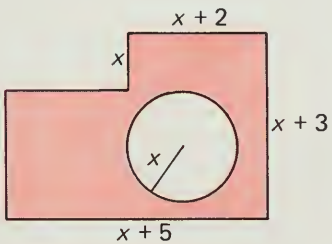
72

6.



286

B **7.** What's the area of the part of the yard that is not in the circular region around the tree? Show your answer in factored form.



$x^2(1 - \pi) + 5(x + 3)$



AL KAUFMAN Cartoon by permission of Masters Agency

SELF-TEST

Find the greatest common factor of the pair.

1. 56, 64 8

2. $24a$, $8b$ 8

3. $36x^2$, $3xy$ $3x$

4. $2mn$, $4mn^2$ $2mn$

Factor.

5. $15x^2y^2 + 25xy^3$ $5xy^2(3x + 5y)$

6. $18mn^2 - 9m^2n^2$ $9mn^2(2 - m)$

7. $8a^2b^3 + 56a^2c^3$ $8a^2(b^3 + 7c^3)$

4 • Factoring Trinomials—Two Sums

Up to now the only trinomials you have factored are those having a common monomial factor.

$$3x^3 + 2x^2 + x = x(3x^2 + 2x + 1)$$

Common monomial factor

Not all trinomials can be factored, but in the next few lessons you will see that many are the product of two binomials. For example, if $(x + 2)(x + 3) = x^2 + 5x + 6$, then we know that $x^2 + 5x + 6$ has two factors, $x + 2$ and $x + 3$.

Recall how to multiply two binomials at sight.

Step 1: $(x + 2)(x + 3) = x^2 \ldots$

Step 2: $(x + 2)(x + 3) = x^2 + 5x \ldots$

Step 3: $(x + 2)(x + 3) = x^2 + 5x + 6$

Now reverse this process to find the factors.

Step 1: $x^2 + 5x + 6 = (\quad)(\quad)$ ← Draw your parentheses.

Step 2: $x^2 + 5x + 6 = (x \quad)(x \quad)$ ← Your first terms must give you x^2.

Step 3: $x^2 + 5x + 6 = (x \ ? \)(x \ ? \)$

What two numbers have a product of 6?

1 and 6 2 and 3

Which of the pairs of factors of 6 have a sum of 5?

2 and 3

Step 4: $x^2 + 5x + 6 = (x + 2)(x + 3)$

Of course, you should check to see if your answer is correct.

$$(x + 2)(x + 3) = x^2 + 5x + 6 \quad \checkmark$$

Study these two examples.

EXAMPLE 1 Factor $x^2 + 7x + 10$.

Step 1: $x^2 + 7x + 10 = (\quad)(\quad)$
Step 2: $x^2 + 7x + 10 = (x \quad)(x \quad)$
Step 3: $x^2 + 7x + 10 = (x \ ? \)(x \ ? \)$

What two numbers have a product of 10?

1 and 10 2 and 5

Which of the pairs of factors of 10 have a sum of 7?

2 and 5

Step 4: $x^2 + 7x + 10 = (x + 2)(x + 5)$

EXAMPLE 2 Factor $a^2 + 10a + 16$.

Step 1: $a^2 + 10a + 16 = (\quad)(\quad)$
Step 2: $a^2 + 10a + 16 = (a \quad)(a \quad)$
Step 3: $a^2 + 10a + 16 = (a \ ? \)(a \ ? \)$

What two numbers have a product of 16?

1 and 16 2 and 8

4 and 4

Which of the pairs of factors of 16 have a sum of 10?

2 and 8

Step 4: $a^2 + 10a + 16 = (a + 2)(a + 8)$

Classroom Practice

Name the possible pairs of factors.

1. 10 1, 10; 2, 5	**2.** 7 1, 7	**3.** 9 1, 9; 3, 3	**4.** 6 1, 6; 2, 3	**5.** 11 1, 11
6. 12 1, 12; 2, 6; 3, 4	**7.** 15 1, 15; 3, 5	**8.** 17 1, 17	**9.** 16 1, 16; 2, 8; 4, 4	**10.** 20 1, 20; 2, 10; 4, 5
11. 21 1, 21; 3, 7	**12.** 24 1, 24; 2, 12; 3, 8; 4, 6	**13.** 25 1, 25; 5, 5	**14.** 30 1, 30; 2, 15; 3, 10; 5, 6	**15.** 36 1, 36; 2, 18; 3, 12; 4, 9; 6, 6

Factor.

16. $x^2 + 8x + 7$
$(x + 1)(x + 7)$

17. $x^2 + 4x + 3$
$(x + 1)(x + 3)$

18. $a^2 + 6a + 5$
$(a + 1)(a + 5)$

19. $a^2 + 5a + 4$
$(a + 4)(a + 1)$

20. $x^2 + 4x + 4$
$(x + 2)(x + 2)$

21. $y^2 + 8y + 15$
$(y + 3)(y + 5)$

22. $n^2 + 8n + 12$
$(n + 6)(n + 2)$

23. $x^2 + 6x + 8$
$(x + 4)(x + 2)$

24. $n^2 + 11n + 24$
$(n + 8)(n + 3)$

25. $x^2 + 13x + 42$
$(x + 6)(x + 7)$

26. $x^2 + 13x + 30$
$(x + 3)(x + 10)$

27. $y^2 + 13y + 40$
$(y + 5)(y + 8)$

Written Exercises

Complete.

		If the product of two numbers is:	→ and their sum is: →	then the two numbers are:
Sample		18	11	2 and 9
A	1.	5	6	1, 5 ?
	2.	6	5	3, 2 ?
	3.	8	6	2, 4 ?
	4.	12	8	2, 6 ?
	5.	12	7	3, 4 ?
	6.	9	6	3, 3 ?
	7.	15	8	3, 5 ?
	8.	14	9	7, 2 ?
	9.	18	9	3, 6 ?
	10.	20	9	4, 5 ?
	11.	21	10	3, 7 ?
	12.	24	14	2, 12 ?
	13.	24	11	3, 8 ?
	14.	36	13	4, 9 ?
	15.	36	12	6, 6 ?

Complete.

16. $x^2 + 8x + 7 = (x \underset{+7}{_?_})(x \underset{+1}{_?_})$

17. $a^2 + 7a + 6 = (a \underset{+6}{_?_})(a \underset{+1}{_?_})$

18. $n^2 + 9n + 20 = (n \underset{+5}{_?_})(n \underset{+4}{_?_})$

19. $x^2 + 9x + 18 = (x \underset{+6}{_?_})(x \underset{+3}{_?_})$

20. $y^2 + 9y + 14 = (y \underset{+7}{_?_})(y \underset{+2}{_?_})$

21. $r^2 + 10r + 21 = (r \underset{+3}{_?_})(r \underset{+7}{_?_})$

Factor. Check by multiplication.

22. $y^2 + 3y + 2$
$(y + 1)(y + 2)$

23. $n^2 + 17n + 16$
$(n + 1)(n + 16)$

24. $y^2 + 16y + 15$
$(y + 1)(y + 15)$

25. $a^2 + 5a + 4$
$(a + 1)(a + 4)$

26. $m^2 + 7m + 6$
$(m + 1)(m + 6)$

27. $x^2 + 11x + 10$
$(x + 1)(x + 10)$

28. $x^2 + 11x + 18$
$(x + 9)(x + 2)$

29. $n^2 + 6n + 8$
$(n + 4)(n + 2)$

30. $y^2 + 6y + 9$
$(y + 3)(y + 3)$

Factor.

31. $x^2 + 9x + 14$
$(x + 7)(x + 2)$

32. $a^2 + 9a + 18$
$(a + 3)(a + 6)$

33. $y^2 + 12y + 11$
$(y + 1)(y + 11)$

34. $r^2 + 11r + 30$
$(r + 6)(r + 5)$

35. $b^2 + 12b + 27$
$(b + 3)(b + 9)$

36. $m^2 + 11m + 28$
$(m + 7)(m + 4)$

37. $x^2 + 12x + 32$
$(x + 4)(x + 8)$

38. $y^2 + 24y + 23$
$(y + 23)(y + 1)$

39. $x^2 + 13x + 30$
$(x + 3)(x + 10)$

40. $a^2 + 9a + 20$
$(a + 4)(a + 5)$

41. $a^2 + 12a + 20$
$(a + 2)(a + 10)$

42. $n^2 + 12n + 35$
$(n + 7)(n + 5)$

B **43.** $x^2 + 27x + 50$
$(x + 25)(x + 2)$

44. $x^2 + 17x + 72$
$(x + 9)(x + 8)$

45. $m^2 + 28m + 75$
$(m + 25)(m + 3)$

46. $x^2 + 20x + 64$
$(x + 16)(x + 4)$

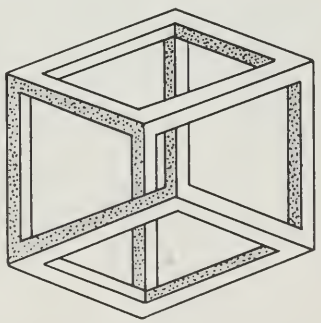
47. $a^2 + 24a + 63$
$(a + 21)(a + 3)$

48. $y^2 + 24y + 44$
$(y + 22)(y + 2)$

Puzzles & Things

Optical Illusion

The museum director asked a carpenter to put glass panes in the showcase shown here. Is it possible? **No, it is not possible.**



5 • Factoring Trinomials—Two Differences

You know that when you want to factor a positive number like 15, you might also consider the negative factors as well as the positive ones.

$$15 = 3 \times 5 \qquad \text{but also} \qquad 15 = (-3) \times (-5)$$

In this section you will use this idea to factor some trinomials.

EXAMPLE 1 Factor $x^2 - 8x + 15$.

Step 1: $x^2 - 8x + 15 = (\quad)(\quad)$
Step 2: $x^2 - 8x + 15 = (x \quad)(x \quad)$
Step 3: $x^2 - 8x + 15 = (x \ ? \)(x \ ? \)$

What two negative numbers have a product of 15?

-1 and -15 $\qquad$ -3 and -5

Which of the pairs of factors of 15 have a sum of -8?

-3 and -5

Step 4: $x^2 - 8x + 15 = (x - 3)(x - 5)$

Check: $(x - 3)(x - 5) = x^2 - 8x + 15$ ✓

EXAMPLE 2 Factor $n^2 - 10n + 16$.

Step 1: $n^2 - 10n + 16 = (\quad)(\quad)$
Step 2: $n^2 - 10n + 16 = (n \quad)(n \quad)$
Step 3: $n^2 - 10n + 16 = (n \ ? \)(n \ ? \)$

You have to find two numbers whose product is 16 and whose sum is -10.

-8 and -2

Step 4: $n^2 - 10n + 16 = (n - 8)(n - 2)$

Check: $(n - 8)(n - 2) = n^2 - 10n + 16$ ✓

Classroom Practice

Tell whether the factors will be sums or differences. Factor.

1. $x^2 - 6x + 9$
differences; $(x - 3)(x - 3)$

2. $x^2 + 2x + 1$
sums; $(x + 1)(x + 1)$

3. $y^2 + 3y + 2$
sums; $(x + 2)(x + 1)$

4. $n^2 - 7n + 10$
differences; $(n - 5)(n - 2)$

5. $n^2 - 10n + 24$
differences; $(n - 6)(n - 4)$

6. $n^2 + 5n + 6$
sums; $(n + 3)(n + 2)$

7. $c^2 + 7c + 6$
sums; $(c + 1)(c + 6)$

8. $x^2 - 4x + 4$
differences; $(x - 2)(x - 2)$

9. $a^2 - 9a + 20$
differences; $(a - 4)(a - 5)$

Written Exercises

Complete.

A

	If the product of two numbers is:	→ and their sum is: →	then the two numbers are:
1.	6	-5	$-3, -2$?
2.	10	-7	$-5, -2$?
3.	18	-9	$-6, -3$?
4.	12	-7	$-3, -4$?
5.	24	-10	$-6, -4$?

Factor.

6. $n^2 + 3n + 2$
$(n + 2)(n + 1)$

7. $x^2 - 7x + 10$
$(x - 5)(x - 2)$

8. $y^2 - 8y + 12$
$(y - 6)(y - 2)$

9. $x^2 - 5x + 6$
$(x - 3)(x - 2)$

10. $n^2 - 10n + 21$
$(n - 3)(n - 7)$

11. $y^2 - 6y + 5$
$(y - 5)(y - 1)$

12. $b^2 - 2b + 1$
$(b - 1)(b - 1)$

13. $x^2 - 11x + 24$
$(x - 3)(x - 8)$

14. $x^2 - 13x + 30$
$(x - 3)(x - 10)$

15. $n^2 + 9n + 18$
$(n + 6)(n + 3)$

16. $n^2 - 11n + 18$
$(n - 9)(n - 2)$

17. $n^2 - 5n + 4$
$(n - 4)(n - 1)$

18. $a^2 - 9a + 14$
$(a - 7)(a - 2)$

19. $x^2 + 6x + 8$
$(x + 4)(x + 2)$

20. $x^2 - 7x + 12$
$(x - 4)(x - 3)$

21. $y^2 - 11y + 28$
$(y - 7)(y - 4)$

22. $x^2 - 12x + 27$
$(x - 3)(x - 9)$

23. $n^2 + 10n + 25$
$(n + 5)(n + 5)$

24. $n^2 - 11n + 24$
$(n - 3)(n - 8)$

25. $y^2 - 11y + 30$
$(y - 6)(y - 5)$

26. $r^2 - 14r + 33$
$(r - 3)(r - 11)$

27. $x^2 - 14x + 49$
$(x - 7)(x - 7)$

28. $a^2 - 12a + 36$
$(a - 6)(a - 6)$

29. $m^2 - 13m + 40$
$(m - 5)(m - 8)$

For more practice, see page 419.

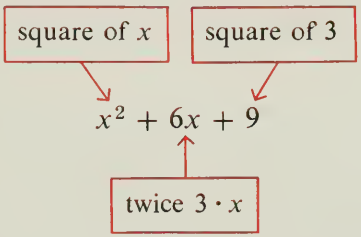
6 • Factoring Trinomial Squares

When the two factors of a trinomial are equal, the trinomial is a **trinomial square.**

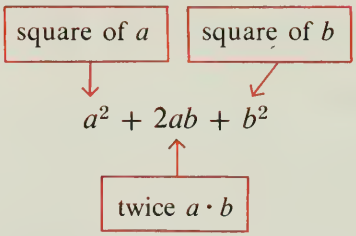
This is a trinomial square because it is the product of two equal factors.

$$x^2 + 6x + 9 = (x + 3)(x + 3) = (x + 3)^2$$

Take a good look at trinomial squares. Learn how to recognize them.

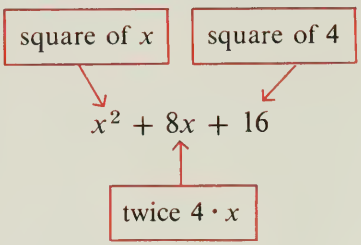


So, $x^2 + 6x + 9 = (x + 3)^2$.



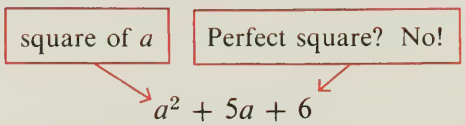
So, $a^2 + 2ab + b^2 = (a + b)^2$.

EXAMPLE 1 Is $x^2 + 8x + 16$ a trinomial square? See if the terms meet the test.



Yes, $x^2 + 8x + 16 = (x + 4)^2$.

EXAMPLE 2 Is $a^2 + 5a + 6$ a trinomial square?

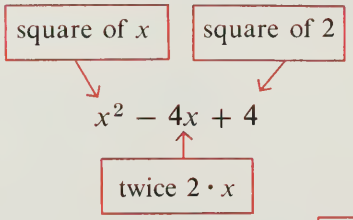


No, $a^2 + 5a + 6$ is not a trinomial square.

Of course, if you forget how to recognize a trinomial square, you can factor using the method you learned in the last two sections. This new method is only a short cut to make work a little easier.

Sometimes a trinomial is the square of a difference. In that case, the middle term will have a minus sign.

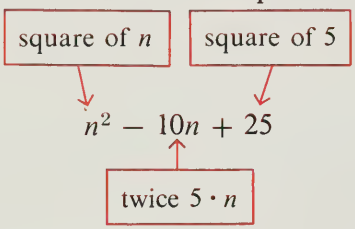
EXAMPLE 3 Is $x^2 - 4x + 4$ a trinomial square?



Yes, $x^2 - 4x + 4 = (x - 2)^2$.

Use $-$ because the middle term in the trinomial is negative.

EXAMPLE 4 Is $n^2 - 10n + 25$ a trinomial square?



Yes, $n^2 - 10n + 25 = (n - 5)^2$.

Written Exercises

Which of the following are trinomial squares?

A

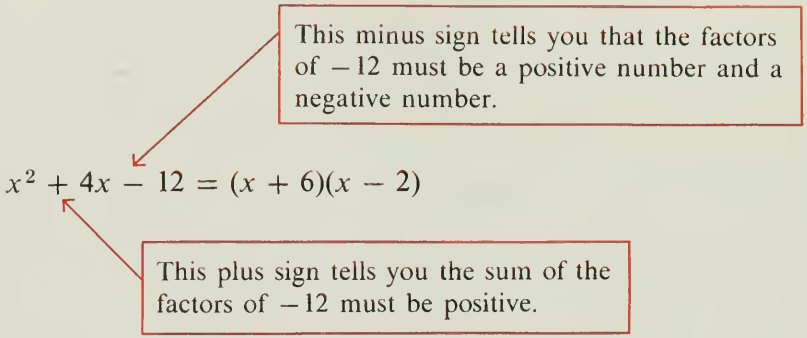
1. $x^2 + 2x + 1$ yes
2. $y^2 + 4y + 4$ yes
3. $a^2 + 6a + 9$ yes
4. $y^2 + y + 4$ no
5. $y^2 + 4y + 1$ no
6. $y^2 + 2y + 4$ no

Factor. Note that four exercises are not trinomial squares. Factor those as you did in the last sections.

7. $a^2 + 10a + 25$ $(a + 5)^2$
8. $n^2 + 4n + 4$ $(n + 2)^2$
9. $x^2 + 8x + 16$ $(x + 4)^2$
10. $n^2 + 12n + 36$ $(n + 6)^2$
11. $x^2 - 4x + 4$ $(x - 2)^2$
12. $a^2 - 9a + 20$ $(a - 4)(a - 5)$
13. $y^2 + 14y + 49$ $(y + 7)^2$
14. $x^2 - 20x + 100$ $(x - 10)^2$
15. $n^2 + 18n + 81$ $(n + 9)^2$
16. $x^2 + 15x + 56$ $(x + 7)(x + 8)$
17. $x^2 - 19x + 90$ $(x - 9)(x - 10)$
18. $b^2 - 15b + 36$ $(b - 3)(b - 12)$
19. $a^2 - 2ab + b^2$ $(a - b)^2$
20. $m^2n^2 + 18mn + 81$ $(mn + 9)^2$
21. $1 - 20x + 100x^2$ $(1 - 10x)^2$

7 • Factoring Trinomials

Here is a trinomial having one factor a sum and one factor a difference. Notice how the signs help you discover the factors.



EXAMPLE 1 Factor $n^2 + 4n - 21$.

Step 1: $n^2 + 4n - 21 = (\quad)(\quad)$
Step 2: $n^2 + 4n - 21 = (n \quad)(n \quad)$
Step 3: $n^2 + 4n - 21 = (n \ ? \)(n \ ? \)$

Look for factors of −21 whose sum is +4.

$3 \times -7 = -21$ and $3 + (-7) = -4$
$-3 \times 7 = -21$ and $-3 + 7 = 4$ ✓

Step 4: $n^2 + 4n - 21 = (n - 3)(n + 7)$

EXAMPLE 2 Factor $x^2 - 2x - 15$.

Step 1: $x^2 - 2x - 15 = (\quad)(\quad)$
Step 2: $x^2 - 2x - 15 = (x \quad)(x \quad)$
Step 3: $x^2 - 2x - 15 = (x \ ? \)(x \ ? \)$

Look for factors of −15 whose sum is −2.

$-3 \times 5 = -15$ and $-3 + 5 = 2$
$3 \times -5 = -15$ and $3 + (-5) = -2$ ✓

Step 4: $x^2 - 2x - 15 = (x + 3)(x - 5)$

In the next example, notice that -24 has many different factors. You might have to check quite a few of these before you find the right ones.

EXAMPLE 3 Factor $x^2 - 2x - 24$.

Step 1: $x^2 - 2x - 24 = (\quad)(\quad)$
Step 2: $x^2 - 2x - 24 = (x \quad)(x \quad)$
Step 3: $x^2 - 2x - 24 = (x \ ? \)(x \ ? \)$

Look for factors of -24 whose sum is -2.

$2 \times -12 = -24$ and $2 + (-12) = -10$
$3 \times -8 = -24$ and $3 + (-8) = -5$
$4 \times -6 = -24$ and $4 + (-6) = -2$

Step 4: $x^2 - 2x - 24 = (x + 4)(x - 6)$

It is always wise to check your answer by multiplying.

1. 1, −6; −1, 6; 2, −3; −2, 3
2. 1, −5; −1, 5
3. 1, −10; −1, 10; 2, −5; −2, 5
4. 1, −8; −1, 8; 2, −4; −2, 4
5. 1, −7; −1, 7
6. 1, −12; −1, 12; 2, −6; −2, 6; 3, −4; −3, 4
7. 1, −16; −1, 16; 2, −8; −2, 8; 4, −4
8. 1, −20; −1, 20; 2, −10; −2, 10; 4, −5; −4, 5
9. 1, −36; −1, 36; 2, −18; −2, 18; 3, −12; −3, 12; 4, −9; −4, 9; 6, −6
10. 1, −24; −1, 24; 2, −12; −2, 12; 3, −8; −3, 8; 4, −6; −4, 6

Classroom Practice

Name the possible pairs of factors.

1. -6 **2.** -5 **3.** -10 **4.** -8 **5.** -7

6. -12 **7.** -16 **8.** -20 **9.** -36 **10.** -24

Factor.

11. $a^2 + 5a - 14$
$(a - 2)(a + 7)$

12. $x^2 + x - 6$
$(x - 2)(x + 3)$

13. $y^2 + 2y - 8$
$(y - 2)(y + 4)$

14. $n^2 - 3n - 10$
$(n - 5)(n + 2)$

15. $x^2 - 6x - 16$
$(x - 8)(x + 2)$

16. $n^2 - 4n - 5$
$(n - 5)(n + 1)$

17. $x^2 + 4x - 12$
$(x - 2)(x + 6)$

18. $a^2 - 3a - 40$
$(a - 8)(a + 5)$

19. $x^2 + 5x - 24$
$(x - 3)(x + 8)$

20. $y^2 + 6y - 16$
$(y - 2)(y + 8)$

21. $b^2 - 5b - 24$
$(b - 8)(b + 3)$

22. $m^2 + 8m - 33$
$(m - 3)(m + 11)$

23. $x^2 + x - 30$
$(x - 5)(x + 6)$

24. $y^2 + y - 56$
$(y - 7)(y + 8)$

25. $n^2 - 2n - 63$
$(n - 9)(n + 7)$

26. $x^2 - 2x - 24$
$(x + 4)(x - 6)$

27. $x^2 + 2x - 15$
$(x - 3)(x + 5)$

28. $a^2 + a - 20$
$(a - 4)(a + 5)$

29. $y^2 - 5y - 36$
$(y - 9)(y + 4)$

30. $n^2 + 2n - 35$
$(n - 5)(n + 7)$

31. $x^2 - 4x - 32$
$(x - 8)(x + 4)$

Written Exercises

Complete.

		If the product of two numbers is:	→ and their sum is: →	then the two numbers are:
A	1.	−10	−3	−5, 2 ?
	2.	−18	7	9, −2 ?
	3.	−4	−3	−4, 1 ?
	4.	−21	−4	−7, 3 ?
	5.	−2	−1	−2, 1 ?
	6.	−6	1	3, −2 ?
	7.	−30	−7	−10, 3?
	8.	−30	−1	−6, 5 ?
	9.	−28	3	7, −4 ?
	10.	−15	2	5, −3 ?
	11.	−14	13	14, −1?
	12.	−24	−2	−6, 4 ?

Factor.

13. $x^2 + 4x - 21$
$(x + 7)(x - 3)$

14. $x^2 - 3x - 4$
$(x - 4)(x + 1)$

15. $x^2 + 7x - 18$
$(x + 9)(x - 2)$

16. $x^2 - 6x - 16$
$(x - 8)(x + 2)$

17. $b^2 + b - 12$
$(b - 3)(b + 4)$

18. $b^2 - 4b - 12$
$(b - 6)(b + 2)$

19. $n^2 - 3n - 18$
$(n - 6)(n + 3)$

20. $y^2 - 9y - 10$
$(y - 10)(y + 1)$

21. $x^2 - x - 20$
$(x - 5)(x + 4)$

22. $a^2 + 3a - 10$
$(a - 2)(a + 5)$

23. $y^2 + 14y - 15$
$(y - 1)(y + 15)$

24. $n^2 - 3n - 28$
$(n - 7)(n + 4)$

25. $b^2 + 2b - 24$
$(b + 6)(b - 4)$

26. $x^2 - x - 30$
$(x - 6)(x + 5)$

27. $b^2 - 7b - 30$
$(b - 10)(b + 3)$

28. $a^2 - 9a - 22$
$(a - 11)(a + 2)$

29. $x^2 + 12x - 28$
$(x + 14)(x - 2)$

30. $a^2 + 11a - 26$
$(a + 13)(a - 2)$

31. $y^2 + 4y - 32$
$(y + 8)(y - 4)$

32. $x^2 - 5x - 36$
$(x - 9)(x + 4)$

33. $y^2 + 6y - 27$
$(y + 9)(y - 3)$

34. $x^2 + 9x - 36$
$(x - 3)(x + 12)$

35. $m^2 + 12m - 64$
$(m - 4)(m + 16)$

36. $x^2 - x - 72$
$(x - 9)(x + 8)$

37. $n^2 - 2n - 63$
$(n - 9)(n + 7)$

38. $b^2 + 10b - 24$
$(b - 2)(b + 12)$

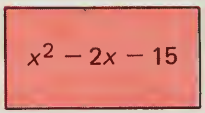
39. $y^2 + y - 42$
$(y - 6)(y + 7)$

Mixed Practice Exercises

Factor.

1. $x^2 + 10x + 9$ $(x + 9)(x + 1)$
2. $x^2 + 8x + 15$ $(x + 3)(x + 5)$
3. $n^2 - 8n + 12$ $(n - 6)(n - 2)$
4. $a^2 - 9a + 20$ $(a - 4)(a - 5)$
5. $y^2 + y - 12$ $(y - 3)(y + 4)$
6. $x^2 + 2x - 15$ $(x - 3)(x + 5)$
7. $y^2 - 4y - 21$ $(y - 7)(y + 3)$
8. $x^2 - 9x - 22$ $(x - 11)(x + 2)$
9. $x^2 + 10x + 21$ $(x + 3)(x + 7)$
10. $a^2 + 4a - 32$ $(a + 8)(a - 4)$
11. $b^2 + 4b - 5$ $(b - 1)(b + 5)$
12. $x^2 - 8x + 16$ $(x - 4)(x - 4)$
13. $b^2 - 8b + 7$ $(b - 7)(b - 1)$
14. $x^2 + 10x - 11$ $(x + 11)(x - 1)$
15. $y^2 + y - 20$ $(y - 4)(y + 5)$
16. $a^2 - 4a - 32$ $(a - 8)(a + 4)$
17. $x^2 + 12x + 35$ $(x + 7)(x + 5)$
18. $y^2 + 5y - 36$ $(y - 4)(y + 9)$
19. $n^2 - 15n + 56$ $(n - 7)(n - 8)$
20. $y^2 - y - 42$ $(y - 7)(y + 6)$
21. $x^2 + 15x + 54$ $(x + 9)(x + 6)$
22. $a^2 + 2a - 48$ $(a - 6)(a + 8)$
23. $n^2 + 14n + 45$ $(n + 9)(n + 5)$
24. $x^2 + x - 72$ $(x + 9)(x - 8)$
25. $n^2 - 5n - 50$ $(n - 10)(n + 5)$
26. $x^2 - 2x - 48$ $(x + 6)(x - 8)$
27. $y^2 + 24y - 52$ $(y + 26)(y - 2)$

28. A rectangular field has an area of $x^2 - 2x - 15$. Find expressions for its length and width. $(x - 5)(x + 3)$



29. What is the perimeter of the field in Exercise 28? $4x - 4$

30. A rectangular field has an area of $y^2 - 2y - 48$. Find expressions for its length and width. $(y + 6)(y - 8)$

31. What is the perimeter of the field in Exercise 30? $4y - 4$

For more practice, see page 420.

SELF-TEST

Factor.

1. $x^2 + 11x + 18$ $(x + 9)(x + 2)$
2. $y^2 + 14y + 24$ $(y + 12)(y + 2)$
3. $a^2 - 6a + 8$ $(a - 4)(a - 2)$
4. $m^2 + 11m - 26$ $(m + 13)(m - 2)$
5. $x^2 - 12x + 36$ $(x - 6)(x - 6)$
6. $x^2 + 8x - 48$ $(x + 12)(x - 4)$
7. $a^2 - 12a + 27$ $(a - 3)(a - 9)$
8. $y^2 + y - 42$ $(y - 6)(y + 7)$
9. $b^2 + 24b - 81$ $(b + 27)(b - 3)$
10. $m^2 - 18m + 56$ $(m - 14)(m - 4)$
11. $x^2 + 20x + 100$ $(x + 10)(x + 10)$
12. $m^2 + 16m + 64$ $(m + 8)(m + 8)$

8 • A Special Product

When you multiply two binomials, the product isn't always a trinomial. When your factors are the sum and difference of two monomials, look what happens to the product.

$$\begin{array}{r} a + 2 \\ a - 2 \\ \hline a^2 + 2a \\ \quad - 2a - 4 \\ \hline a^2 + 0 - 4 \end{array} \qquad \begin{array}{r} a + b \\ a - b \\ \hline a^2 + ab \\ \quad - ab - b^2 \\ \hline a^2 + 0 - b^2 \end{array}$$

$$a^2 - 4 \qquad a^2 - b^2$$

The middle term has disappeared.

Let's look at what the second example above shows us.

$$(a + b)(a - b) = a^2 - b^2$$

(sum of two numbers) **times** (difference of two numbers) = (square of first number) **minus** (square of second number)



To multiply the sum of two monomials by their difference,

1. square the first term,
2. then subtract the square of the second term.

EXAMPLE 1 $(x + 3)(x - 3) = x^2 - 9$

EXAMPLE 2 $(n + 4)(n - 4) = n^2 - 16$

EXAMPLE 3 $(m - n)(m + n) = m^2 - n^2$

EXAMPLE 4 $(2a + b)(2a - b) = (2a)^2 - b^2$
$= 4a^2 - b^2$

Classroom Practice

Complete.

1. $(x + 6)(x - 6) = (\underset{x}{?})^2 - (\underset{6}{?})^2$
2. $(a - 5)(a + 5) = (\underset{a}{?})^2 - (\underset{5}{?})^2$
3. $(n + 3)(n - 3) = (\underset{n}{?})^2 - (\underset{3}{?})^2$
4. $(x - 7)(x + 7) = (\underset{x}{?})^2 - (\underset{7}{?})^2$
5. $(y + 10)(y - 10) = (\underset{y}{?})^2 - (\underset{10}{?})^2$
6. $(b - 9)(b + 9) = (\underset{b}{?})^2 - (\underset{9}{?})^2$

Written Exercises

Multiply.

A
1. $(n - 7)(n + 7)$ $n^2 - 49$
2. $(x - 8)(x + 8)$ $x^2 - 64$
3. $(a + 10)(a - 10)$ $a^2 - 100$
4. $(y - 9)(y + 9)$ $y^2 - 81$
5. $(m + n)(m - n)$ $m^2 - n^2$
6. $(a + c)(a - c)$ $a^2 - c^2$
7. $(x + y)(x - y)$ $x^2 - y^2$
8. $(1 - 3b)(1 + 3b)$ $1 - 9b^2$
9. $(1 - 2x)(1 + 2x)$ $1 - 4x^2$
10. $(m + 4n)(m - 4n)$ $m^2 - 16n^2$
11. $(3x - 2)(3x + 2)$ $9x^2 - 4$
12. $(6x + 1)(6x - 1)$ $36x^2 - 1$
13. $(5y - 2)(5y + 2)$ $25y^2 - 4$
14. $(7x - 1)(7x + 1)$ $49x^2 - 1$
15. $(9m - n)(9m + n)$ $81m^2 - n^2$
16. $(4a - b)(4a + b)$ $16a^2 - b^2$
17. $(5x + 6)(5x - 6)$ $25x^2 - 36$
18. $(10x - 8)(10x + 8)$ $100x^2 - 64$

B
19. $(5x + 3y)(5x - 3y)$ $25x^2 - 9y^2$
20. $(2x - 6y)(2x + 6y)$ $4x^2 - 36y^2$
21. $(10x + 5y)(10x - 5y)$ $100x^2 - 25y^2$
22. $(x^2 - 1)(x^2 + 1)$ $x^4 - 1$
23. $(a^2 - 2)(a^2 + 2)$ $a^4 - 4$
24. $(b^3 - 5)(b^3 + 5)$ $b^6 - 25$

Find the product by using the rule $(a + b)(a - b) = a^2 - b^2$.

Sample Find (23)(17).

$$\begin{aligned}(23)(17) &= (20 + 3)(20 - 3)\\ &= 400 - 9\\ &= 391\end{aligned}$$

C
25. (21)(19) 399
26. (51)(49) 2499
27. (52)(48) 2496
28. (18)(22) 396
29. (32)(28) 896
30. (38)(42) 1596
31. (93)(87) 8091
32. (83)(77) 6391
33. (24)(36) 864
34. (94)(86) 8084
35. (55)(65) 3575
36. (95)(85) 8075
37. (84)(76) 6384
38. (59)(61) 3599
39. (48)(52) 2496
40. (73)(67) 4891

9 • Factoring the Difference of Squares

In the last section you saw that the following is true.

$$(a + b)(a - b) = a^2 - b^2$$

If this equation is turned around, we have the following.

$$a^2 - b^2 = (a + b)(a - b)$$

The last equation shows the pattern to use when you want to factor the difference of two squares.

EXAMPLE 1 Factor $c^2 - 25$.

Step 1: $c^2 - 25 = (c \quad)(c \quad)$
Step 2: $c^2 - 25 = (c + 5)(c - 5)$

Check: $(c + 5)(c - 5) = c^2 - 25$ ✓

EXAMPLE 2 Factor $a^2 - y^2$.

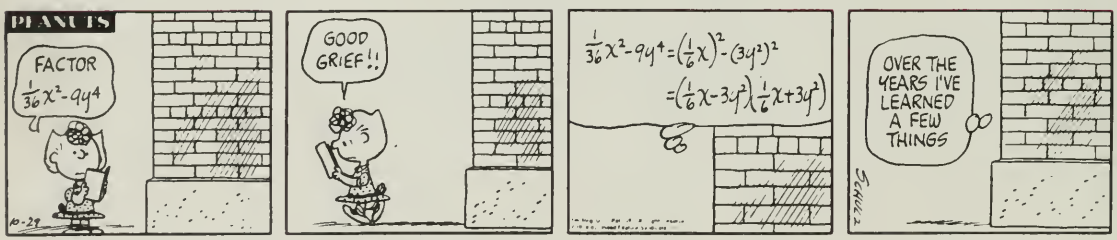
Step 1: $a^2 - y^2 = (a \quad)(a \quad)$
Step 2: $a^2 - y^2 = (a + y)(a - y)$

Check: $(a + y)(a - y) = a^2 - y^2$ ✓

EXAMPLE 3 Factor $x^2 - 4y^2$.

Step 1: $x^2 - 4y^2 = (x \quad)(x \quad)$
Step 2: $x^2 - 4y^2 = (x + 2y)(x - 2y)$

Check: $(x + 2y)(x - 2y) = x^2 - 4y^2$ ✓



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Written Exercises

Which of the expressions are the difference of two squares?

A **1.** $n^2 - 9$ yes **2.** $x^2 - 16$ yes **3.** $3x^2 - y^2$ no **4.** $2n^2 - 9$ no

5. $25m^2 - n^2$ yes **6.** $1 - 10x^2$ no **7.** $a^2 - 3b^2$ no **8.** $36m^2 - 1$ yes

Factor.

9. $x^2 - 16$
$(x - 4)(x + 4)$

10. $x^2 - 64$
$(x - 8)(x + 8)$

11. $n^2 - 9$
$(n - 3)(n + 3)$

12. $y^2 - 81$
$(y - 9)(y + 9)$

13. $a^2 - 9b^2$
$(a - 3b)(a + 3b)$

14. $b^2 - 4a^2$
$(b - 2a)(b + 2a)$

15. $x^2 - 4y^2$
$(x - 2y)(x + 2y)$

16. $r^2 - 16t^2$
$(r - 4t)(r + 4t)$

17. $b^2 - 64c^2$
$(b - 8c)(b + 8c)$

18. $9n^2 - 49$
$(3n - 7)(3n + 7)$

19. $4a^2 - 25b^2$
$(2a - 5b)(2a + 5b)$

20. $49m^2 - n^2$
$(7m - n)(7m + n)$

21. $x^2 - 121$
$(x - 11)(x + 11)$

22. $y^2 - 144$
$(y - 12)(y + 12)$

23. $64x^2 - 9y^2$
$(8x - 3y)(8x + 3y)$

24. $81a^2 - 49b^2$
$(9a - 7b)(9a + 7b)$

Factor. You may need to use the commutative property to rearrange the terms first.

Samples $-x^2 + 1 = 1 - x^2$
$= (1 + x)(1 - x)$

$-1 + 4x^4 = 4x^4 - 1$
$= (2x^2 + 1)(2x^2 - 1)$

B **25.** $-x^2 + 4$
$(2 + x)(2 - x)$

26. $-1 + 4y^2$
$(2y - 1)(2y + 1)$

27. $-x^2 + 64$
$(8 - x)(8 + x)$

28. $-4x^2 + 9$
$(3 - 2x)(3 + 2x)$

29. $-y^2 + x^2$
$(x - y)(x + y)$

30. $4y^4 - 1$
$(2y^2 - 1)(2y^2 + 1)$

31. $4x^6 - 9$
$(2x^3 - 3)(2x^3 + 3)$

32. $-1 + 49y^4$
$(7y^2 - 1)(7y^2 + 1)$

33. $144 - 121x^2$
$(12 - 11x)(12 + 11x)$

34. $225a^2 - 1$
$(15a - 1)(15a + 1)$

35. $-4a^2 + 36b^2c^2$
$(6bc - 2a)(6bc + 2a)$

36. $-a^2 + 100b^2$
$(10b - a)(10b + a)$

You may be able to factor more than once in the following exercises.

Sample $x^4 - 16 = (x^2 + 4)(x^2 - 4)$
$= (x^2 + 4)(x + 2)(x - 2)$

38. $(4y^2 + 9z^2)(2y + 3z)(2y - 3z)$
39. $(4a^2 + 1)(2a + 1)(2a - 1)$
42. $(a^4 + b^4)(a^2 + b^2)(a + b)(a - b)$
43. $(x^4y^2 + 1)(x^2y + 1)(x^2y - 1)$

C **37.** $y^4 - 81$
$(y^2 + 9)(y + 3)(y - 3)$

38. $16y^4 - 81z^4$

39. $16a^4 - 1$

40. $-1 + 81a^4$
$(9a^2 + 1)(3a + 1)(3a - 1)$

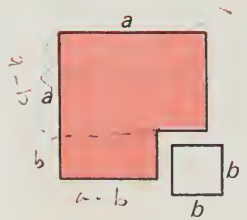
41. $256 - x^4y^4$
$(16 + x^2y^2)(4 + xy)(4 - xy)$

42. $a^8 - b^8$

43. $x^8y^4 - 1$

44. $81a^8b^4 - c^8$
$(9a^4b^2 + c^4)(3a^2b + c^2)(3a^2b - c^2)$

45. Make a square a units on each side. Cut out a small corner square, b units on a side. Can you cut the shaded region into two pieces and rearrange the pieces to show the area is $(a + b)(a - b)$? See page 8 of Answers to Odd-Numbered Exercises in the back of the text.



46. Suppose x and y are both even or both odd numbers. Show that $x^2 - y^2$ has 4 as a factor.
even: let $x = 2a$ and $y = 2b$; $x^2 - y^2 = (2a)^2 - (2b)^2 = 4a^2 - 4b^2 = 4(a^2 - b^2)$
odd: let $x = 2a + 1$ and $y = 2b + 1$; $x^2 - y^2 = (2a + 1 + 2b + 1)(2a + 1 - 2b - 1)$
$= 4(a + b + 1)(a - b)$

For more practice, see page 420.

10 • Many Types of Factoring

Sometimes you'll see an expression which can be factored several times. In this section you will have to look very closely at the terms of an expression to see what is the easiest way to factor completely.

EXAMPLE 1 Factor $3x^2 + 15x + 18$.

$$3x^2 + 15x + 18$$
$$= 3(x^2 + 5x + 6)$$
$$= 3(x + 2)(x + 3)$$

Each term has a common monomial factor, 3.

Next the $x^2 + 5x + 6$ is factored to get $(x + 2)(x + 3)$.

EXAMPLE 2 Factor $x^3 - 9x$.

$$x^3 - 9x$$
$$= x(x^2 - 9)$$
$$= x(x + 3)(x - 3)$$

Each term has a common monomial factor, x.

Next the $x^2 - 9$ is factored to get $(x + 3)(x - 3)$.

EXAMPLE 3 Factor $-x^2 + 6x - 8$.

$$-x^2 + 6x - 8$$
$$= -1(x^2 - 6x + 8)$$
$$= -1(x - 4)(x - 2)$$

Each term has a common monomial factor, -1.

Next the $x^2 - 6x + 8$ is factored to get $(x - 4)(x - 2)$.

Follow these steps in factoring an expression.

Guide for Factoring An Expression

1. Do the terms have a common monomial factor? Factor.
2. Now is one of the factors the difference between two squares? If so, factor using the rule $a^2 - b^2 = (a + b)(a - b)$.
3. After Step 1, is one of the factors a trinomial? Factor.

Classroom Practice

Factor.

1. $2a^2 + 14a + 20$
 $2(a + 5)(a + 2)$
2. $2x^2 - 14x + 24$
 $2(x - 3)(x - 4)$
3. $3y^2 + 3y - 18$
 $3(y - 2)(y + 3)$
4. $4x^2 + 4x - 80$
 $4(x + 5)(x - 4)$
5. $3x^2 - 27$
 $3(x - 3)(x + 3)$
6. $9a^2 - 4$
 $(3a - 2)(3a + 2)$
7. $x^3 - 16x$
 $x(x - 4)(x + 4)$
8. $y^3 - x^2y$
 $y(y - x)(y + x)$
9. $2r^2s - 8st^2$
 $2s(r - 2t)(r + 2t)$

Written Exercises

Factor.

A
1. $3y^2 + 18y + 24$
 $3(y + 4)(y + 2)$
2. $4x^2 + 24x - 64$
 $4(x - 2)(x + 8)$
3. $2y^2 - 16y + 30$
 $2(y - 5)(y - 3)$
4. $2b^2 - 4b - 48$
 $2(b - 6)(b + 4)$
5. $5a^2 - 20a - 60$
 $5(a - 6)(a + 2)$
6. $3x^2 + 30x + 27$
 $3(x + 9)(x + 1)$
7. $4x^2 + 8x - 60$
 $4(x - 3)(x + 5)$
8. $3y^2 - 12y - 63$
 $3(y - 7)(y + 3)$
9. $2x^2 - 16x + 32$
 $2(x - 4)^2$
10. $5a^2 - 5b^2$
 $5(a - b)(a + b)$
11. $8x^2 - 32$
 $8(x - 2)(x + 2)$
12. $x^2y - y^3$
 $y(x - y)(x + y)$
13. $4x^2 - 36$
 $4(x - 3)(x + 3)$
14. $3a^2 - 27b^2$
 $3(a - 3b)(a + 3b)$
15. $6x^2 - 24y^2$
 $6(x - 2y)(x + 2y)$

B
16. $-x^2 - 2xy - y^2$
 $-1(x + y)^2$
17. $-x^2 + 4x - 3$
 $-1(x - 3)(x - 1)$
18. $2a^3 - 8a$
 $2a(a - 2)(a + 2)$
19. $-162 + 2x^2$
 $2(x - 9)(x + 9)$
20. $-48 + 3x^2$
 $3(x - 4)(x + 4)$
21. $4a - 4a^2 + 8$
 $-4(a - 2)(a + 1)$
22. $-3x^2 + 30x - 75$
 $-3(x - 5)^2$
23. $-200b^2 + 2a^2$
 $2(a - 10b)(a + 10b)$
24. $-2x^2 + 24x - 72$
 $-2(x - 6)^2$

For more practice, see page 421.

SELF-TEST

Multiply.

1. $(m - 13)(m + 13)$
 $m^2 - 169$
2. $(3 + 4x)(3 - 4x)$
 $9 - 16x^2$
3. $(5a - 7)(5a + 7)$
 $25a^2 - 49$

Factor.

4. $a^2 - 64$
 $(a - 8)(a + 8)$
5. $m^2 - 36n^2$
 $(m - 6n)(m + 6n)$
6. $16x^2 - 9y^2$
 $(4x - 3y)(4x + 3y)$
7. $3x^2 + 15x + 18$
 $3(x + 3)(x + 2)$
8. $2y^2 - 4y - 30$
 $2(y - 5)(y + 3)$
9. $2m^2 - 8n^2$
 $2(m - 2n)(m + 2n)$

11 • More Difficult Factoring (Optional)

Trinomials like $2x^2 + 7x + 3$ can often be factored, even though factoring is more difficult. When the coefficient of the first term is not 1, you have to work by trial and error.

EXAMPLE 1 Factor $2x^2 + 7x + 3$.

Step 1: $2x^2 + 7x + 3 = (\quad)(\quad)$

Step 2: $2x^2 + 7x + 3 = (2x \quad)(x \quad)$ ← Factor the first term. $2x$ and x

Step 3: $2x^2 + 7x + 3 = (2x \ ? \)(x \ ? \)$ What two numbers have a product of 3? 3 and 1 $\quad -3$ and -1

Choose the factors of 3 so that the middle term of the trinomial product will be $+7x$.

$(2x + 3)(x + 1)$ $\quad$ **$(2x + 1)(x + 3)$** ✓

$(2x - 3)(x - 1)$ $\quad (2x - 1)(x - 3)$

Step 4: $2x^2 + 7x + 3 = (2x + 1)(x + 3)$

Check: $(2x + 1)(x + 3) = 2x^2 + 7x + 3$ ✓

EXAMPLE 2 Factor $3a^2 - 5a - 2$.

Step 1: $3a^2 - 5a - 2 = (\quad)(\quad)$

Step 2: $3a^2 - 5a - 2 = (3a \quad)(a \quad)$

Step 3: $3a^2 - 5a - 2 = (3a \ ? \)(a \ ? \)$ What two numbers have a product of -2? -1 and 2 $\quad 1$ and -2

Choose the factors of -2 so that the middle term of the trinomial product will be $-5a$.

$(3a - 1)(a + 2)$ $\quad (3a + 2)(a - 1)$

$(3a + 1)(a - 2)$ ✓ $\quad (3a - 2)(a + 1)$

Step 4: $3a^2 - 5a - 2 = (3a + 1)(a - 2)$

Check: $(3a + 1)(a - 2) = 3a^2 - 5a - 2$ ✓

Written Exercises

Factor.

1. $3x^2 + 4x + 1$ $(3x + 1)(x + 1)$
2. $2x^2 + 5x + 2$ $(2x + 1)(x + 2)$
3. $2a^2 + 9a + 9$ $(2a + 3)(a + 3)$
4. $3y^2 + 7y + 2$ $(3y + 1)(y + 2)$
5. $2b^2 + 5b + 3$ $(2b + 3)(b + 1)$
6. $2x^2 + 7x + 3$ $(2x + 1)(x + 3)$
7. $3a^2 - 8a + 5$ $(3a - 5)(a - 1)$
8. $5a^2 + 4a - 1$ $(5a - 1)(a + 1)$
9. $3x^2 - 10x + 8$ $(3x - 4)(x - 2)$
10. $5x^2 - 7x + 2$ $(5x - 2)(x - 1)$
11. $3y^2 - 5y - 2$ $(3y + 1)(y - 2)$
12. $3x^2 + x - 2$ $(3x - 2)(x + 1)$
13. $2x^2 + 5x - 3$ $(2x - 1)(x + 3)$
14. $7x^2 + 13x - 2$ $(7x - 1)(x + 2)$
15. $2x^2 - 3x - 9$ $(2x + 3)(x - 3)$
16. $4x^2 - 7x - 2$ $(4x + 1)(x - 2)$
17. $4a^2 + 5a - 9$ $(4a + 9)(a - 1)$
18. $6x^2 - 19x + 3$ $(6x - 1)(x - 3)$
19. $6b^2 - 5b - 25$ $(3b + 5)(2b - 5)$
20. $8x^2 - 14x - 4$ $2(4x + 1)(x - 2)$
21. $4a^2 - a - 5$ $(4a - 5)(a + 1)$
22. $6a^2 - 7a + 2$ $(3a - 2)(2a - 1)$
23. $4x^2 - 8x - 5$ $(2x - 5)(2x + 1)$
24. $6y^2 + y - 15$ $(3y + 5)(2y - 3)$
25. $8r^2 - 6r - 9$ $(4r + 3)(2r - 3)$
26. $6r^2 - 23r + 7$ $(3r - 1)(2r - 7)$
27. $8x^2 + 2x - 3$ $(4x + 3)(2x - 1)$
28. $8c^2 - 10c + 3$ $(4c - 3)(2c - 1)$
29. $4c^2 + 6c + 2$ $2(2c + 1)(c + 1)$
30. $6x^2 + 21x - 90$ $3(2x - 5)(x + 6)$
31. $9y^2 + 15y + 6$ $3(3y + 2)(y + 1)$
32. $6x^2 + 11x - 10$ $(3x - 2)(2x + 5)$
33. $6x^2 + 15x + 6$ $3(2x + 1)(x + 2)$
34. $10a^2 + 14a + 4$ $2(5a + 2)(a + 1)$
35. $10y^2 - 29y + 10$ $(5y - 2)(2y - 5)$
36. $12n^2 - 11n + 2$ $(4n - 1)(3n - 2)$
37. $6x^2 + 11x - 121$ $(3x - 11)(2x + 11)$
38. $10x^2 - 31x - 14$ $(5x + 2)(2x - 7)$
39. $12x^2 + 7x - 12$ $(3x + 4)(4x - 3)$
40. $6x^2 + 19x + 15$ $(3x + 5)(2x + 3)$
41. $15n^2 + 17n - 4$ $(5n - 1)(3n + 4)$
42. $20y^2 - 11y - 4$ $(4y + 1)(5y - 4)$

Puzzles & Things

Some statements that seem logical have built-in contradictions. Such statements are called paradoxes. Here's a famous example.

None of the men in a certain town wear beards. The town's only barber shaves only those men who don't shave themselves. Since the barber is a man, who shaves him?



CONSUMER CORNER



Small pocket calculators, like the one at the left, can save a great deal of time. You just enter your problem into the machine by pressing the buttons. The answer appears instantly.

EXAMPLE 1 $261 + 498 = _?_$

You'll want to be sure to always clear the calculator before you begin.

Enter	Display
C	0.
261	261.
+	261.
498	498.
=	759.

Check the display when you enter something new. Be sure you pressed the correct buttons.

EXAMPLE 2 $\$84.45 \div 3 + \$2.83 = _?_$

Be sure to enter the decimal points.

On most calculators, you don't have to press = when you have more than one operation.

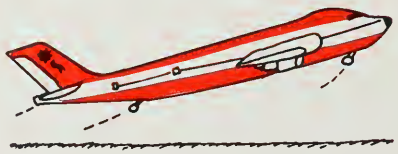
Enter	Display
C	0.
84.45	84.45
÷	84.45
3	3.
+	28.15
2.83	2.83
=	30.98

Here's the answer to the division.

Your calculator can help you find interesting information you probably never would figure if you had to do all the arithmetic in your head.

1. Calculate the total amount this 747 must lift off the ground.

number of passengers: 285
crew members: 11
ave. person: 68 kg
ave. baggage per person: 18 kg
cargo: 2808 kg
aircraft itself: 158,223 kg **total: 186,487 kg**



2. If you are between 14 and 18 years old, your daily calorie intake should be close to the figure given below.

women: 2400 men: 3000

Here is a list of what two people ate in one day. The number of calories per serving are given also. What's each person's total?

Cheryl **2374 calories**	Pete **2181 calories**
1 donut (125)	1 cereal with milk (130)
3 milks (150 ea.)	1 orange juice (90)
1 hamburger with roll (369)	2 pieces pizza (185 ea.)
ketchup (20)	1 fruit drink (126)
potato chips (158)	1 candy bar (130)
2 cola (150 ea.)	2 servings meatloaf (280 ea.)
1 ice cream stick (162)	1 tossed salad (90)
1 serving beef (275)	2 baked potatoes (90 ea.)
1 serving noodles (200)	2 lo-fat milks (80 ea.)
1 serving string beans (23)	3 pieces fudge (115 ea.)
2 pieces cake (146 ea.)	

Reviewing the Chapter

Find the greatest common factor of each pair. *(See pp. 170–171.)*

1. $6, 9$ 3
2. $16, 4$ 4
3. $15, 3$ 3
4. $24, 32$ 8
5. $14, 28$ 14
6. $56, 72$ 8
7. $18, 24$ 6
8. $30, 27$ 3
9. $3y, 9y$ $3y$
10. $16n, 8n^3$ $8n$
11. $4x^2, x^3$ x^2
12. $4y, 12y^3$ $4y$
13. $6a, 18b$ 6
14. $24x, 6x^2$ $6x$
15. $20x, 4y$ 4
16. $5x^3, 30x^2$ $5x^2$

Factor. Look for a common monomial factor. *(See pp. 172–173.)*

17. $2n^2 + 4$ $2(n^2 + 2)$
18. $3x^2 - 9$ $3(x^2 - 3)$
19. $ab^2 + a^2b$ $ab(b + a)$
20. $7x^2 - 21x$ $7x(x - 3)$
21. $2y^3 + 8y^2$ $2y^2(y + 4)$
22. $3xy - 27$ $3(xy - 9)$
23. $2x^2 - 8x^3$ $2x^2(1 - 4x)$
24. $18n^2 + 9$ $9(2n^2 + 1)$
25. $36x + 6$ $6(6x + 1)$
26. $5w - 25w^2$ $5w(1 - 5w)$
27. $x^2 + 4x$ $x(x + 4)$
28. $a - 4a^2$ $a(1 - 4a)$

Factor. *(See pp. 176–181.)*

29. $n^2 + 6n + 9$ $(n + 3)^2$
30. $x^2 + 8x + 7$ $(x + 7)(x + 1)$
31. $x^2 - 10x + 25$ $(x - 5)^2$
32. $x^2 - 9x + 14$ $(x - 7)(x - 2)$
33. $n^2 + 9n + 20$ $(n + 4)(n + 5)$
34. $n^2 + 12n + 20$ $(n + 10)(n + 2)$
35. $y^2 - 9y + 8$ $(y - 8)(y - 1)$
36. $y^2 - 11y + 30$ $(y - 6)(y - 5)$
37. $n^2 - 13n + 22$ $(n - 11)(n - 2)$
38. $y^2 + 17y + 30$ $(y + 15)(y + 2)$
39. $n^2 + 12n + 35$ $(n + 7)(n + 5)$
40. $y^2 + 12y + 32$ $(y + 4)(y + 8)$
41. $a^2 - 18a + 81$ $(a - 9)^2$
42. $x^2 + 27x + 50$ $(x + 25)(x + 2)$
43. $m^2 + 8m + 16$ $(m + 4)^2$
44. $x^2 - 10x + 21$ $(x - 3)(x - 7)$
45. $n^2 + 28n + 75$ $(n + 25)(n + 3)$
46. $a^2 - 14a + 49$ $(a - 7)^2$
47. $y^2 - 13y + 40$ $(y - 5)(y - 8)$
48. $n^2 - 18n + 32$ $(n - 16)(n - 2)$
49. $y^2 + 9y + 18$ $(y + 6)(y + 3)$
50. $y^2 + 11y + 18$ $(y + 9)(y + 2)$
51. $x^2 + 11x + 30$ $(x + 6)(x + 5)$
52. $h^2 - 24h + 23$ $(h - 23)(h - 1)$
53. $n^2 + 20n + 64$ $(n + 4)(n + 16)$
54. $b^2 - 25b + 100$ $(b - 5)(b - 20)$
55. $x^2 + 12x + 27$ $(x + 3)(x + 9)$
56. $m^2 - 11m + 28$ $(m - 7)(m - 4)$
57. $x^2 - 24x + 80$ $(x - 4)(x - 20)$
58. $b^2 - 15b + 56$ $(b - 7)(b - 8)$
59. $x^2 + 18x + 32$ $(x + 16)(x + 2)$
60. $m^2 + 17m + 72$ $(m + 9)(m + 8)$
61. $y^2 - 16y + 63$ $(y - 9)(y - 7)$

Factor. Watch the signs. *(See pp. 182–187.)*

62. $x^2 + 3x - 18$
$(x + 6)(x - 3)$

63. $n^2 - 7n - 18$
$(n - 9)(n + 2)$

64. $y^2 - 2y - 24$
$(y - 6)(y + 4)$

65. $m^2 + 6m - 16$
$(m + 8)(m - 2)$

66. $x^2 - 5x - 24$
$(x + 3)(x - 8)$

67. $y^2 - 10y - 24$
$(y - 12)(y + 2)$

68. $x^2 - x - 12$
$(x - 4)(x + 3)$

69. $y^2 - 12y + 36$
$(y - 6)^2$

70. $n^2 - n - 20$
$(n - 5)(n + 4)$

71. $a^2 + 3a - 10$
$(a + 5)(a - 2)$

72. $a^2 - 9a - 10$
$(a - 10)(a + 1)$

73. $b^2 + 5b - 6$
$(b + 6)(b - 1)$

Multiply. *(See pp. 188–189.)*

74. $(x - 2)(x + 2)$
$x^2 - 4$

75. $(y + 4)(y - 4)$
$y^2 - 16$

76. $(n + 5)(n - 5)$
$n^2 - 25$

77. $(1 + 2a)(1 - 2a)$
$1 - 4a^2$

78. $(2y + 3)(2y - 3)$
$4y^2 - 9$

79. $(4n + 1)(4n - 1)$
$16n^2 - 1$

80. $(x + y)(x - y)$
$x^2 - y^2$

81. $(3x - y)(3x + y)$
$9x^2 - y^2$

82. $(2x + 5)(2x - 5)$
$4x^2 - 25$

83. $(xy + 1)(xy - 1)$
$x^2y^2 - 1$

84. $(a + 2b)(a - 2b)$
$a^2 - 4b^2$

85. $(4x - 3y)(4x + 3y)$
$16x^2 - 9y^2$

Factor. *(See pp. 190–191.)*

86. $a^2 - 1$ $(a - 1)(a + 1)$

87. $a^2 - b^2$ $(a - b)(a + b)$

88. $x^2 - 4y^2$
$(x - 2y)(x + 2y)$

89. $25 - n^2$ $(5 - n)(5 + n)$

90. $n^2 - 25$ $(n - 5)(n + 5)$

91. $4x^2 - y^2$
$(2x - y)(2x + y)$

92. $9n^2 - 1$ $(3n - 1)(3n + 1)$

93. $16a^2 - b^2$ $(4a - b)(4a + b)$

94. $9a^2 - b^2$
$(3a - b)(3a + b)$

95. $49a^2 - b^2$ $(7a - b)(7a + b)$

96. $x^2y^2 - 1$ $(xy - 1)(xy + 1)$

97. $1 - a^2b^2$
$(1 - ab)(1 + ab)$

Factor. Look for a common monomial factor first. *(See pp. 192–193.)*

98. $n^3 - n$
$n(n - 1)(n + 1)$

99. $3a^2 + 6a + 3$
$3(a + 1)^2$

100. $2n^2 + 6n - 20$
$2(n + 5)(n - 2)$

101. $4x^2 - 16$
$4(x - 2)(x + 2)$

102. $2y^2 - 18$
$2(y + 3)(y - 3)$

103. $3a^2 + 15a + 18$
$3(a + 3)(a + 2)$

104. $2y^2 - 4y - 48$
$2(y - 6)(y + 4)$

105. $3x^2 - 3x - 36$
$3(x - 4)(x + 3)$

106. $x^3 - xy^2$
$x(x - y)(x + y)$

107. $2n^2 + 8n - 42$
$2(n + 7)(n - 3)$

108. $4a^2 - 20a - 24$
$4(a - 6)(a + 1)$

109. $ab^2 - 4ac^2$
$a(b - 2c)(b + 2c)$

110. $3y^2 - 9y - 30$
$3(y - 5)(y + 2)$

111. $2a^2 + 12a + 16$
$2(a + 4)(a + 2)$

112. $3n^2 - 3n - 60$
$3(n - 5)(n + 4)$

A permission-to-reproduce chapter test can be found on page T16.

Cumulative Review

Solve.

1. $x + 5 = 19$ 14
2. $b - 11 = 21$ 32
3. $13 - m = 4$ 9
4. $6n = 54$ 9
5. $7x = -35$ -5
6. $2m + 3 = 17$ 7
7. $5x - 9 = -14$ -1
8. $3(y + 2) = 15$ 3
9. $5(x - 4) = -35$ -3
10. $4m - 9m = 75$ -15
11. $5x + 24 = 9x$ 6
12. $3y - 20 = 7y$ -5
13. $5(a + 3) = 2a$ -5
14. $3m - 2 = 6m - 11$ 3
15. $-a + 8 = 4a + 3$ 1

Simplify.

16. $m(-2m)(3n)$ $-6m^2n$
17. $5(a^2 + 3) - 3a^2$ $2a^2 + 15$
18. $-a^2(-a)(3b)$ $3a^3b$
19. $(3x - 4) + (x + 5)$ $4x + 1$
20. $(4a + 7) - (2a - 3)$ $2a + 10$
21. $(a^2 - 3b^2) + 2a^2$ $3a^2 - 3b^2$
22. $4x - 3y - (x - y)$ $3x - 2y$
23. $(5x^2 + 6) + (x - 10)$ $5x^2 + x - 4$
24. $(a^2 + 4a) - (5a + 7)$ $a^2 - a - 7$
25. $\begin{array}{r} (m^2 - 3n^2) \\ -(3m^2 + n^2) \\ \hline \end{array}$ $4m^2 - 2n^2$
26. $\begin{array}{r} (4x - 5y) \\ -(3x - 2y) \\ \hline \end{array}$ $x - 3y$
27. $\begin{array}{r} (m^2 + 3m - 7) \\ -(3m^2 \quad - 4) \\ \hline \end{array}$ $-2m^2 + 3m - 3$
28. $\dfrac{25m^2n^3}{5m^2}$ $5n^3$
29. $\dfrac{2a^2 - 6ab}{2a}$ $a - 3b$
30. $\dfrac{3xy^2 - 18x^2y^2}{3xy}$ $y - 6xy$

Multiply.

31. $(ab^2)^2$ a^2b^4
32. $y(-3y + 4)$ $-3y^2 + 4y$
33. $x(2x^2 + 3x - 4)$ $2x^3 + 3x^2 - 4x$
34. $(m + 3)(m + 4)$ $m^2 + 7m + 12$
35. $(n - 6)(n - 8)$ $n^2 - 14n + 48$
36. $(2x + 3)(x + 4)$ $2x^2 + 11x + 12$
37. $(3b + 7)(b - 2)$ $3b^2 + b - 14$
38. $(4m + 7)(4m - 7)$ $16m^2 - 49$
39. $(3y + 1)(2y - 3)$ $6y^2 - 7y - 3$
40. $(4x - 3)(3x + 4)$ $12x^2 + 7x - 12$
41. $(5a - b)(5a + b)$ $25a^2 - b^2$
42. $(2m - 3n)(4m + n)$ $8m^2 - 10mn - 3n^2$

Factor.

43. $a^2 + 6ab^2$ $a(a + 6b^2)$
44. $5x^2 - 10x$ $5x(x - 2)$
45. $3mn + 12m^2n$ $3mn(1 + 4m)$
46. $x^2 + 5x + 4$ $(x + 1)(x + 4)$
47. $a^2 - 3a + 2$ $(a - 2)(a - 1)$
48. $m^2 - 4m - 5$ $(m - 5)(m + 1)$
49. $n^2 - 7n + 12$ $(n - 3)(n - 4)$
50. $25m^2 - 4$ $(5m - 2)(5m + 2)$
51. $x^2 + 6x - 16$ $(x + 8)(x - 2)$
52. $x^2 + 6x + 9$ $(x + 3)^2$
53. $4x^2 - 4x - 24$ $4(x - 3)(x + 2)$
54. $3x^2 - 12y^2$ $3(x - 2y)(x + 2y)$

 A more extensive cumulative review for Chapters 1–6 can be found on page 440.

UNIT D

Here's what you'll learn in this chapter:

1. To read and draw a bar graph.
2. To read and draw a broken line graph.
3. To read and graph ordered pairs of numbers on a coordinate plane.
4. To draw the graph of an equation when the graph is a line.
5. To find the slope of a line.



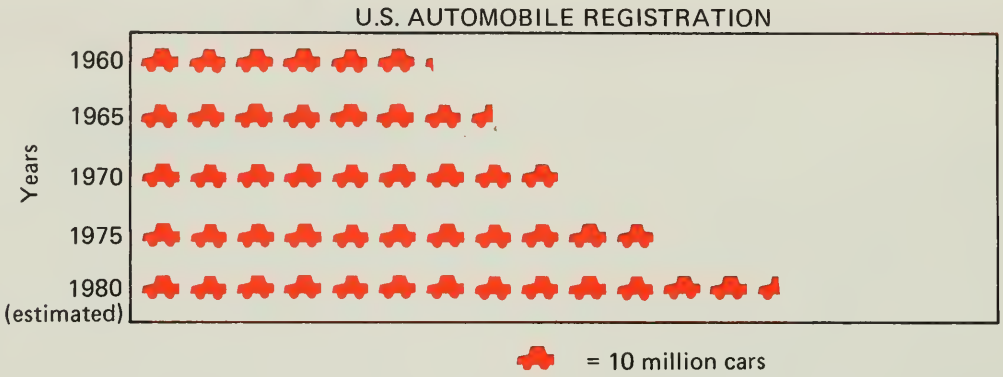
Chapter 7

Graphs



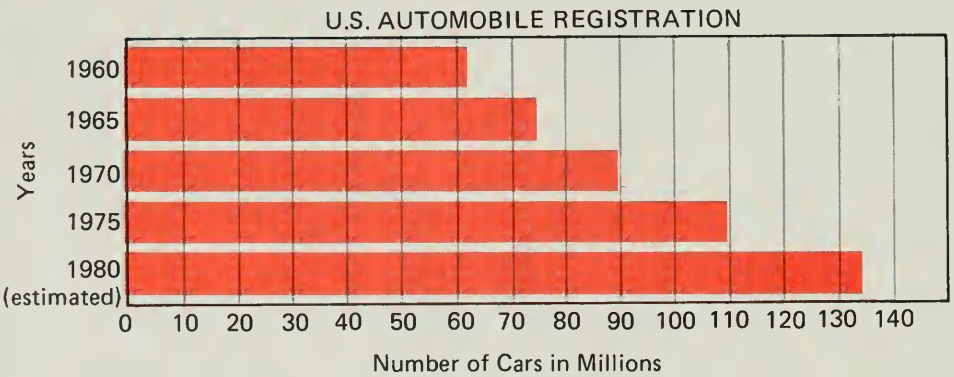
1 • Graphs You Often See

Many of the graphs in this chapter are like those you find in newspapers and magazines. Here is a **pictograph** showing the number of cars registered in the United States.



The key below the graph tells you that each symbol represents 10 million cars. You can see that in 1960 there were a little more than 60 million cars registered.

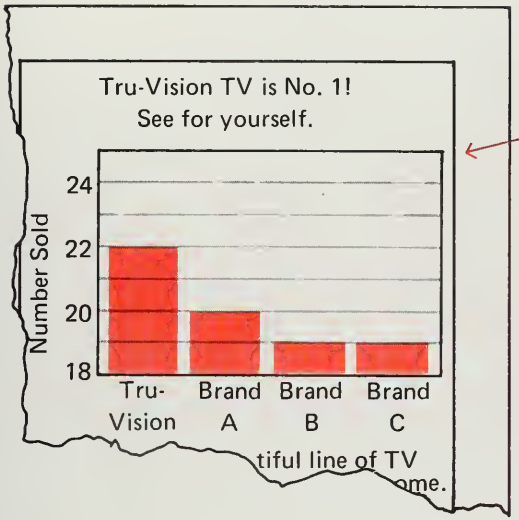
Here's a type of graph that is closely related to the pictograph. It is called a **bar graph.** Each row of car symbols in the graph above is replaced by a bar.



You can use the graphs above to find the increase in the number of cars between 1960 and 1975.

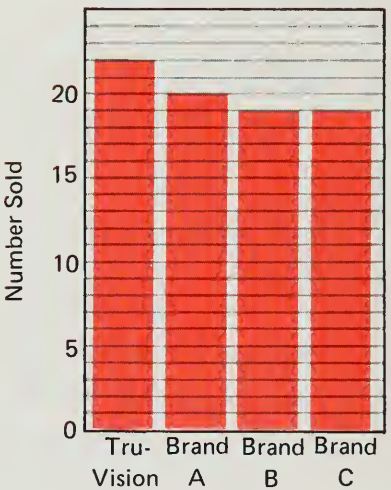
$$\begin{aligned} \text{Cars in 1975} &= \text{about } 110 \text{ million} \\ \text{Cars in 1960} &= \underline{\text{about } 62 \text{ million}} \\ \text{Increase in cars} &= \text{about } 48 \text{ million} \end{aligned}$$

The bar graph in the advertisement below is misleading. Do you see why?



At first glance it looks as if Tru-Vision TV is twice as popular as Brand A. This is because the scale on the side of the graph does not start with 0. Instead, it starts with 18.

Perhaps the graph at the right is more honest.



Classroom Practice

1. Study the pictograph on the opposite page. About how many cars were registered in the U.S. in 1965? How many in 1970?
 75 million, 90 million
2. Study the bar graph on the opposite page. About how many cars were registered in the U.S. in 1975?
 110 million
3. About what was the increase in registered cars between 1970 and 1975? 20 million
4. Study the Tru-Vision TV ad. How many of the TV sets sold were Tru-Vision? 22
5. How many of the TV sets sold were Brand A? 20

Written Exercises

The graph at the right shows the increase in sales for a newspaper in one year.

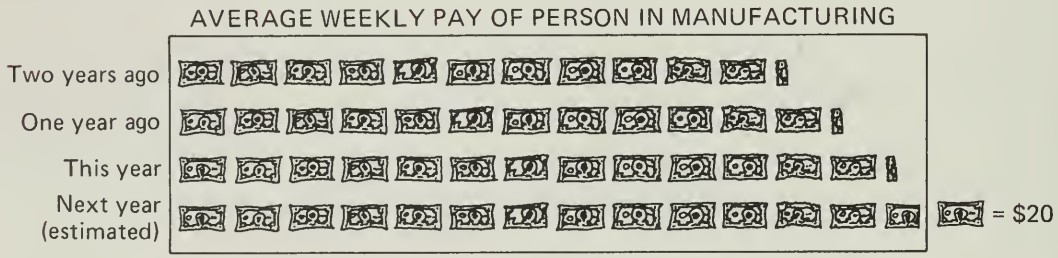


A **1.** During which months did the sales decrease?
July, Aug

2. The bar for December is twice as long as the bar for January. Does this mean that sales doubled during the year? No

3. What was the increase in sales from January to December? 2500

Study the pictograph below. Use it for Exercises 4 and 5.

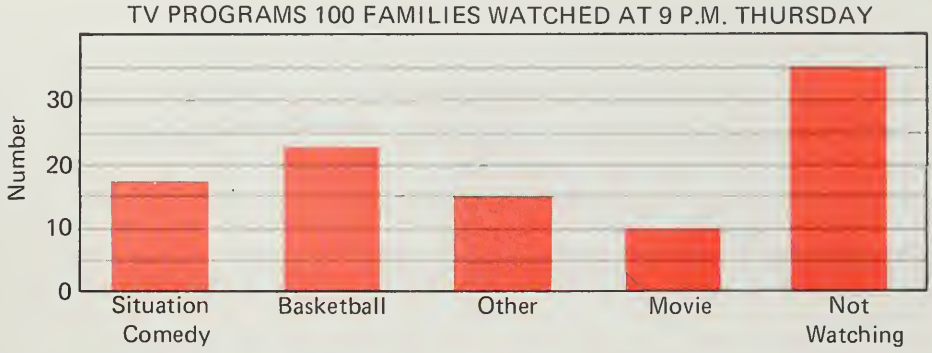


4. Complete the table.

Time	Average Weekly Pay
Two years ago	$225 ?
One year ago	$245 ?
This year	$265 ?
Next year (estimated)	$275 ?

5. Make a bar graph using the information in your table in Exercise 4.
Check students' graphs.

In a TV survey, 100 families were asked what TV show they watched at 9 P.M. last Thursday. The results are shown in the graph below.



6. How many families were not watching TV? 35

7. Which TV program had the most viewers? How many viewers? Basketball, 23

8. Which TV program had the fewest viewers? How many viewers? Movie, 10

9. Make a bar graph, like the one above, which shows which TV programs members of your class watched last Friday at 9 P.M. Check student's graphs.

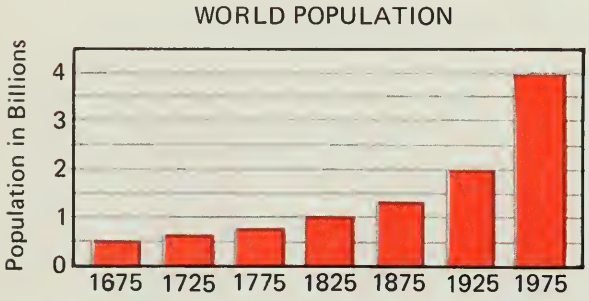
A newspaper advertisement shows the cost of several brands of hair dryers.



10. Zip Dry *looks* a lot less expensive. How much less expensive is the Zip Dry than the Easy Dry? $1.00

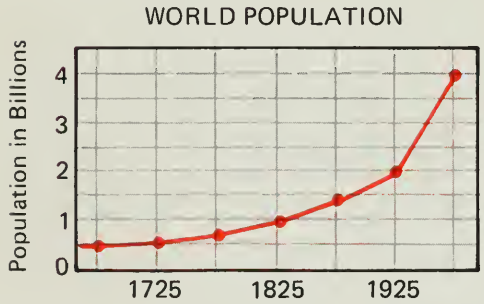
11. Redraw the graph. This time make the scale at the side of the graph start at $0 instead of $20. Your new graph will better show how the prices of the different brands of hair dryers compare. Check student's graphs. Zip 21, Style 24, Fashion 23, Easy 22

For more practice, see page 422.

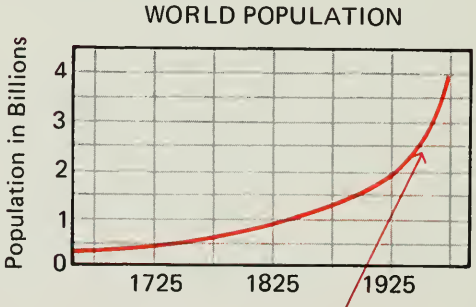


From 1675 to 1975 the world population increased by leaps and bounds! Read the figures on the bar graph at the left.

If you join the tops of the bars, you will get a **broken line graph** like this.



If you "smooth out" the broken line graph, you'll get a curve like this.

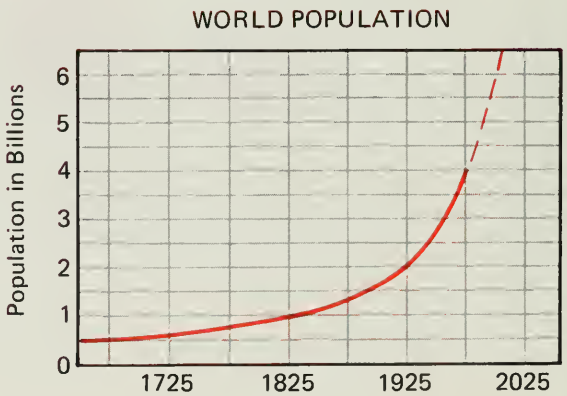


We can use the graph to estimate the population for years not labeled at the bottom of the graph. In 1950 the population was about $2\frac{1}{2}$ billion.

If the world population continues to grow at the *same* rate, we can extend the graph and answer questions about the next century.

Question: What will the world population be in the year 2000?

Answer: About 6 billion people.



Classroom Practice

Refer to the graphs on the opposite page.

1. The world population was $1\frac{1}{2}$ billion in the year ___?___. 1885
2. The world population in 1975 was about ___?___ billion. 4
3. The world population in 1990 may be about ___?___ billion. 6

Written Exercises

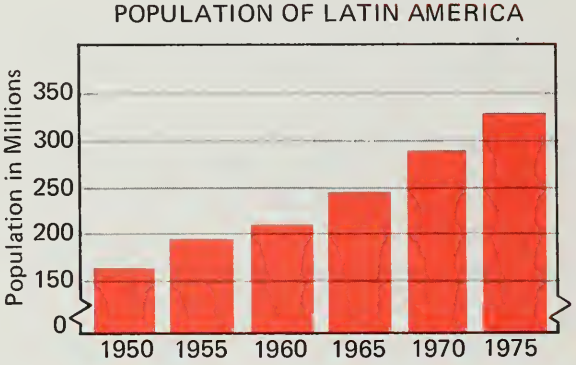
Refer to the graphs on the opposite page.

A 1. The world population in 1875 was about ___?___ billion. $1\frac{1}{2}$

2. The estimated world population in 2025 is about ___?___ billion. 8
3. The world population may reach $4\frac{1}{2}$ billion in the year ___?___. 1985

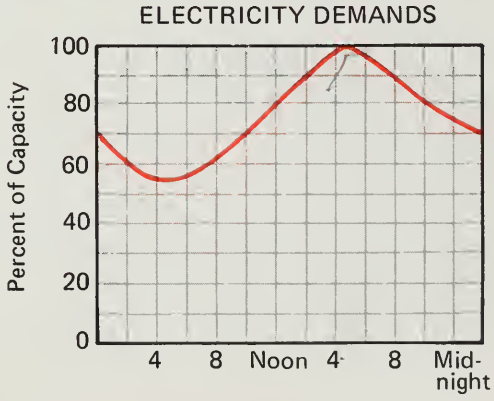
The graph at the right shows some population figures for Latin America.

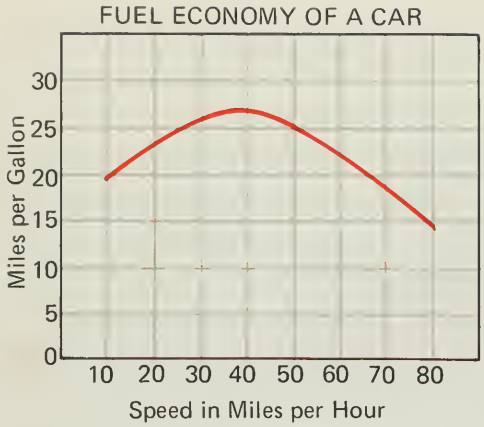
4. Use this bar graph to make a broken line graph of Latin American population. Check student's graphs.
5. Estimate the population of Latin America in 1985. about 425 million
6. Estimate Latin America's population in the year 1990. about 475 million



People's need for electricity changes during a typical day. At certain times of the day, electricity plants must produce 90%–100% of all they can produce. Examine the graph at the right.

7. At midnight, the plant needs to produce ___?___ percent of the electricity it can produce. about 75
8. At what time does the plant need to produce the most electricity? Can the plant produce more electricity at that time? Why? 5:00 p.m.; the plant could not produce more since it is already producing at 100% of its capacity.





The graph at the left shows the fuel economy of a certain car. Complete the sentences.

9. The car gets the most miles per gallon at about __?__ miles per hour. 40

10. At about 60 miles per hour the car gets __?__ miles per gallon. 22.5

11. The car will get 20 miles per gallon at about 10 __?__ miles per hour and also at __?__ miles per hour. about 65

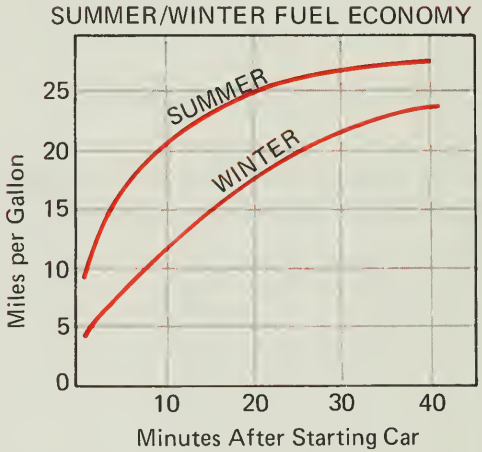
Study the graph at the right. Complete the following sentences.

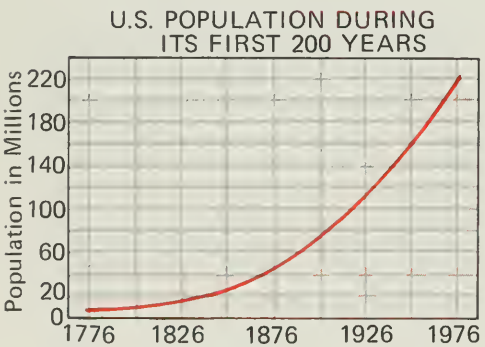
12. After 30 minutes in the summer, the car will get __?__ miles per gallon. about 26

13. After 30 minutes in the winter, the car will get __?__ miles per gallon. about 21

14. After starting the car in the summer it takes __?__ minutes before you get 20 miles per gallon. about 9

15. After starting the car in the winter, it takes __?__ minutes before you get 20 miles per gallon. about 26





Study the graph at the left.

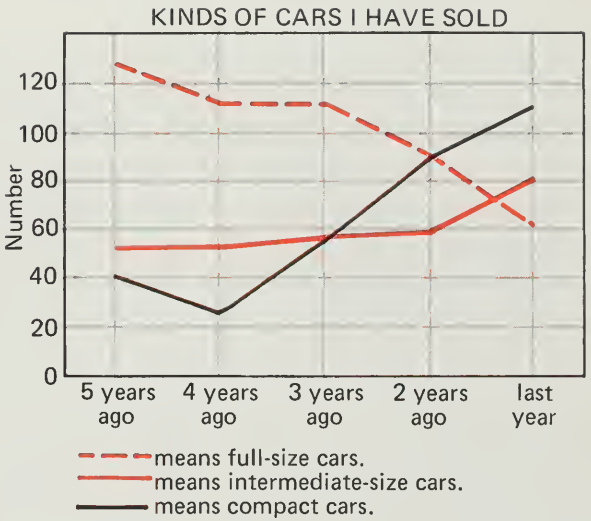
16. In 1876 the population of the United States was about __?__ million. 40

17. The population of the United States was 160 million in the year __?__. 1951

18. About what was the increase in population from 1776 to 1976? about 210 million

19. Estimate the United States' population in the year 2026. about 370 million

Here are three graphs that a car salesperson drew.



20. Which type of car was the salesperson's best seller five years ago? How many cars was that? full size; about 127

21. What was the total number of cars this person sold last year? 250 cars

22. When did the compact cars begin to sell better than the full-size cars? about 2 years ago

23. When did the compact cars begin to sell better than the intermediate-size cars? about 3 years ago

Suppose gasoline costs $1.00 per gallon and you drive a car which averages 20 miles per gallon.

24. How much does it cost you for gasoline to go 1 mile? 5¢ per mile

25. Now complete the following table.

Miles traveled per year	10,000	20,000	30,000
Gasoline cost in dollars per year	$500	?	?

$1000 $1500

26. Use the table in Exercise 25 to draw a graph. Your points should lie along a straight line. Do they? Check student's graphs.



Suppose gasoline costs $1.00 per gallon, but you drive a car which averages 25 miles per gallon.

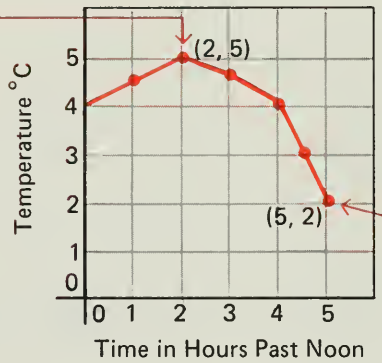
27. How much does it cost for gasoline to drive 1 mile? How much does it cost for gasoline to drive 2 miles? 4¢ per mile

28. Make a table and draw a graph like the one in Exercise 26. 10,000–$400, 20,000–$800, 30,000–$1200
Check student's graphs.

3 • Points on a Graph

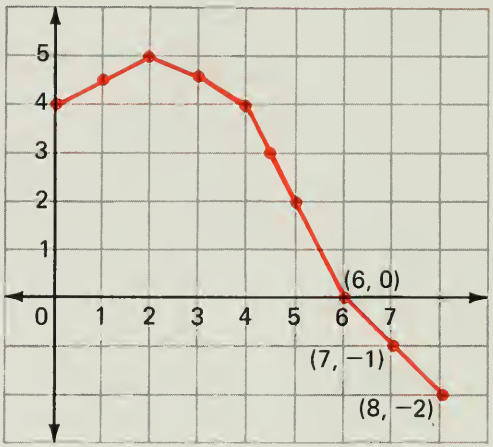
A temperature-time graph is shown below. Notice that the points (2,5) and (5,2) are not the same. You can see that the order in which you write the pair of numbers is very important!

The temperature at 2 o'clock was 5°. We label the point (2,5) and call it **point (2,5).**



The temperature at 5 o'clock was 2°. We label the point (5,2) and call it point (5,2).

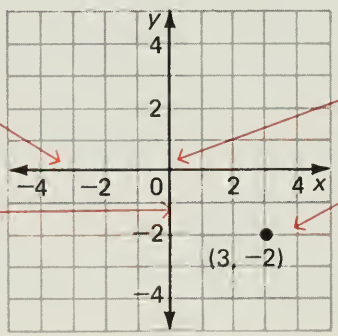
If the temperature drops below zero we shall need to extend our graph as shown at the right.



In drawing or reading graphs, you ought to know a few terms.

This number line is called the **horizontal axis.**

This number line is called the **vertical axis.**



The point (0,0) is called the **origin.**

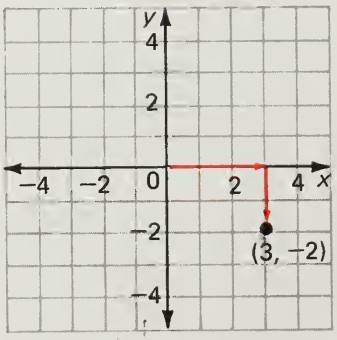
This is point (3, −2). The numbers 3 and −2 are called the **coordinates** of point (3, −2).

When you want to **plot** (or **locate**) points like $(3,-2)$ and $(-4,1)$, you follow these steps.

To plot $(3,-2)$

Step 1: Go 3 units right.

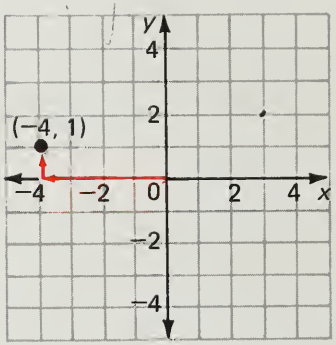
Step 2: Go 2 units down.



To plot $(-4,1)$

Step 1: Go 4 units left.

Step 2: Go 1 unit up.

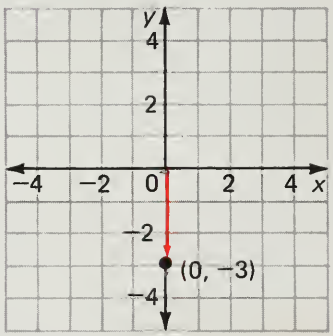


If a point has a 0 coordinate, it lies on one of the axes.

To plot $(0,-3)$

Step 1: Go 0 units right or left.

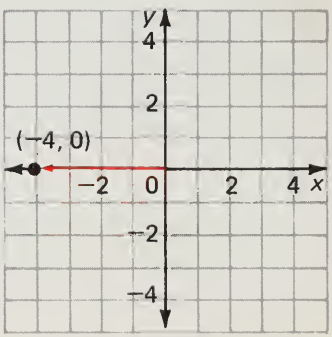
Step 2: Go 3 units down.



To plot $(-4,0)$

Step 1: Go 4 units left.

Step 2: Go 0 units up or down.

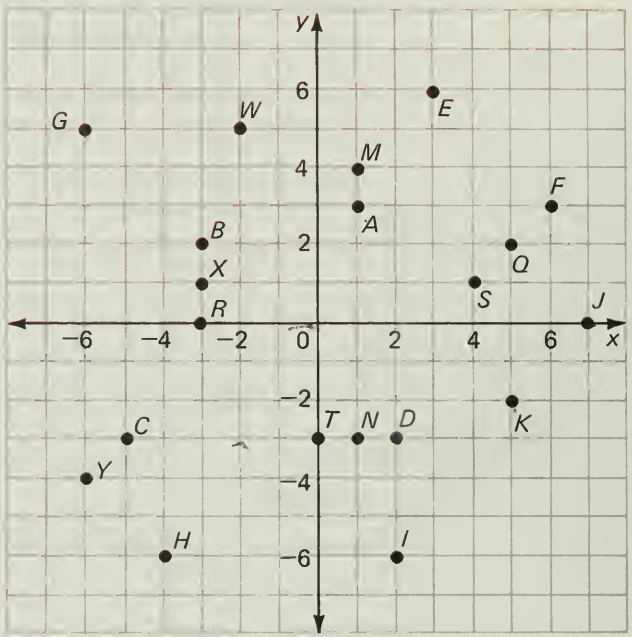


Classroom Practice

Give the coordinates of each point.

Sample Point A has coordinates (1,3).

1. B (−3, 2)
2. C (−5, −3)
3. D (2, −3)
4. E (3, 6)
5. F (6, 3)
6. G (−6, 5)
7. H (−4, −6)
8. I (2, −6)
9. J (7, 0)



Name the point by its letter.

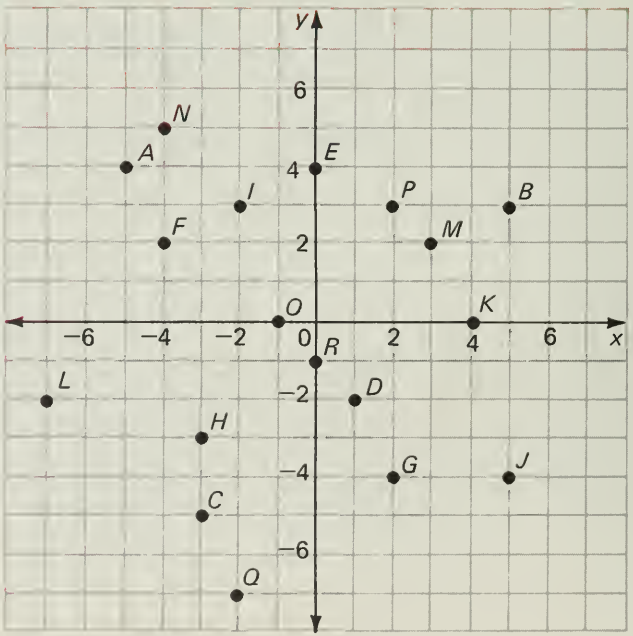
10. (4,1) S
11. (1,4) M
12. (5,−2) K
13. (−2,5) W
14. (−3,0) R
15. (0,−3) T
16. (−3,1) X
17. (1,−3) N
18. (−6,−4) Y
19. (5,2) Q

Written Exercises

Give the coordinates of each point.

A

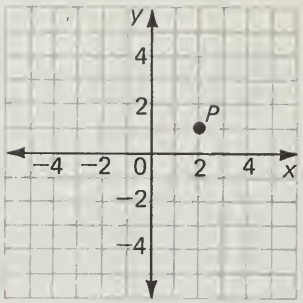
1. A (−5, 4)
2. B (5, 3)
3. C (−3, −5)
4. D (1, −2)
5. E (0, 4)
6. F (−4, 2)
7. G (2, −4)
8. H (−3, −3)
9. I (−2, 3)



Name the point by its letter.

10. (2,3) P
11. (3,2) M
12. (−1,0) O
13. (0,−1) R
14. (5,−4) J
15. (−4,5) N
16. (−7,−2) L
17. (−2,−7) Q
18. (4,0) K

Refer to the graph at the right.



19. How many units is P from the vertical axis? 2

20. How many units is P from the horizontal axis? 1

Using the graph at the right, give the coordinates of the following points.

21. The point 3 units above P (2, 4)

22. The point 3 units below P (2, −2)

23. The point 3 units to the right of P (5, 1)

24. The point 3 units to the left of P (−1, 1)

In each of Exercises 25–28 plot the points in order. Connect each point to the next one in the list by drawing a line. Check student's graphs.

25. (4,2); (4,−4); (−5,−4); (−5,2); (4,2) You should get a rectangle.

26. (5,0); (0,5); (−5,0); (0,−5); (5,0) You should get a square.

27. (3,−2); (6,8); (−4,6); (−7,−4); (3,−2) This figure is called a parallelogram.

B **28.** (−3,0); (0,3); (3,3); (4,0); (6,0); (7,−1); (7,−2); (6,−2); (5,−3); (4,−3); (3,−2); (−1,−2); (−2,−3); (−3,−3); (−4,−2); (−5,−2); (−5,−1); (−4,0); (−3,0)

For more practice, see page 422.

SELF-TEST

Give the coordinates of each point.



1. A (0, −2) **2.** B (−2, 1) **3.** C (−2, 0)

4. D (−1, −2) **5.** E (2, −2) **6.** F (−2, −2)

Name the point by its letter.

7. (2,1) H **8.** (1,2) J **9.** (3,−2) I

10. (−2,3) L **11.** (0,2) K **12.** (2,0) G

4 • Solution Pairs

Some equations have only one variable. Equations of this type have solutions which are single numbers.

$$5x = 10$$
$$x = 2$$

Some equations have two variables. Their solutions are pairs of numbers. In the equation below, $x = 5$ and $y = 1$ is a solution because $5 - 1 = 4$.

$$x - y = 4$$

The solution $x = 5$ and $y = 1$ of the equation above is sometimes called a **solution pair.** It is written as an **ordered pair** of numbers, (5,1).

(5,1) ← x value first, y value second

Here are a few more solution pairs of the equation $x - y = 4$.

(6,2) is a solution because $6 - 2 = 4$
(7,3) is a solution because $7 - 3 = 4$
(3, −1) is a solution because $3 - (-1) = 4$

(9,2) is NOT a solution because $9 - 2 \neq 4$

EXAMPLE 1 Find four solution pairs for the equation $x + y = 6$.

$x + y = 6$	
If x is	then y is
0	6 → (0,6) is a solution.
1	5 → (1,5) is a solution.
2	4 → (2,4) is a solution.
4	2 → (4,2) is a solution.

(2,4) and (4,2) are different solutions.

Some equations are a little more difficult to solve in your head. In that case, just solve for y before you begin to look for solution pairs.

EXAMPLE 2 Find four solution pairs for $y - 2x = 1$.

Step 1: Solve the equation for y.

$$\begin{aligned} y - 2x &= 1 \\ y - 2x + 2x &= 1 + 2x \\ y &= 1 + 2x \end{aligned}$$

Step 2: Replace x with a value. Find the value of y.

$y = 1 + 2x$	
If x is	then y is
0	1 → (0,1)
1	3 → (1, 3)
-1	-1 → $(-1,-1)$
2	5 → (2,5)

EXAMPLE 3 Find four solution pairs for $2x + y = 10$.

Step 1: Solve the equation for y.

$$\begin{aligned} 2x + y &= 10 \\ 2x + y - 2x &= 10 - 2x \\ y &= 10 - 2x \end{aligned}$$

Step 2: Replace x with a value. Find the value of y.

$y = 10 - 2x$	
x	y
0	10 → (0,10)
1	8 → (1,8)
-1	12 → $(-1,12)$
2	6 → (2,6)

Classroom Practice

Which choices are solution pairs for the equation?

1. $x + y = 4$	**a.** (3,1)	**b.** (6,−2)	**c.** (−2,2)
2. $y = 3x + 1$	**a.** (2,7)	**b.** (0,1)	**c.** (1,0)
3. $xy = 6$	**a.** (1,5)	**b.** (−3,−2)	**c.** (6,0)
4. $y = x - 2$	**a.** (2,4)	**b.** (−2,4)	**c.** (1,−1)

Find four solution pairs for the equation. Answers may vary.

5. $y = x + 2$
(0, 2)(1, 3)
(2, 4)(3, 5)

6. $y = 3x - 1$
(0, −1)(1, 2)
(3, 8)(−3, −10)

7. $2x + y = 8$
(0, 8)(1, 6)
(2, 4)(−2, 12)

8. $y - 2x = 3$
(0, 3)(1, 5)
(2, 7)(−2, −1)

Written Exercises

Which choices are solution pairs for the equation?

A **1.** $x + y = 8$	**a.** (5,3)	**b.** (−5,−3)	**c.** (−4,12)
2. $x - y = 8$	**a.** (10,2)	**b.** (−4,4)	**c.** (−6,−2)
3. $y = 3x - 2$	**a.** (0,2)	**b.** (−2,0)	**c.** (2,4)
4. $y = 5 - 2x$	**a.** (3,1)	**b.** (1,3)	**c.** (−1,7)
5. $y - 3x = 10$	**a.** (0,4)	**b.** (−1,7)	**c.** (2,10)
6. $2x - y = 8$	**a.** (1,−6)	**b.** (0,−8)	**c.** (−8,8)

Complete.

7. $y = 2x$

x	y	
3	?	6
5	?	10
7	?	14
8	?	16

8. $y = -3x$

x	y	
2	?	−6
−4	?	12
3	?	−9
−3	?	9

9. $y = x + 3$

x	y	
0	?	3
2	?	5
−4	?	−1
5	?	8

10. $y = 2x - 1$

x	y	
0	?	−1
−1	?	−3
1	?	1
2	?	3

11. $y = -x + 2$

x	y	
0	?	2
1	?	1
2	?	0
−2	?	4

12. $y = 9 - 2x$

x	y	
0	?	9
2	?	5
3	?	3
−3	?	15

13. $s = 3t + 6$

t	s	
0	?	6
−1	?	3
1	?	9
−2	?	0

14. $b = 4a - 1$

a	b	
0	?	−1
1	?	3
−1	?	−5
2	?	7

Solve the equation for y.

15. $x + y = 9$ — $9 - x$

16. $y + 2x = 4$ — $4 - 2x$

17. $y - x = 6$ — $6 + x$

18. $3x + y = 7$ — $7 - 3x$

19. $4x + y = 5$ — $5 - 4x$

20. $y - 3x = 7$ — $7 + 3x$

21. $y + 4x = 9$ — $9 - 4x$

22. $y + 5 = 4x$ — $4x - 5$

23. $7 + y = 9x$ — $9x - 7$

24. $x - y = 9$ — $x - 9$

25. $2x - y = 5$ — $2x - 5$

26. $3x - y = 6$ — $3x - 6$

Find three solution pairs for the equation. Answers may vary.

27. $y = x - 1$ — (1, 0)(2, 1)(−1, −2)

28. $y = 2x + 5$ — (0, 5)(1, 7)(2, 9)

29. $y + x = 0$ — (1, −1)(2, −2)(−1, 1)

30. $y + x = 7$ — (0, 7)(1, 6)(−1, 8)

31. $y - x = 6$ — (0, 6)(1, 7)(−1, 5)

32. $y - x = 8$ — (0, 8)(2, 10)(−2, 6)

33. $y - 2x = 4$ — (0, 4)(2, 8)(−2, 0)

34. $y - 3x = 2$ — (0, 2)(1, 5)(−1, −1)

35. $y - 4x = 7$ — (0, 7)(1, 11)(−1, 3)

36. $y + 5x = 0$ — (0, 0)(1, −5)(−1, 5)

37. $2x + y = 2$ — (0, 2)(1, 0)(−1, 4)

38. $y - 3x = 9$ — (0, 9)(1, 12)(−3, 0)

Can you guess an equation to go with the table?

B **39.**

x	y
1	4
2	8
3	12
4	16

$y = 4x$

40.

x	y
1	2
2	3
4	5
5	6

$y = x + 1$

41.

x	y
1	5
2	9
3	13
4	17

$y = 4x + 1$

42.

x	y
0	−1
1	1
2	3
3	5

$y = 2x - 1$

Which choices are solution pairs for the inequality?

43. $y > 2x$ — **a.** (2,8) **b.** (3,8) **c.** (4,8)

44. $y < 3x$ — **a.** (2,5) **b.** (2,6) **c.** (2,7)

45. $y < x - 2$ — **a.** (5,1) **b.** (5,2) **c.** (5,3)

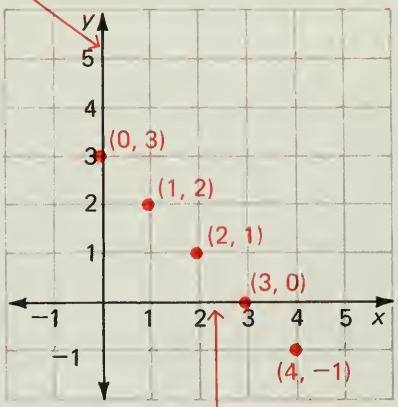
46. $y > 2x - 3$ — **a.** (0,1) **b.** (0,0) **c.** (15,8)

For more practice, see page 422.

5 • Graphs of Equations

We call this the *y*-axis.

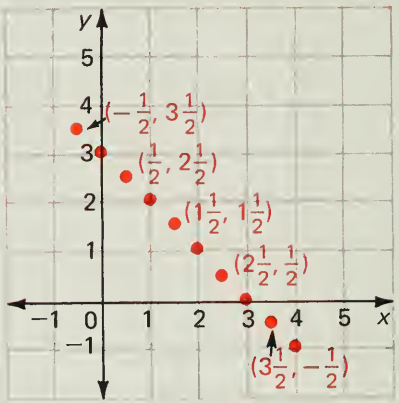
This graph shows some of the solutions of the equation $x + y = 3$. Since each solution is an ordered pair of numbers, we can picture each solution as a point.



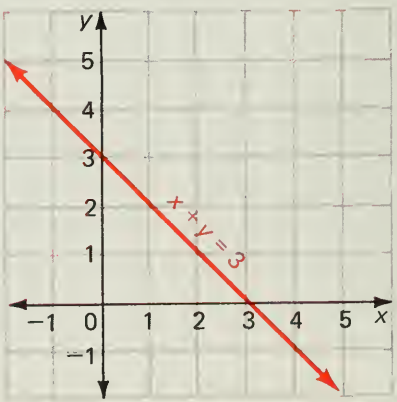
This is the *x*-axis.

$x + y = 3$		
x	y	
0	3	(0,3)
1	2	(1,2)
2	1	(2,1)
3	0	(3,0)
4	−1	(4,−1)

This graph shows some more solutions of the equation $x + y = 3$.



If we could graph *all* the solutions of the equation $x + y = 3$, we would get the line shown. This line is called **the graph of the equation $x + y = 3$.**



In this section the equations will have graphs which are straight lines. You will find later, however, that not all equations have straight line graphs.

Classroom Practice

Draw the graph of the equation. Check student's graphs.

1. $y = 2x$
(1, 2)(0, 0)

2. $y = x + 2$
(1, 3)(0, 2)

3. $y = -x + 3$
(1, 2)(0, 3)

4. $y = -2x + 1$
(1, −1)(0, 1)

5. $x + y = 3$
(1, 2)(0, 3)

6. $y - x = 1$
(1, 2)(0, 1)

7. $y - 2x = 2$
(1, 4)(0, 2)

8. $2x + y = 7$
(1, 5)(0, 7)

9. $x + y = -5$
(1, −6)(0, −5)

10. $y - 3x = -2$
(1, 1)(0, −2)

11. $3x + y = 4$
(1, 1)(0, 4)

12. $y - 4x = -1$
(1, 3)(0, −1)

Written Exercises

Draw the graph of the equation. Check student's graphs.

A

1. $y = 3x$
(1, 3)(0, 0)

2. $y = 4x$
(1, 4)(0, 0)

3. $y = x + 3$
(1, 4)(0, 3)

4. $y = 3x + 3$
(1, 6)(0, 3)

5. $y = 4x - 2$
(1, 2)(0, −2)

6. $y = -x + 1$
(1, 0)(0, 1)

7. $y = 2x + 3$
(1, 5)(0, 3)

8. $y = 3x - 4$
(1, −1)(0, −4)

9. $y = -2x + 3$
(1, 1)(0, 3)

10. $x + y = 4$
(1, 3)(0, 4)

11. $x + y = 5$
(1, 4)(0, 5)

12. $y - x = 5$
(1, 6)(0, 5)

13. $2x + y = 6$
(1, 4)(0, 6)

14. $y - 5x = 0$
(1, 5)(0, 0)

15. $y - 2x = 7$
(1, 9)(0, 7)

16. $x - y = 9$
(1, −8)(0, −9)

17. $3x - y = 5$
(1, −2)(0, −5)

18. $2x - y = 5$
(1, −3)(0, −5)

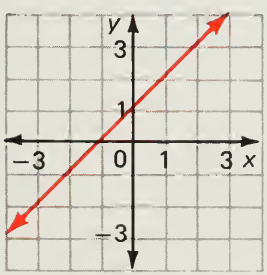
19. $2x + y = -3$
(1, −5)(0, −3)

20. $y - 4x = -1$
(1, 3)(0, −1)

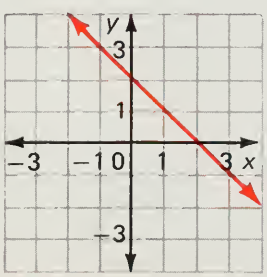
21. $3x + y = 0$
(1, −3)(0, 0)

Can you guess the equation of the graph? Find some solution pairs and make a chart for help.

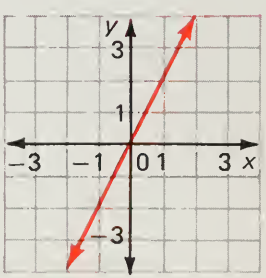
B

22. 

$y = x + 1$

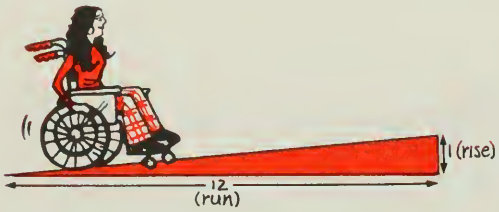
23. 

$y = -x + 2$

24. 

$y = 2x$

6 • Slope of a Line

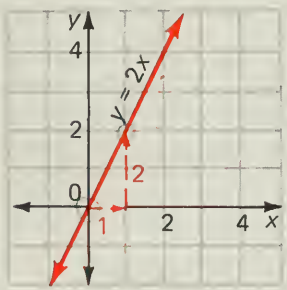


Architects know that wheelchair ramps should not be too steep. They also know that if the ramp is not steep enough, then the ramp might be much too long. The slope of a ramp is very important.

$$\text{slope} = \frac{\text{rise of ramp}}{\text{run of ramp}} = \frac{1}{12}$$

The slope of a line is measured in the same way. It is given as the ratio of the rise to the run. Study the three graphs below. Notice how you can determine the slope of each line. Notice also that you can determine the slope by examining the equation of the graph.

Line 1

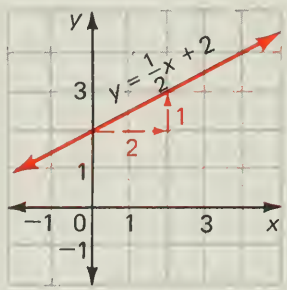


$\text{slope} = \frac{2}{1} = 2$

equation: $y = 2x$

slope: 2

Line 2

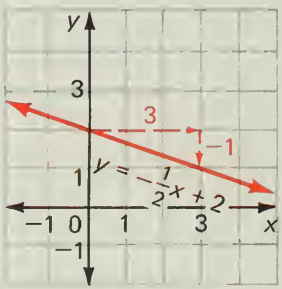


$\text{slope} = \frac{1}{2}$

equation: $y = \frac{1}{2}x + 2$

slope: $\frac{1}{2}$

Line 3



$\text{slope} = \frac{-1}{3} = -\frac{1}{3}$

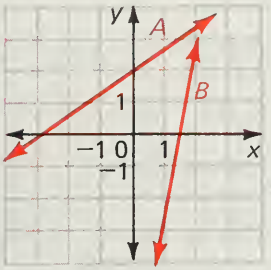
equation: $y = -\frac{1}{3}x + 2$

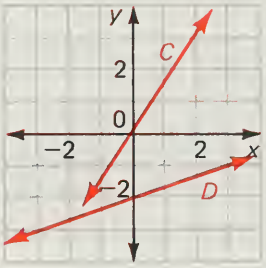
slope: $-\frac{1}{3}$

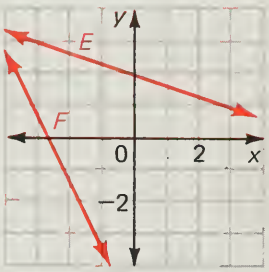
To find the slope *before* you draw the graph, just look at the coefficient of x after the equation is solved for y. Note that the slope can be a positive or negative number.

Written Exercises

Find the slope.







A

1. line A $\frac{2}{3}$
2. line B 5
3. line C $\frac{3}{2}$
4. line D $\frac{1}{3}$
5. line E $-\frac{1}{3}$
6. line F -2

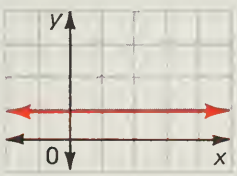
What will be the slope of the line when the graph of the equation is drawn?

7. $y = 3x$ 3
8. $y = 4x - 2$ 4
9. $y = 2x + 3$ 2
10. $y = 4x + 1$ 4
11. $y = -x + 4$ -1
12. $y = -x - 7$ -1
13. $y = -3x$ -3
14. $y = -5x + 4$ -5
15. $y = \frac{1}{3}x + 7$ $\frac{1}{3}$
16. $y = \frac{3}{4}x$ $\frac{3}{4}$
17. $y = -\frac{1}{5}x + 5$ $-\frac{1}{5}$
18. $y = -\frac{2}{3}x + 1$ $-\frac{2}{3}$

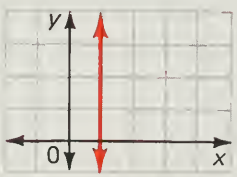
19. Draw a line having a slope of $\frac{3}{4}$. Check student's graphs.

20. Draw a line having a slope of $\frac{4}{3}$. Check student's graphs.

A horizontal line is shown below.



A vertical line is shown below.



B

21. What is its rise? 0
22. What is its run? It varies.
23. What is its slope? 0
24. What is its rise? It varies.
25. What is its run? 0
26. Does the line have a slope? No

For more practice, see page 423.

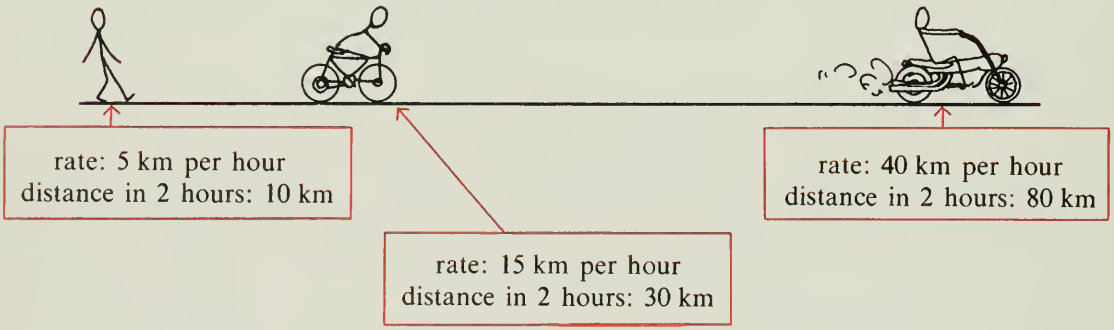
7 • Functions

Often in everyday life one quantity depends on another. When this happens we say that the first quantity is a function of the other.

EXAMPLE 1 The distance you can travel in two hours depends on how fast you are going (your rate).

Your distance is a function of your rate.

$$D = 2r$$



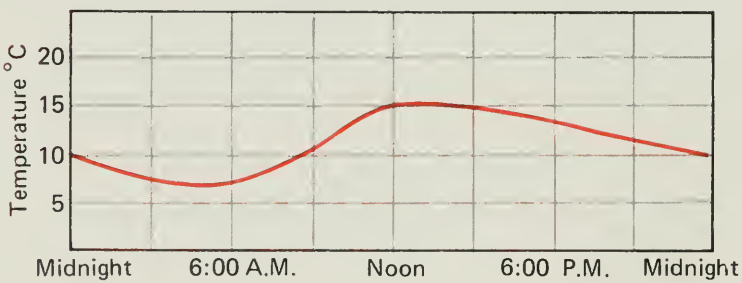
EXAMPLE 2 The value of a car depends upon its age (usually).

The value of a car is a function of its age.

Age of car	new	1 yr	2 yr	3 yr	4 yr
Value of car	$6000	$4800	$3600	$2700	$2025

EXAMPLE 3 The temperature outdoors often depends upon the time of day.

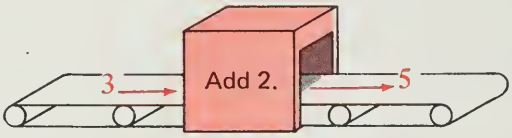
The temperature outdoors is a function of the time of day.



Examples 1–3 show that a function can be described by an equation, a table, or a graph. Let's consider one more function and describe it in all three ways.

EXAMPLE 4 The OUTPUT of a number machine depends upon the INPUT.

The OUTPUT *depends upon the* INPUT.



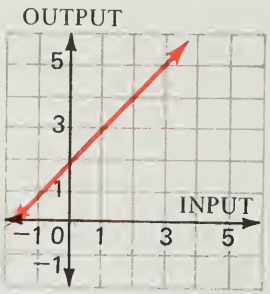
Equation

$y = x + 2$

Table

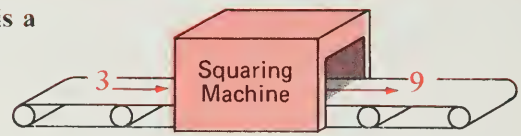
INPUT x	OUTPUT y
0	2
1	3
2	4
3	5

Graph



Classroom Practice

The OUTPUT of the machine at the right is a function of the INPUT.



1. Complete the table.

x = INPUT	3	2	0	−2	−4
y = OUTPUT	9	?	?	?	?

4 0 4 16

2. An equation relating x and y in Exercise 1 is: $y =$?. x^2

The cost of several oranges at 10¢ each is a function of the number of oranges you buy.

3. Complete the table.

n = number you buy	1	2	6	12
C = cost	10	?	?	?

20 60 120

4. An equation relating n and C in Exercise 3 is: $C =$?. $10n$

5. If $y = 3x$, then y is a function of __?__. *x*

6. If $D = 2r$, then D is a function of __?__. *r*

7. The time it takes to travel by bus from one part of town to another is a function of which of the following?
 a. How often the buses run
 b. The traffic
 c. Both the traffic and how often the buses run

Written Exercises

In Exercises 1–4 the OUTPUT is a function of the INPUT.

A 1. Complete the table.

x = INPUT	2	3	4	−2	?
y = OUTPUT	4	6	8	?	10

5 −4

2. An equation relating x and y in Exercise 1 is: $y =$ __?__. *2x*

3. Complete the table.

x = INPUT	4	9	−7	?
y = OUTPUT	5	10	−6	−8

−9

4. An equation relating x and y in Exercise 3 is: $y =$ __?__. *x + 1*

The cost of several grapefruit which sell for 18¢ each is a function of the number you buy.

5. Complete the table.

n = number you buy	1	2	3	12	?
C = cost	18	?	?	?	180

10 36 54 216

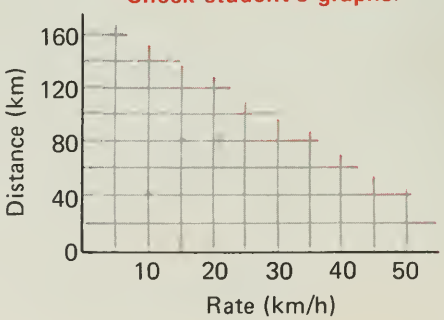
6. An equation relating n and C in Exercise 5 is: $C =$ __?__. *18n*

The distance you travel in 4 hours is a function of your rate.

7. Complete the table.

Rate (km/h)	Distance (km)
10	40
20	80 ?
30	120 ?
40	160 ?

8. Copy and complete the graph.
Check student's graphs.



The wind-chill factor is a function of temperature and wind speed.

Wind Speed (km/h)

	10	20	30	40	50	60
0°	−3°	−9°	−13°	−16°	−18°	−19°
−5°	−9°	−17°	−21°	−23°	−26°	−27°
−10°	−14°	−24°	−27°	−31°	−33°	−35°
−15°	−19°	−29°	−35°	−38°	−41°	−42°

If the temperature is −5°C and the wind is blowing 30 km/h, the wind-chill factor is −21°C. This means that it *feels* as though it is −21°C.

9. Find the wind-chill factor when the temperature is −5°C and the wind speed is 20 km/h. −17° C

10. Find the wind-chill factor when the temperature is −15°C and the wind speed is 50 km/h. −41° C

11. If the temperature is 0°C and the wind-chill factor is −16°, then the wind speed is ___?___ km/h. 40

Scientists have discovered that a cricket's rate of chirping is a function of the temperature of the air!



Number of chirps per minute = 7 × Celsius temperature − 30

$$N = 7C - 30$$

B **12.** Make a graph of this equation. Label the horizontal axis C and the vertical axis N. Check student's graphs. (10, 40)(20, 110)

13. Suppose a cricket chirps 75 times a minute. About what temperature is it? 15° C

For more practice, see page 423.

SELF-TEST

Find three solution pairs for the equation. Answers may vary.

1. $y = 5x$ (0, 0)(1, 5)(2, 10)

2. $y = 3x - 6$ (0, −6)(1, −3) (2, 0)

3. $y = 4x + 1$ (0, 1)(1, 5) (2, 9)

Give the slope of the line when the graph of the equation is drawn.

4. $y = 9x - 4$ 9

5. $y = -3x + 2$ −3

6. $y = -x$ −1

Draw the graph of the equation. Check student's graphs.

7. $y = -4x$

8. $y - 7x = 0$

9. $y = 5x - 6$

Reviewing Arithmetic Skills

Rename.

Sample $\frac{1}{2} = \frac{?}{10} \longrightarrow \frac{1}{2} = \frac{5}{10}$

1. $\frac{1}{4} = \frac{?}{12}$ 3
2. $\frac{1}{5} = \frac{?}{20}$ 4
3. $\frac{1}{3} = \frac{?}{9}$ 3
4. $\frac{1}{2} = \frac{?}{8}$ 4
5. $\frac{1}{6} = \frac{?}{18}$ 3
6. $\frac{2}{3} = \frac{?}{6}$ 4
7. $\frac{3}{8} = \frac{?}{48}$ 18
8. $\frac{3}{4} = \frac{?}{16}$ 12
9. $\frac{4}{7} = \frac{?}{35}$ 20
10. $\frac{5}{6} = \frac{?}{24}$ 20

In each exercise, rename the fractions so that they have the same denominator.

Sample $\frac{1}{2}, \frac{1}{5} \longrightarrow \frac{5}{10}, \frac{2}{10}$

11. $\frac{1}{3}, \frac{1}{4}$ $\frac{4}{12}, \frac{3}{12}$
12. $\frac{1}{2}, \frac{1}{7}$ $\frac{7}{14}, \frac{2}{14}$
13. $\frac{1}{4}, \frac{1}{8}$ $\frac{2}{8}, \frac{1}{8}$
14. $\frac{1}{3}, \frac{1}{5}$ $\frac{5}{15}, \frac{3}{15}$
15. $\frac{1}{4}, \frac{1}{5}$ $\frac{5}{20}, \frac{4}{20}$
16. $\frac{2}{5}, \frac{1}{3}$ $\frac{6}{15}, \frac{5}{15}$
17. $\frac{1}{6}, \frac{4}{9}$ $\frac{3}{18}, \frac{8}{18}$
18. $\frac{2}{3}, \frac{1}{4}$ $\frac{8}{12}, \frac{3}{12}$
19. $\frac{3}{7}, \frac{1}{14}$ $\frac{6}{14}, \frac{1}{14}$
20. $\frac{1}{8}, \frac{5}{6}$ $\frac{3}{24}, \frac{20}{24}$
21. $\frac{3}{4}, \frac{3}{7}$ $\frac{21}{28}, \frac{12}{28}$
22. $\frac{4}{7}, \frac{2}{9}$ $\frac{36}{63}, \frac{14}{63}$
23. $\frac{7}{10}, \frac{2}{3}$ $\frac{21}{30}, \frac{20}{30}$
24. $\frac{5}{6}, \frac{2}{5}$ $\frac{25}{30}, \frac{12}{30}$
25. $\frac{4}{9}, \frac{3}{8}$ $\frac{32}{72}, \frac{27}{72}$

In each exercise, rename the fractions so that they have the same denominator. Then add or subtract.

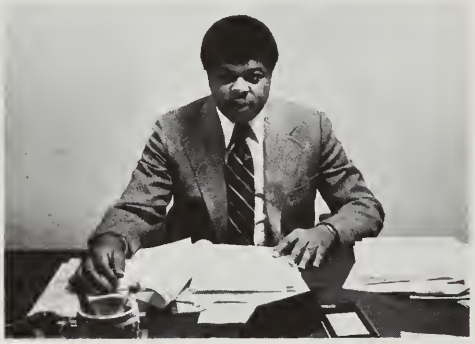
26. $\frac{1}{2} + \frac{1}{7}$ $\frac{9}{14}$
27. $\frac{1}{6} + \frac{1}{5}$ $\frac{11}{30}$
28. $\frac{1}{4} + \frac{1}{6}$ $\frac{5}{12}$
29. $\frac{1}{9} + \frac{1}{3}$ $\frac{4}{9}$
30. $\frac{1}{4} + \frac{1}{7}$ $\frac{11}{28}$
31. $\frac{1}{2} - \frac{1}{3}$ $\frac{1}{6}$
32. $\frac{1}{3} - \frac{1}{8}$ $\frac{5}{24}$
33. $\frac{1}{4} - \frac{1}{5}$ $\frac{1}{20}$
34. $\frac{1}{2} - \frac{1}{6}$ $\frac{1}{3}$
35. $\frac{1}{3} - \frac{1}{7}$ $\frac{4}{21}$
36. $\frac{2}{3} + \frac{1}{4}$ $\frac{11}{12}$
37. $\frac{5}{6} + \frac{1}{7}$ $\frac{41}{42}$
38. $\frac{3}{8} + \frac{1}{6}$ $\frac{13}{24}$
39. $\frac{4}{5} + \frac{3}{7}$ $\frac{43}{35}$ or $1\frac{8}{35}$
40. $\frac{2}{9} + \frac{3}{10}$ $\frac{47}{90}$
41. $\frac{3}{4} - \frac{1}{8}$ $\frac{5}{8}$
42. $\frac{7}{8} - \frac{2}{5}$ $\frac{19}{40}$
43. $\frac{9}{11} - \frac{1}{3}$ $\frac{16}{33}$
44. $\frac{5}{7} - \frac{2}{5}$ $\frac{11}{35}$
45. $\frac{2}{3} - \frac{5}{8}$ $\frac{1}{24}$

CAREER NOTEBOOK

Opportunities in Department Store Work

When you think of jobs in a department store you probably think of sales people and store managers. There are many, many other careers you may not have thought of. Here are just a few.





Accountants work with the financial records of the store.

Buyers select and buy the goods for one or several departments in the store.



Computer programmers design programs that handle billing, payroll, inventory control, and so on.

CONSUMER CORNER



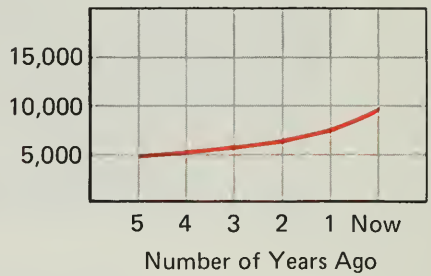
Advertising

There are lots of gimmicks that advertisers use to promote their products. Don't be mistaken. Gimmicks are not necessarily dishonest. As you probably know, some are really clever! Gimmicks are just methods advertisers use to get attention and to interest the consumer in buying the product.

Let's consider a couple of these gimmicks that are used in advertising.

If *Better Living* Magazine wants to show its increased sales, it could do it with a graph like this.

You can see that the sales have doubled in the last 5 years.



But look how much more impressive this graph is. It looks as though the number of sales has really grown by leaps and bounds!

All you have to do is to change the vertical scale and chop off the bottom of the graph.



The advertisers of *Better Living* Magazine could also show the increase in picture form like this.

Since the sales have doubled, the dimensions in the second picture were drawn twice as large as the first.



5 years ago



Today

Measures 1.5 cm by 2 cm.
Area: $1.5 \times 2 = 3$ cm^2

Measures 3 cm by 4 cm.
Area: $3 \times 4 = 12$ cm^2

Since the second area is 4 times the first, it *looks* as if sales have more than doubled!

1. Take a look at the graph about newspaper subscriptions on page 206 in this chapter. Make a line graph showing the same facts. Stretch the vertical scale so that it looks as though subscriptions have *really* climbed. Check student's graphs.

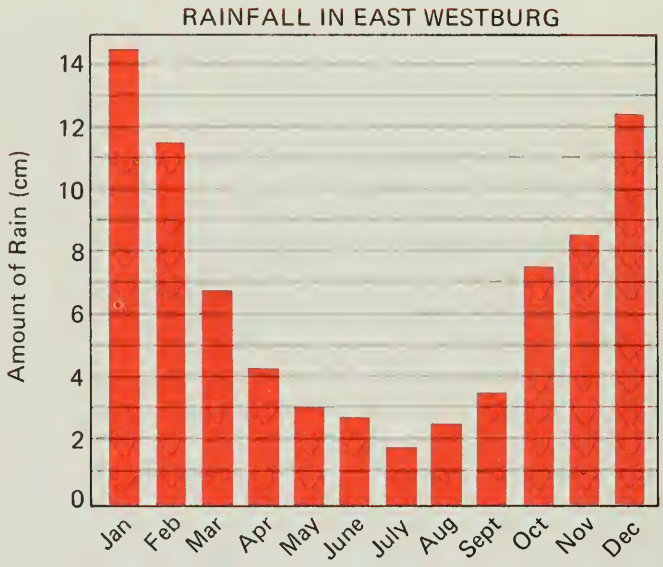
2. Sales have gone down for Gas Guzzler Motor Company. They don't want to scare people. Draw a graph which shows the following facts, but try to make the decrease in sales look very small. Check student's graphs.

 Sales: 5 years ago, 23,000; 4 years ago, 25,000; 3 years ago, 20,000; 2 years ago, 17,000; 1 year ago, 15,000; now, 13,000.

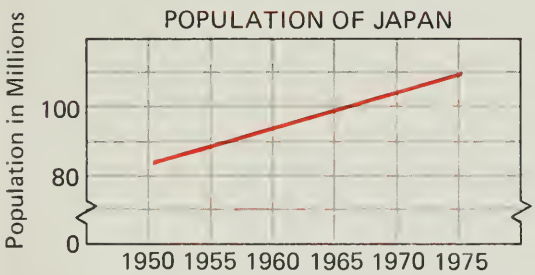
3. Money seems to "shrink" when it can buy less. Suppose the dollar could buy twice as much 5 years ago as it can now. Show the shrinking of the dollar in picture form. Check student's pictures.

Reviewing the Chapter

The graph at the right shows the amount of rainfall in East Westburg last year. *(See pp. 204–207.)*



1. About how much rain fell in October? 7.5 cm
2. Which month had the greatest amount of rain? Jan.
3. Which month had the least amount of rain? July
4. About how much more rain fell in February than in September? 8 cm
5. During the spring months, the amount of rain __?__ (increased/decreased).



The graph at the left shows the population of Japan from 1950 to 1975. *(See pp. 208–211.)*

6. Estimate the population in the year 1970. about 104 million
7. The population was 100 million in the year __?__. 1966
8. Extend the graph and estimate the population in the year 1985. about 120 million

Complete. *(See pp. 216–219.)*

9. $y = -4x$

x	y	
0	?	0
1	?	−4
2	?	−8
−2	?	8

10. $y = 4x + 3$

x	y	
0	?	3
1	?	7
2	?	11
3	?	15

11. $x + y = 10$

x	y	
0	?	10
1	?	9
2	?	8
−1	?	11

12. $y = 5 - 3x$

x	y	
0	?	5
1	?	2
2	?	−1
−2	?	11

Draw a graph of the equation. *(See pp. 220-221.)*

13. $y = x - 6$
(3, −3)(2, −4)(1, −5)

14. $y = 3x + 4$
(0, 4)(−1, 1)(−2, −2)

15. $y - 4x = 3$
(0, 3)(−1, −1)(−2, −5)

16. $y + x = 3$
(1, 2)(0, 3)(−1, 4)

17. $y = -2x + 4$
(2, 0)(1, 2)(0, 4)

18. $y + 3x = 2$
(1, −1)(0, 2)(−1, 5)

Give the slope of the line when the graph of the equation is drawn. *(See pp. 222-223.)*

19. $y = 3x + 4$ 3

20. $y = -x - 4$ −1

21. $y = \frac{3}{4}x - 6$ $\frac{3}{4}$

Hamburger sells for $2.95 a kilogram at a local market. *(See pp. 224-227.)*

22. The total cost of several kilograms of hamburger is a function of ___?___. number of kilograms

23. Complete the table.

n = number of kilograms	1	2	3	5	?
C = Cost	?	?	?	?	$23.60

8

2.95 5.90 8.85 14.75

24. An equation relating n and C in Exercise 23 is: $C =$ ___?___. $2.95n$

The Cooler Soft Drink Company will give you one free bottle of root beer for every 12 bottles you buy. *(See pp. 224-227.)*

25. The number of free bottles you receive is a function of ___?___. number you buy

26. Complete the table.

n = number you buy	12	24	36	60	108
F = number of free bottles	?	?	?	?	?

1 2 3 5 9

27. How many free bottles do you receive if you buy 15 bottles of root beer? 1

The Booster Club earns 50¢ for each bumper sticker it sells. *(See pp. 224-227.)*

28. The amount of money the club earns is a function of ___?___. number sold

29. Complete the table.

n = number sold	10	15	20	50	?
A = Amount earned	?	?	?	?	55

110

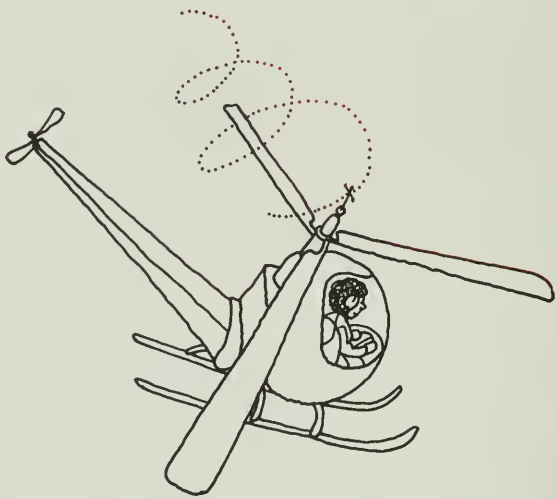
5.00 7.50 10.00 25.00

30. Use the table in Exercise 29 to draw a graph. Check student's graphs.

A permission-to-reproduce chapter test can be found on page T17.

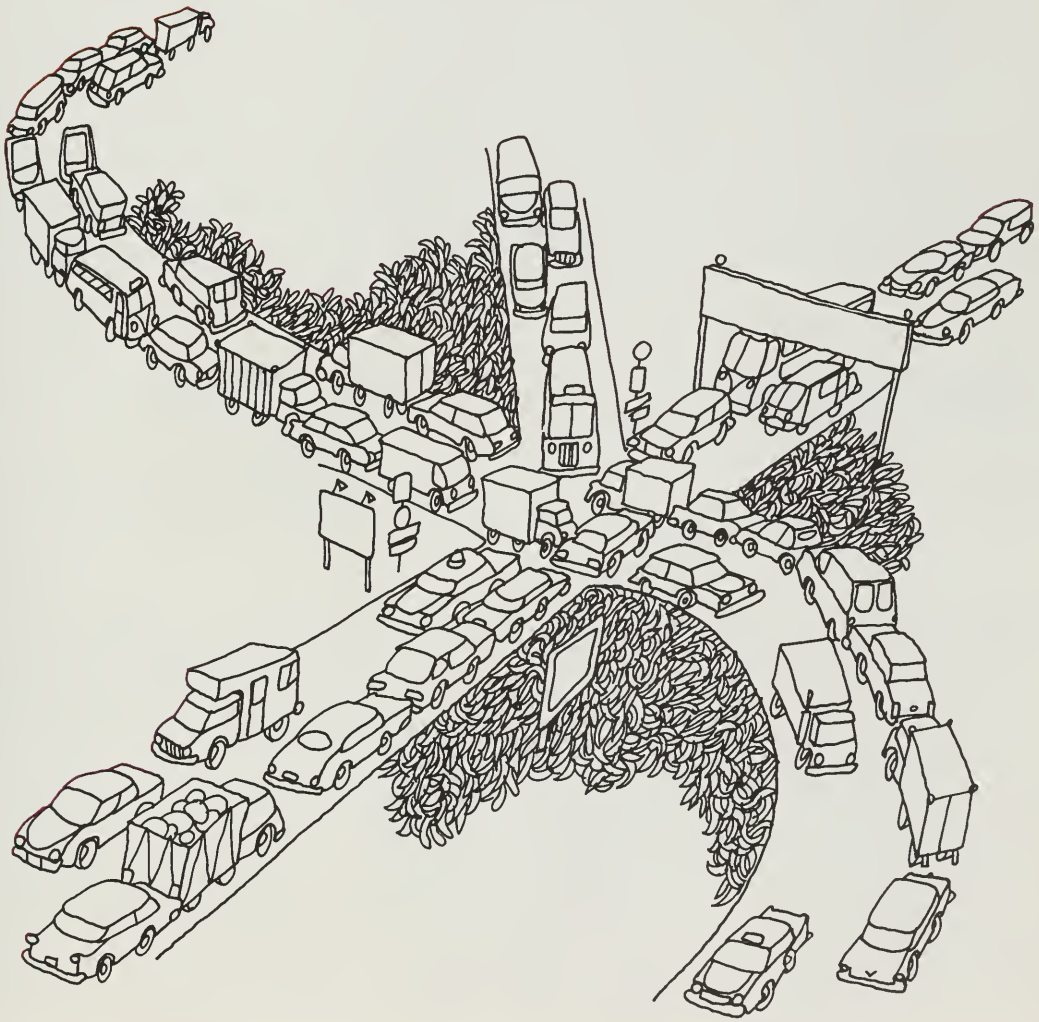
Here's what you'll learn in this chapter:

1. To find the solution of two equations by graphing.
2. To determine whether two equations share one solution pair, no solution pairs, or all solution pairs.
3. To find the solution of two equations by substitution.
4. To find the solution of two equations by addition or subtraction.
5. To solve word problems with two equations and two variables.



Chapter 8

Equations With Two Variables



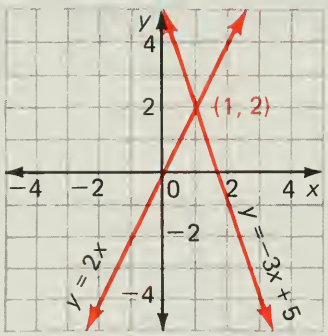
1 • The Graphing Method



When two or more streets cross, we call the place where they cross their **intersection.**

When two lines cross, the point where they cross is their intersection.

$y = 2x$	
x	y
0	0
1	2
2	4
3	6



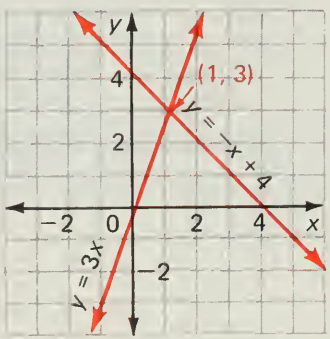
$y = -3x + 5$	
x	y
0	5
1	2
2	−1
3	−4

The graphs of $y = 2x$ and $y = -3x + 5$ intersect at (1,2). This means that (1,2) is a solution of *both* equations.

EXAMPLE 1 Find the ordered pair which is a solution of both $y = 3x$ and $y = -x + 4$.

Step 1

$y = 3x$	
x	y
0	0
1	3
−1	−3



Step 2

$y = -x + 4$	
x	y
0	4
1	3
2	2

Answer: (1,3)

Sometimes you'll have to solve an equation for y before you can draw the graph of the equation.

EXAMPLE 2 Find the ordered pair which is a solution of both $y - 2x = 8$ and $2x + y = 4$.

Step 1: Solve each equation for y.

$y - 2x = 8$

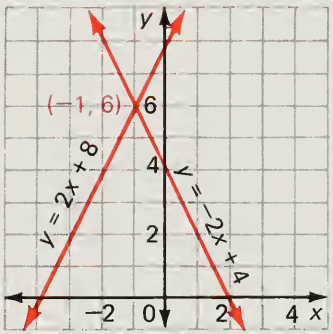
$y = 2x + 8$

$2x + y = 4$

$y = -2x + 4$

Step 2

$y = 2x + 8$	
x	y
0	8
−1	6
−2	4



Step 3

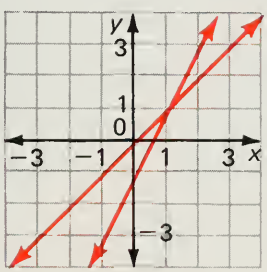
$y = -2x + 4$	
x	y
0	4
1	2
2	0

Answer: (−1,6)

Classroom Practice

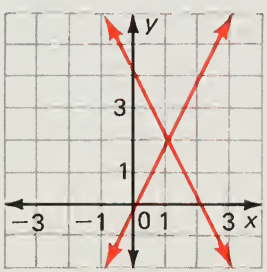
Name the ordered pair which is a solution of the two equations.

1.



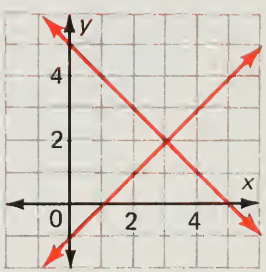
(1, 1)

2.



(1, 2)

3.



(3, 2)

By graphing, find the ordered pair which is a solution of both equations.

4. $y = x$
 $y = 2x + 3$
 $(-3, -3)$

5. $y = x + 4$
 $y = -x$
 $(-2, 2)$

6. $x + y = 2$
 $y = 2x + 5$
 $(-1, 3)$

7. $3x + y = 1$
 $y - x = -3$
 $(1, -2)$

8. $y = x - 4$
 $y + 2x = 2$
 $(2, -2)$

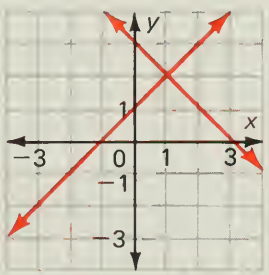
9. $y - 2x = 7$
 $y = -3x - 3$
 $(-2, 3)$

10. $y = -x - 3$
 $y - 3x = 5$
 $(-2, -1)$

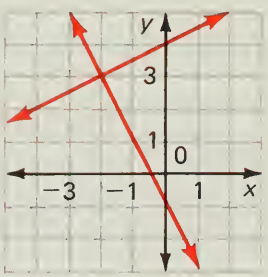
11. $y + x = 6$
 $y - 2x = 3$
 $(1, 5)$

Written Exercises

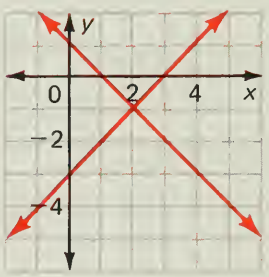
Name the ordered pair which is a solution of the two equations.

A 1. 

$(1, 2)$

2. 

$(-2, 3)$

3. 

$(2, -1)$

Answers to Exs. 4–7 may vary.

4. Name two points on the graph of $y = 4 - x$. $(0, 4)(2, 2)$

5. Name two points on the graph of $x + y = 12$. $(0, 12)(6, 6)$

6. Name two points on the graph of $y - x = 6$. $(0, 6)(-6, 0)$

7. Name two points on the graph of $2x + y = 1$. $(0, 1)(1, -1)$

By graphing, find the ordered pair which is a solution of both equations.

8. $y = -x$
 $y = 3x - 4$ $(1, -1)$

9. $y = x + 1$
 $y = -x + 3$ $(1, 2)$

10. $y = -x - 1$
 $y = 3x + 7$ $(-2, 1)$

11. $y = 2x$
 $y = -2x - 4$ $(-1, -2)$

12. $y = -3x + 2$
 $y = 2x - 3$ $(1, -1)$

13. $y = 2x + 6$
 $y = -x - 3$ $(-3,$

14. $y - x = 0$
 $y + x = 4$ $(2, 2)$

15. $y - x = 3$
 $x + y = -1$ $(-2, 1)$

16. $x + y = -4$
 $y - 2x = 5$ $(-3, -$

17. $y - x = 0$
 $y + 3x = -4$ $(-1, -1)$

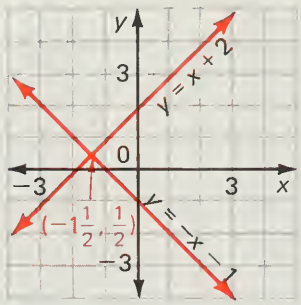
18. $y - 3x = -1$
 $y - 2x = 1$ $(2, 5)$

19. $y - 2x = -5$
 $y - x = -3$ $(2, -$

Sometimes the solution pair is a pair of fractions instead of a pair of integers. Find the solution pair. Your answer does not have to be exact.

Sample

$y = x + 2$	
x	y
0	2
1	3
-1	1



$y = -x - 1$	
x	y
0	-1
1	-2
-2	1

Answer: $\left(-1\frac{1}{2}, \frac{1}{2}\right)$

B **20.** $y = -x - 1$, $y = x + 4$ $\left(-2\frac{1}{2}, 1\frac{1}{2}\right)$

21. $2x + y = 2$, $y - 3x = 9$ $\left(-1\frac{2}{5}, 4\frac{4}{5}\right)$

22. $y - 2x = 4$, $x + y = 5$ $\left(\frac{1}{3}, 4\frac{2}{3}\right)$

23. $y - 2x = 8$ $\left(-3\frac{1}{3}, 1\frac{1}{3}\right)$, $x + y = -2$

24. $y = x + 7$ $\left(-2\frac{1}{2}, 4\frac{1}{2}\right)$, $y = -\frac{1}{5}x + 4$

25. $y = \frac{1}{4}x - 3$, $y = -\frac{1}{4}x - 2$ $\left(2, -2\frac{1}{2}\right)$

26. a. Draw the graphs of the equations $y = x + 1$ and $y = x - 1$ on the same set of axes. Check student's graphs.

b. Do these graphs intersect? no

c. Do the two equations share a solution pair? no

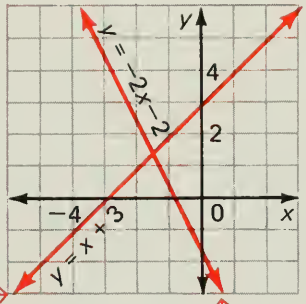
For more practice, see page 424.

Puzzles & Things

Believe it or not, there's a tree that grows its leaves according to a mathematical formula. The tree is *Schumanniophyton problematicum*. Its formula is $L = 12a + 4$. L represents the number of leaves produced, while a represents the age of the tree. Smart tree!



2 • No Solution, Many Solutions



In the last lesson you saw how you could find a solution pair for two equations whose graphs intersect. Look at the graphs at the left. Notice that the two lines slant in different ways. You can see that the **slopes** are not the same, so the lines intersect.

slope $= \frac{\text{rise}}{\text{run}} = \frac{-2}{1} = -2$

slope $= \frac{\text{rise}}{\text{run}} = \frac{1}{1} = 1$

Of course, you don't have to draw the graphs to see whether their slopes are the same or not. You can check by the equations instead.

Make sure both equations are solved for y.
Look at the coefficient of x.

$y = x + 3$ — Slope is 1.

$y = -2x - 2$ — Slope is -2.

Now let's think of lines whose slopes are the same. There are two possible cases.

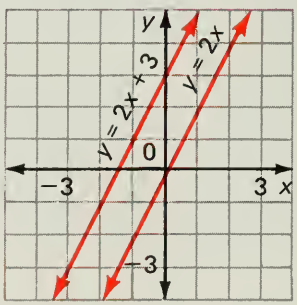
CASE 1: The lines will not intersect. Let's consider $y - 2x = 3$ and $y - 2x = 0$.

$y - 2x = 3$ $\qquad$ $y - 2x = 0$

$y = 2x + 3$ $\qquad$ $y = 2x$

The slopes are the same.

The graphs will never intersect. They are **parallel lines.**



The equations *do not* share a solution pair.

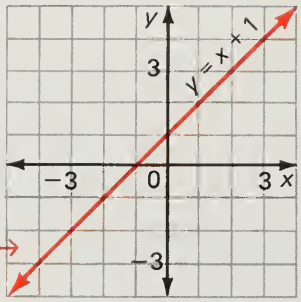
CASE 2: The lines will overlap. Consider $y - 1 = x$ and $y - x = 1$.

$y - 1 = x$ $\qquad$ $y - x = 1$

$y = x + 1$ $\qquad$ $y = x + 1$

The slopes are the same. The equations are the same too.

The graph of each equation is the same line!



As you can see, both equations share *all* solutions.

Now let's use these ideas to check equations to see if they share solution pairs.

EXAMPLE 1 Do $x + y = 4$ and $y - 3x = 2$ share any solution pairs?

$x + y = 4$ $\qquad$ $y - 3x = 2$

$y = -x + 4$ $\qquad$ $y = 3x + 2$

Slope is -1. $\qquad$ Slope is 3.

Answer: Yes, the equations share *one* solution pair.

EXAMPLE 2 Do $7 + y = 2x$ and $y - 2x = 5$ share any solution pairs?

$7 + y = 2x$ $\qquad$ $y - 2x = 5$

$y = 2x - 7$ $\qquad$ $y = 2x + 5$

Slope is 2. $\qquad$ Slope is 2.

Answer: No, the equations *do not* share any solution pairs.

EXAMPLE 3 Do $y - 4 = 2x$ and $y - 2x = 4$ share any solution pairs?

$y - 4 = 2x$ $\qquad$ $y - 2x = 4$

$y = 2x + 4$ $\qquad$ $y = 2x + 4$

The slopes are the same, but so are the equations!

Answer: The equations share *all* solution pairs.

Classroom Practice

Find the slope of the lines whose equations are given.

1. $y = 3x$ 3 **2.** $y = 3x - 5$ 3 **3.** $3x + y = 5$ −3 **4.** $y - 3x = 7$ 3

5. Which equations in Exercises 1–4 name parallel lines? 1, 2, 4

Tell whether the equations share *one* solution pair, *no* solution pair, or *all* solution pairs.

6. $y = x + 6$, $y = -2x$ one **7.** $y = 3x - 9$, $y = 3x + 1$ no **8.** $y - 3x = 3$, $y - 2x = 4$ one **9.** $y - 2x = 3$, $-3 + y = 2x$ all

Written Exercises

Find the slope of the lines whose equations are given.

A **1.** $y = 4x$ 4 **2.** $y = 4x + 3$ 4 **3.** $y = x + 1$ 1 **4.** $y = \frac{1}{3}x + 2$ $\frac{1}{3}$

5. $y - 3x = 6$ 3 **6.** $y + 2x = 4$ −2 **7.** $4x - y = 0$ 4 **8.** $4x + y = 0$ −4

9. $2x - y = 3$ 2 **10.** $y + 5x = 2$ −5 **11.** $y = 8 - 7x$ −7 **12.** $y = x - 5$ 1

Tell whether the equations share *one* solution pair, *no* solution pair, or *all* solution pairs.

13. $y = 2x$, $y = 2x + 3$ no **14.** $y = \frac{1}{3}x$, $y = \frac{1}{3}x + 2$ no **15.** $y + x = 8$, $y + 2x = 8$ one **16.** $x - y = 4$, $x - y = 6$ no

17. $x - y = 8$, $x + y = 6$ one **18.** $2x + y = 9$, $y + 4x = 6$ one **19.** $3x + y = 4$, $y - 3x = 2$ one **20.** $4x - y = 6$, $y - 4x = 7$ no

21. $3x + y = 7$, $y + 3x = 7$ all **22.** $y + 2x = 4$, $2x + y = 8$ no **23.** $y - 4x = 9$, $y - 9 = 4x$ all **24.** $x + y = -3$, $y - 2x = -6$ one

25. $2x + y = 9$, $y + 2x = 7$ no **26.** $y = -3x + 3$, $y - x = 5$ one **27.** $3x - y = 4$, $y + 4 = -3x$ one **28.** $y + 2x = 4$, $y - 2x = -4$ one

B **29. a.** Will the graphs of these equations intersect? no

$$y = 2x + 3$$
$$y = 2x - 1$$

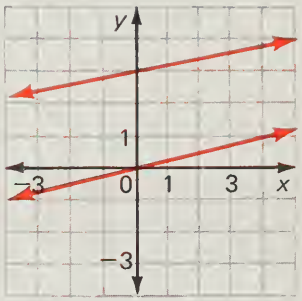
b. Will the graph of $y = -x + 1$ intersect either graph? yes

c. Can the three equations share a solution pair? no

30. a. Are the lines in color at the right parallel? (Check their slopes.) no

b. Will they intersect? yes

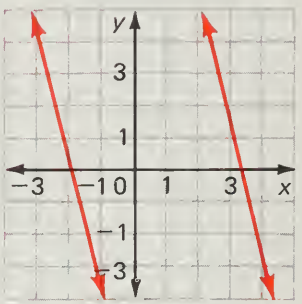
c. Do the equations of the lines share a solution pair? yes



31. a. Are the lines in color at the right parallel? (Check their slopes.) yes

b. Will they intersect? no

c. Do the equations of the lines share a solution pair? no



For more practice, see page 424.

SELF-TEST

By graphing, find the ordered pair which is a solution of both equations. Check student's graphs.

1. $y = 2x + 3$ (3, 9)
 $y = 3x$

2. $y = x + 3$ $\left(1\frac{2}{3}, 4\frac{2}{3}\right)$
 $y = -2x + 8$

3. $y + x = 0$
 $y - x = -2$
 (1, −1)

Find the slope of the lines whose equations are given.

4. $y = 3x + 7$ 3

5. $y - 3x = 0$ 3

6. $4 + y = -x$
 −1

Tell whether the equations share *one* solution pair, *no* solution pair, or *all* solution pairs.

7. $y = 3x - 4$
 $y - 3x = 0$
 no

8. $y - x = 2$
 $y = 2x - 1$
 one

9. $y - 6 = 3x$
 $y - 3x = 6$
 all

3 · The Substitution Method

There are more exact ways of solving a pair of equations than by graphing. Here is one method.

EXAMPLE 1 Solve $y + 1 = 3x$ and $2x + y = 9$.

Step 1: Solve one equation for y. →

$$y + 1 = 3x$$
$$y = 3x - 1$$

Step 2: Substitute the value of y in the other equation. Solve for x. →

$$2x + y = 9$$
$$2x + (3x - 1) = 9$$
$$5x - 1 = 9$$
$$5x = 10$$
$$x = 2$$

Step 3: Substitute your x value in the equation in Step 1. Find the value of y. →

$$y = 3x - 1$$
$$y = 3 \cdot 2 - 1$$
$$y = 5$$

Answer: The solution pair is (2,5).

Check: Is (2,5) a solution of *both* equations? Yes!

$y + 1 = 3x$	
$5 + 1$	$3 \cdot 2$
6	6 ✓

$2x + y = 9$	
$2 \cdot 2 + 5$	9
$4 + 5$	
9	✓

EXAMPLE 2 Solve $x = 2y$ and $x + y = 6$.

The first equation says that $x = 2y$. Substitute $2y$ for x. →

$$x + y = 6$$
$$2y + y = 6$$
$$3y = 6$$
$$y = 2$$

Substitute.

Substitute 2 for y in one of your original equations. →

$$x = 2y$$
$$x = 2 \cdot 2$$
$$x = 4$$

Answer: The solution pair is (4,2). You may check it.

Classroom Practice

Solve by the substitution method.

1. $y = 4x$
$x + y = 10$
(2, 8)

2. $y = 3x$
$x + y = 20$
(5, 15)

3. $y = x - 2$
$x + y = 12$
(7, 5)

4. $x = 3y$
$5y - x = 8$
(12, 4)

5. $x = 2y + 1$
$x + 3y = 16$
(7, 3)

6. $x - y = 2$
$3y + x = 14$
(5, 3)

7. $x + y = 3$
$5x = 3y - 1$
(1, 2)

8. $x - 2y = 5$
$3x + y = -6$
(−1, −3)

Written Exercises

Solve by the substitution method.

A

1. $y = 2x$
$x + y = 9$
(3, 6)

2. $y = 5x$
$y - x = 8$
(2, 10)

3. $x = y + 3$
$x + y = 7$
(5, 2)

4. $y = x + 1$
$x + y = 5$
(2, 3)

5. $y = x + 4$
$x + y = 22$
(9, 13)

6. $x = 2 - y$
$2y + x = 9$
(−5, 7)

7. $y = 3x + 1$
$4x + y = 8$
(1, 4)

8. $y = 2x - 3$
$4x + y = 9$
(2, 1)

9. $x + y = 2$
$3x + y = 8$
(3, −1)

10. $2x + y = 5$
$4x - y = 1$
(1, 3)

11. $3x - y = 13$
$2x + 3y = 16$
(5, 2)

12. $2x - y = 9$
$5x + 2y = 27$
(5, 1)

13. $x = 5y$
$3x = 7y + 16$
(10, 2)

14. $x - 5y = 8$
$4x + 2y = 10$
(3, −1)

15. $a + 2b = 7$
$2a = 3b$
(3, 2)

16. $p - 5q = 6$
$3p - 2q = 5$
(1, −1)

Solve.

Sample $D = rt$
$r = 5t$
Find D in terms of t.

Answer: $D = rt$
$D = (5t)t$ ← Substitute.
$D = 5t^2$

B

17. $A = bh$
$b = 2h$
Find A in terms of h.
$A = 2h^2$

18. $V = Bh$
$B = 2h^2$
Find V in terms of h.
$V = 2h^3$

19. $V = lwh$
$l = 3h$
$w = 2h$
Find V in terms of h.
$V = 6h^3$

Solve for x, y, and z.

C

20. $x + y + z = 180$
$y = 3x$ $x = 20$
$z = 5x$ $y = 60$
$z = 100$

21. $x + y + z = 62$
$x = 2z - 5$ $x = 19$
$y = 3z - 5$ $y = 31$
$z = 12$

22. $x + 2y + 3z = 0$
$2x + y = 6$ $x = 3$
$3x + z = 8$ $y = 0$
$z = -1$

4 • The Add-or-Subtract Method

When solving a pair of equations, you can often add or subtract the equations to get a new equation with just one variable.

EXAMPLE 1 Solve: $\begin{cases} 2x + y = 7 \\ 3x - y = 8 \end{cases}$

Step 1: Add. This makes the y term drop out. →

$$\begin{array}{r} 2x + y = 7 \\ 3x - y = 8 \\ \hline 5x + 0 = 15 \end{array}$$

Step 2: Solve the new equation. →

$$\begin{aligned} 5x &= 15 \\ x &= 3 \end{aligned}$$

Step 3: Substitute 3 for x in one equation. Find the value of y. →

$$\begin{aligned} 2x + y &= 7 \\ 2 \cdot 3 + y &= 7 \\ 6 + y &= 7 \\ y &= 1 \end{aligned}$$

Answer: The solution pair is (3,1). You may check it.

EXAMPLE 2 Solve: $\begin{cases} 5x + 6y = 3 \\ 5x + 2y = 11 \end{cases}$

Step 1: Subtract to make the x term drop out. →

$$\begin{array}{r} 5x + 6y = 3 \\ 5x + 2y = 11 \\ \hline 0 + 4y = -8 \end{array}$$

Step 2: Solve the new equation. →

$$\begin{aligned} 4y &= -8 \\ y &= -2 \end{aligned}$$

Step 3: Substitute -2 for y in one equation. Find the value of x. →

$$\begin{aligned} 5x + 6y &= 3 \\ 5x + 6(-2) &= 3 \\ 5x - 12 &= 3 \\ 5x &= 15 \\ x &= 3 \end{aligned}$$

Answer: The solution pair is $(3,-2)$. You may check it.

Classroom Practice

Would you add or would you subtract the two equations? Does the x term or does the y term drop out?

1. $x + y = 6$ add; $x - y = 2$ y

2. $2x + y = 5$ add; $2x - y = 3$ y

3. $5x + 2y = 9$ sub; $3x + 2y = 7$ y

4. $x + 4y = 5$ sub; $x + y = 2$ x

5. $2x + 3y = 6$ sub; $2x + y = 2$ x

6. $x + 2y = 7$ add; $3x - 2y = 5$ y

7. $x + 3y = 0$ add; $2x - 3y = 9$ y

8. $4x + 2y = 6$ sub; $4x - y = 3$ x

9–16. Solve the pairs of equations in Exercises 1–8.

9. $x = 4$; $y = 2$ 10. $x = 2$; $y = 1$ 11. $x = 1$; $y = 2$ 12. $x = 1$; $y = 1$

13. $x = 0$; $y = 2$ 14. $x = 3$; $y = 2$ 15. $x = 3$; $y = -1$ 16. $x = 1$; $y = 1$

Written Exercises

Solve by the addition method.

A **1.** $x + y = 2$, $x - y = 10$ (6, −4)

2. $5x + 4y = 1$, $3x - 4y = 7$ (1, −1)

3. $4x - y = 8$, $2x + y = -2$ (1, −4)

4. $5p + 3q = 10$, $2p - 3q = 4$ (2, 0)

5. $2a - b = 3$, $4a + b = 9$ (2, 1)

6. $2x + y = 10$, $3x - y = 5$ (3, 4)

7. $3x - 2y = 8$, $x + 2y = 8$ (4, 2)

8. $4r - 7s = 13$, $4r + 7s = -29$ (−2, −3)

Solve by the subtraction method.

9. $x + 6y = 10$, $x + 2y = 2$ (−2, 2)

10. $x - y = 20$, $x - 3y = 10$ (25, 5)

11. $3x + y = 7$, $-2x + y = -8$ (3, −2)

12. $3x - 4y = 21$, $2x - 4y = 18$ (3, −3)

13. $3x + 2y = 18$, $x + 2y = 14$ (2, 6)

14. $2x - 5y = 14$, $2x - 3y = 10$ (2, −2)

15. $2a + 3b = 7$, $2a - b = -5$ (−1, 3)

16. $2s - 5r = 17$, $6s - 5r = 1$ (−4, −5)

Solve by either the addition or subtraction method.

17. $x + y = 9$, $x - y = 5$ (7, 2)

18. $x + 2y = 7$, $3x - 2y = 5$ (3, 2)

19. $2x + 3y = 8$, $2x - y = -8$ (−2, 4)

20. $a + 3b = 6$, $2a + 3b = 9$ (3, 1)

21. $3x - 2y = 13$, $4x + 2y = 8$ (3, −2)

22. $4x - 3y = 9$, $2x - 3y = 3$ (3, 1)

23. $3x + y = 0$, $6x - y = 18$ (2, −6)

24. $4a - 7b = 13$, $2a - 7b = 3$ (5, 1)

25. $4x - 2y = 10$, $-x - 2y = 0$ (2, −1)

26. $-4x + y = 7$, $4x + 3y = 5$ (−1, 3)

27. $8x + 3y = 5$, $x - 3y = 4$ (1, −1)

28. $5x - 2y = 7$, $5x + 3y = 2$ (1, −1)

29. $3a - 12b = 9$, $a - 12b = 11$ (−1, −1)

30. $2x - 11y = 18$, $6x - 11y = 10$ (−2, −2)

31. $6x - 2y = 8$, $5x - 2y = 5$ (3, 5)

32. $4x - y = 15$, $-4x + 3y = -5$ (5, 5)

Can you work with fractions? The following exercises have fractions in the solutions.

Sample

$$\begin{array}{r} 2x + 4y = 2 \\ 6x - 4y = 2 \\ \hline 8x \quad\quad = 4 \\ x = \frac{1}{2} \end{array}$$

Substitute. →

$$\begin{array}{r} 2x + 4y = 2 \\ 2 \cdot \frac{1}{2} + 4y = 2 \\ 1 + 4y = 2 \\ 4y = 1 \\ y = \frac{1}{4} \end{array}$$

Answer: $\left(\frac{1}{2}, \frac{1}{4}\right)$

B **33.** $6x + 9y = 4$
$6x + 3y = 0$
$\left(-\frac{1}{3}, \frac{2}{3}\right)$

34. $5a - b = 3$
$10a + b = 3$
$\left(\frac{2}{5}, -1\right)$

35. $8a + 6b = -1$
$8a - 4b = 4$
$\left(\frac{1}{4}, -\frac{1}{2}\right)$

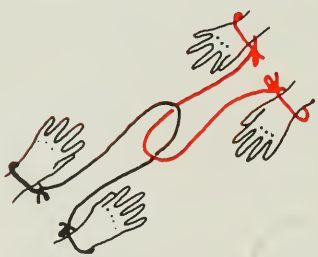
Puzzles & Things

A String Duet

Here is a stunt you might like to try with one of your friends.

Tie a piece of string to your wrists. Tie another piece of string to your friend's wrists so that the two pieces of string interlock as shown. Now try to separate yourself from your friend without cutting the string, untying the knots, or taking the string off your wrists. It can be done!

Pass the center of one string under the string circling the other person's wrist, over that person's hand, then back under the string again.



Topology is the branch of mathematics that can explain why this stunt can be done. Topology deals with figures and how they can be bent and stretched.

Mixed Practice

Solve by the graphing method.

1. $y = -x + 3$
$y = 2x - 1$
$\left(1\frac{1}{3}, 1\frac{2}{3}\right)$

2. $x + y = 0$
$3x + y = -8$
$(-4, 4)$

3. $y + 3x = -7$
$y - 4x = 7$
$(-2, -1)$

4. $x + y = 4$
$2x + y = 4$
$(0, 4)$

Solve by the substitution method.

5. $x = 3y$
$x + y = 12$ $(9, 3)$

6. $y = x + 1$
$x + y = 5$ $(2, 3)$

7. $x + 2y = 2$
$x + 3y = 13$ $(-20, 11)$

8. $y - 2x = 4$
$y + 4x = 16$
$(2, 8)$

Solve by either the addition or subtraction method.

9. $x + y = 7$
$2x - y = 5$ $(4, 3)$

10. $2a + 3b = 5$
$2a + b = 3$ $(1, 1)$

11. $3x - 2y = 1$
$-3x + 4y = 7$ $(3, 4)$

12. $4x - 7y = 9$
$6x - 7y = 3$
$(-3, -3)$

Solve by the method which seems easiest.

13. $y = x + 2$
$2x + y = 11$ $(3, 5)$

14. $x + y = 8$
$x - 3y = 4$ $(7, 1)$

15. $2x + y = 9$
$3x - y = 6$ $(3, 3)$

16. $x + 2y = 5$
$3x + y = 10$
$(3, 1)$

17. $2x - 3y = 8$
$x + 3y = 7$ $\left(5, \frac{2}{3}\right)$

18. $x = 2y$
$x + 3y = 5$ $(2, 1)$

19. $y - 2x = 7$
$y + 3x = 2$ $(-1, 5)$

20. $y - x = 3$
$y = 2x + 4$
$(-1, 2)$

21. $3x - y = 8$
$x + 2y = -2$
$(2, -2)$

22. $2a - 4b = 6$
$-a - 3b = 7$
$(-1, -2)$

23. $x + y = -2$
$2x + y = 4$
$(6, -8)$

24. $x - 5y = 2$
$2x + y = 4$
$(2, 0)$

For more practice, see page 424.

SELF-TEST

Solve by the substitution method.

1. $y = 2x - 1$
$x + 3y = 11$ $(2, 3)$

2. $x = 3y$
$2x = y + 5$ $(3, 1)$

3. $4p = 3q$
$p - q = 2$
$(-6, -8)$

Solve by the addition method.

4. $3x - y = 2$
$x + y = 6$ $(2, 4)$

5. $2a + 3b = -1$
$a - 3b = 4$ $(1, -1)$

6. $3r + 2s = 4$
$4r - 2s = 10$
$(2, -1)$

Solve by the subtraction method.

7. $x + y = 2$
$x - 3y = -6$ $(0, 2)$

8. $s + 2t = 8$
$3s + 2t = 4$ $(-2, 5)$

9. $3x - y = 10$
$x - y = 4$
$(3, -1)$

5 • Word Problems, Two Variables

You know how to solve problems by using one variable. Now you can use two variables in problem solving.

EXAMPLE 1 Art and Lynn were partners in a bowling tournament. In the first game, Lynn's score was 10 more than Art's. Their combined score was 330. Find each person's score.



Let a = Art's score
Let b = Lynn's score

Lynn's score was 10 more than Art's.
$b = 10 + a$

Their combined score was 330.
$a + b = 330$

Now solve the two equations. We'll use the substitution method.

Take your second equation.

$$\begin{cases} b = 10 + a \\ a + b = 330 \end{cases}$$

$$a + b = 330$$

Then substitute.

$$a + (10 + a) = 330$$
$$2a + 10 = 330$$
$$2a = 320$$
$$a = 160$$

Now use one of your original equations.

$$b = 10 + a$$

Then substitute.

$$b = 10 + 160$$
$$b = 170$$

Answer: Art scored 160. Lynn scored 170.

Check: $170 - 160 = 10$ *and* $160 + 170 = 330$ ✓

EXAMPLE 2 The sum of two numbers is 10. ← Fact 1

One number is 4 more than twice the other number. ← Fact 2

Find the numbers.

Let x = one number.
Let y = the other number.

using Fact 1 → $x + y = 10$

using Fact 2 → $x = 4 + 2y$

Take your first equation.

Then substitute.

$$x + y = 10$$
$$(4 + 2y) + y = 10$$
$$4 + 3y = 10$$
$$3y = 6$$
$$y = 2$$

Now use one of your original equations.

Then substitute.

$$x = 4 + 2y$$
$$x = 4 + 2 \cdot 2$$
$$x = 8$$

Answer: 8 and 2
Check: $8 + 2 = 10$ *and* $8 = 4 + 2 \cdot 2$ ✓

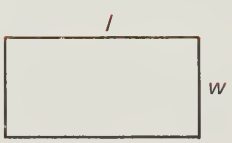
Classroom Practice

Solve. Use two variables and two equations.

1. One number is 4 more than another.
The sum of the numbers is 20.
Find the numbers. 8, 12

2. The sum of two numbers is 18.
The difference of the numbers is 12.
Find the numbers. 15, 3

3. The length of a rectangle is twice the width.
The perimeter is 24.
What are the length and the width? length: 8
width: 4



$l = 2w$
$2l + 2w = 24$
$6w = 24$
$w = 4$ $l = 8$

4. Cindy is 5 years older than Marie.
Their ages total 33.
How old is each person? Cindy: 19; Marie: 14

5. The Jets scored 4 points more than the Vets.
The total of their scores was 38.
How many points did each team score?
Jets: 21; Vets: 17



Written Exercises

Solve. Use two variables and two equations.

A

1. The sum of two numbers is 21.
Their difference is 13.
Find the numbers. 17, 4

2. The sum of two numbers is 35.
One number is 4 times the other.
Find the numbers. 28, 7

3. One number is 5 more than another number.
The sum of the two numbers is 43.
Find the numbers. 24, 19

4. One number is 5 times another number.
It is also 3 more than twice the other number.
Find the numbers. 5, 1

5. Porky is 30 kilograms more than Bugs.
Together they are 100 kilograms.
How much is each? Bugs: 35 kg; Porky: 65 kg

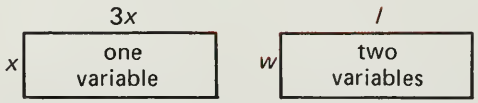
6. Ellen is 5 years older than Trudy.
Their ages total 39.
How old is each person? Ellen: 22; Trudy: 17

7. In a game of bowling, Bud's score was 12 more than Jan's.
Their combined score was 328.
Find their scores. Bud: 170; Jan: 158

8. My English class has 29 students.
There are 3 more girls than boys.
How many girls are there? 16 girls

Solve, using one variable. Then solve again using two variables.

B **9.** The length of a rectangle is three times the width.
The perimeter is 48.
Find the length and the width. length: 18; width: 6

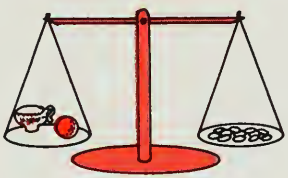


10. The Dodgers beat the Expos by 3 runs.
There was a total of 19 runs scored in the game.
How many runs did each team score? Dodgers: 11 runs; Expos: 8 runs

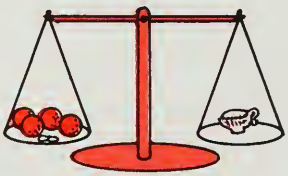
11. A bottle of cola costs 25¢.
The deposit on the bottle is 15¢ less than the price of the cola.
How much is the deposit? 5¢

12. I'm thinking of two numbers whose difference is 20.
Twice one number equals three times the other.
Find the numbers. 40, 60

Puzzles & Things



1 ball and 1 cup
balance 12 pennies.



4 balls and 2 pennies
balance the cup.



How many pennies will balance the cup? 10

6 • Using Multiplication

In solving pairs of equations, when the terms in one variable are the same, you know that you can use the addition-or-subtraction method of solving.

Use addition to make the y terms drop out.

$$\begin{aligned} x + 2y &= 7 \\ 3x - 2y &= 5 \end{aligned}$$

Use subtraction to make the x terms drop out.

$$\begin{aligned} 5x + 6y &= 2 \\ 5x + 3y &= -4 \end{aligned}$$

Here's what to do if the terms aren't alike. Just use multiplication to rename one or both equations.

EXAMPLE Solve: $\begin{cases} 2x + y = 5 \\ 5x + 3y = 12 \end{cases}$

Multiply both sides of the equation by 3. This matches up the y coefficients so you can subtract.

$$\begin{aligned} 6x + 3y &= 15 \\ 5x + 3y &= 12 \\ \hline x + 0 &= 3 \\ x &= 3 \end{aligned}$$

Now use one of your original equations.

Then substitute.

$$\begin{aligned} 2x + y &= 5 \\ 2 \cdot 3 + y &= 5 \\ 6 + y &= 5 \\ y &= -1 \end{aligned}$$

Answer: The solution pair is $(3, -1)$.

Check: Is $(3, -1)$ a solution of *both* equations?

$2x + y = 5$	
$2 \cdot 3 + (-1)$	5
$6 - 1$	
5	

$5x + 3y = 12$	
$5 \cdot 3 + 3(-1)$	12
$15 - 3$	
12	

Classroom Practice

Solve.

1. $2x + y = 8$
$3x - 2y = 5$ (3, 2)

2. $3x + 5y = 3$
$x + 2y = 13$ (−59, 36)

3. $2x + y = 5$
$3x - 2y = 4$ (2, 1)

4. $a - 3b = 5$
$3a + 2b = 4$
(2, −1)

5. $3a + b = 4$
$a - 2b = 6$ (2, −2)

6. $x + y = 7$
$3x - 2y = 11$ (5, 2)

7. $3c - 4d = -7$
$-c + 5d = 6$ (−1, 1)

8. $4x - y = 6$
$2x - 3y = 8$
(1, −2)

Written Exercises

Solve.

A

1. $a + b = 9$
$2a - 3b = 3$
(6, 3)

2. $x - 4y = 5$
$2x - 7y = 9$
(1, −1)

3. $y - 3x = 9$
$2y + x = 4$
(−2, 3)

4. $a + 3b = 16$
$2a - b = 4$
(4, 4)

5. $3m + n = 10$
$m + 2n = 10$
(2, 4)

6. $3x - y = 5$
$x + 2y = 11$
(3, 4)

7. $3x - 4y = 1$
$-x - 2y = 3$
(−1, −1)

8. $x - 4y = 3$
$-2x + y = 8$
(−5, −2)

9. $2x - 3y = 6$
$x - 2y = 3$
(3, 0)

10. $6x - 5y = 7$
$2x + 2y = -16$
(−3, −5)

11. $9x + 6y = 0$
$3x + 5y = -9$
(2, −3)

12. $5m - 2n = 14$
$3m + 4n = -2$
(2, −2)

13. $3a + 2b = 6$
$-4a + 6b = 44$
(−2, 6)

14. $5a + 4b = 7$
$3a + 2b = 3$
(−1, 3)

15. $5x - 9y = -3$
$4x - 3y = 6$
(3, 2)

16. $5m + 3n = 2$
$10m - 2n = 12$
(1, −1)

Solve. You'll need to multiply *both* equations, instead of one.

Sample $2x + 7y = 17$ — Multiply by 3. → $6x + 21y = 51$

$3x + 5y = 9$ — Multiply by 2. → $6x + 10y = 18$

$11y = 33$

$y = 3$

You may finish the solution.

B

17. $2c - 3d = -1$
$3c - 4d = -3$ (−5, −3)

18. $3r - 2s = 15$
$7r - 3s = 15$ (−3, −12)

19. $2p + 3q = -1$
$3p + 5q = -2$
(1, −1)

20. $5a - 2b = 1$
$-2a + 3b = 4$ (1, 2)

21. $2x + 4y = 2$
$3x + 5y = 2$ (−1, 1)

22. $3x - 2y = -5$
$-4x + 3y = 8$
(1, 4)

7 • Cost Problems

The cost problems in Chapter 4 involved buying or selling just one thing, like hot dogs. Now that you can work with 2 variables, you can solve cost problems involving 2 things, like hot dogs and root beer.

EXAMPLE 1 Bill buys 2 hot dogs and a root beer for \$1.20.
Rosie buys 3 hot dogs and 2 root beers for \$1.95.
How much does each item cost?



Let h = cost of a hot dog, in cents.
Let r = cost of a root beer, in cents.

2 hot dogs + 1 root beer cost 120¢. → $2h + r = 120$

3 hot dogs + 2 root beers cost 195¢. → $3h + 2r = 195$

Multiply both sides by 2.

$$\begin{aligned} 4h + 2r &= 240 \\ 3h + 2r &= 195 \\ \hline h + 0 &= 45 \\ h &= 45 \end{aligned}$$

Now use one of your original equations. → $2h + r = 120$

Then substitute.

$$\begin{aligned} 2 \cdot 45 + r &= 120 \\ 90 + r &= 120 \\ r &= 30 \end{aligned}$$

Answer: A hot dog costs 45¢. A root beer costs 30¢.

Check: 2 hot dogs + 1 root beer: $90 + 30 = 120$ ✓
3 hot dogs + 2 root beers: $135 + 60 = 195$ ✓

Written Exercises

A **1.** 2 hot dogs and 1 cola cost \$1.40. ⟶ $2h + c = 140$
1 hot dog and 2 colas cost \$1.30. ⟶ ___?___ $h + 2c = 130$
How much does each item cost? hot dog: 50¢; cola: 40¢

2. 3 hamburgers and a milkshake cost \$2.30. ⟶ $3h + m = 230$
2 hamburgers and a milkshake cost \$1.70. ⟶ ___?___ $2h + m = 170$
How much does each item cost? hamburger: 60¢; milkshake: 50¢

3. Two adult tickets and 3 student tickets cost \$12. $\longrightarrow$? $2a + 3s = 12$
One adult ticket and 2 student tickets cost \$7. $\longrightarrow$? $a + 2s = 7$
How much does each kind of ticket cost? adult: \$3; student: \$2

4. Two bags of popcorn and 3 bags of potato chips cost \$1.80. $2p + 3c = 180$
Three bags of popcorn and 2 bags of potato chips cost \$1.70. $3p + 2c = 170$
Find the price of the items. popcorn: 30¢; chips: 40¢

5. Four ice cream cones and 3 milkshakes cost \$3.15. $4i + 3m = 315$
Six ice cream cones and 1 milkshake cost \$2.45. $6i + m = 245$
Find the cost of the items. ice cream: 30¢; milkshake: 65¢

6. Lea paid \$5 for two adult tickets and 1 student ticket. $2a + s = 5$
Paid paid \$6 for 1 adult ticket and 4 student tickets. $a + 4s = 1$
Find the price of each kind of ticket. adult: \$2; student: \$1

B 7. A baseball team bought 20 baseballs.
Some cost \$2 each, and some cost \$3 each.
The total cost was \$48.
Complete the table below. Find x and y. $x = 8, y = 12$

	price × number = cost		
Expensive balls	3	x	$3x$?
Cheaper balls	2	y	$2y$?
		20	48 ?

For more practice, see page 425.

SELF-TEST

Solve.

1. $4x + 4y = 8$
$2x + 3y = 4$ (2, 0)

2. $3n - 12m = 18$
$2n + 2m = 0$ $\left(-1\frac{1}{5}, 1\frac{1}{5}\right)$

3. $x + 2y = 0$
$-3x + 4y = 10$ (−2, 1)

Solve. Use two variables and two equations.

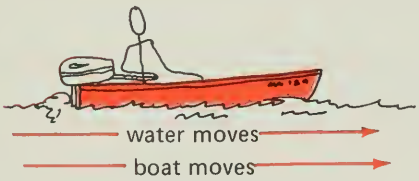
4. Yvette is 3 years younger than Lauren.
Their ages total 33.
How old is each person? Yvette: 15; Lauren: 18

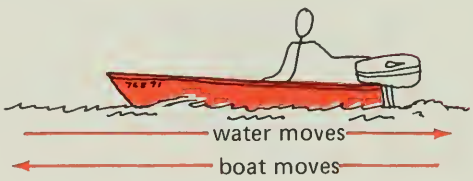
5. Four hamburgers and 2 milkshakes cost \$5.00.
Two hamburgers and 3 milkshakes cost \$3.70.
How much does each item cost? hamburgers: 95¢; milkshake: 60¢

8 • Boat and Aircraft Problems (Optional)

You probably know that it is easier for a boat to move downstream on a river than it is for it to go upstream. This is because of the moving water.

EXAMPLE A motorboat went 18 km downstream in 1 hour. The return trip took 3 hours. Find the rate of the flowing water.





Let x = rate of the boat in still water.
Let y = rate of the flowing water.

	rate	× time	= Distance	
Downstream	$x + y$	1	18	$1(x + y) = 18$
Upstream	$x - y$	3	18	$3(x - y) = 18$

$$x + y = 18$$
$$3x - 3y = 18$$

Simplify the equations.

Multiply both sides of the first equation by 3.

$$3x + 3y = 54$$
$$3x - 3y = 18$$
$$6x + 0 = 72$$
$$x = 12$$

Now use one of your original equations.

Then substitute.

$$x + y = 18$$
$$12 + y = 18$$
$$y = 6$$

Answer: The water was flowing at 6 km/h.

Check: downstream rate:
$x + y = 12 + 6 = 18$
or 18 km in 1 hour ✓

upstream rate:
$x - y = 12 - 6 = 6$
or 6 km in 1 hour
or 18 km in 3 hours ✓

Written Exercises

1. A motorboat can go 12 kilometers downstream in 2 hours.
The return trip takes 3 hours.
Find the speed of the boat in still water. 5 km/hr

2. A motorboat can go 30 kilometers downstream in 3 hours.
The return trip takes 5 hours.
Find the speed of the boat in still water. 8 km/hr

Aircraft problems are like motorboat problems. Flying *with* the wind is easier than flying *into* the wind.

3. An airplane takes 3 hr flying 1200 km into the wind.
With the same wind, the return trip takes 2 hr.
What is the speed of the wind? 100 km/hr

Let x = airplane's rate with no wind.
Let y = the wind speed.

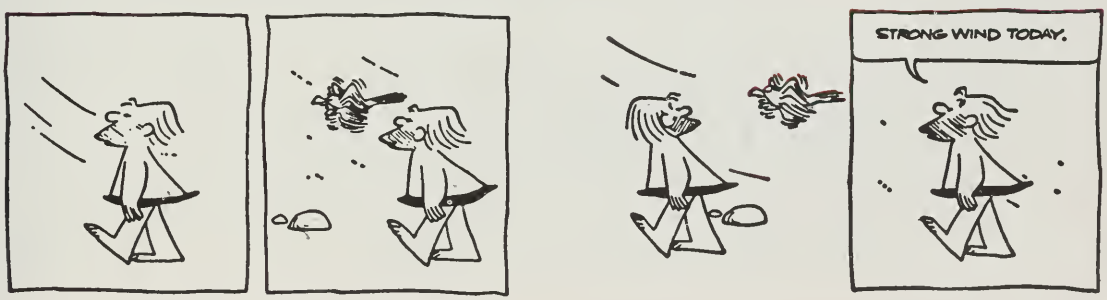
	rate × time = Distance		
Flying *with* the wind	$x + y$	2	1200
Flying *into* the wind	$x - y$	3	1200

4. Look at the cartoon below.

Let x = speed of bird if there were no wind.
Let y = speed of wind.

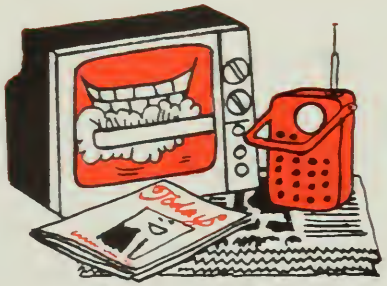
a. Is $x - y$ a positive number or a negative number? negative

b. What would the cartoon be like if $x - y = 0$? The bird would remain in one place.



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CONSUMER CORNER



Polls and Surveys

We have all heard advertising claims like this one:

"The results of a test by an independent laboratory show that 60% of Glisten users have fewer cavities."

Can you believe the claim? Not all polls, surveys, and tests are done in valid ways. Some results, therefore, are worthless. Here is how the tests for Glisten toothpaste might have been conducted.

Test 1

The Test
Have 10 people keep count of their cavities for six months.
Have them switch to Glisten for six months and count their cavities.

The Results
6 people have the same number of cavities.
2 people have fewer cavities with Glisten.
2 people have more cavities with Glisten.

The Report
File the report in the wastebasket and start over again!

Test 2

The Test
Conduct the same test as Test 1, with 10 different people.

The Results
3 people have the same number of cavities.
6 people have fewer cavities.
1 person has more cavities.

The Report
"60% of Glisten users have fewer cavities."

The Moral of the Story

The chances are very good that if you keep testing groups of only 10 people, sooner or later you will find a test group with results favorable for Glisten toothpaste. This is true, even if Glisten is worthless. If the test group has a large number of people, however, Glisten is unlikely to show up better unless it really is better.

The importance of large numbers for accurate results also applies to political polls and surveys.

	Poll 1	Poll 2
Results	6 people favor Chung 4 people favor Bryant	6000 people favor Chung 4000 people favor Bryant
Claim	60% of the population favors Chung.	60% of the population favors Chung.

Worthless claim!
Only 10 people were polled.

Valuable claim.
A total of 10,000 people were polled.

Of course, companies that take nationwide polls must consider the *whole* country, different age groups, and so on.

Try this experiment.

Toss ten coins in the air. Let each "head" be a vote for Glisten toothpaste. Each "tail" can then be a vote against Glisten. How many times do you have to toss the ten coins until Glisten scores 7 out of 10 votes? (Have you ever heard the claim, "Seven out of 10 doctors recommend. . ."?)

Reviewing the Chapter

Solve by the graphing method. *(See pp. 236–239.)* Check student's graphs.

1. $y = x + 2$
 $y = 2x$ (2, 4)

2. $y + x = 3$
 $y - x = 1$ (1, 2)

3. $y = 2x + 5$
 $y = -x - 1$ (−2, 1)

4. $3x + y = 0$
 $y = 2x + 5$ (−1, 3)

5. $x + y = -2$
 $y - x = -6$ (2, −4)

6. $y = 4x + 1$
 $y - 5x = 1$ (0, 1)

Find the slope of the line whose equation is given. *(See pp. 240–243.)*

7. $y = 9x$ 9

8. $y = -x + 3$ −1

9. $y = \frac{1}{2}x + 4$ $\frac{1}{2}$

10. $y + 2x = 0$ −2

11. $y - x = 9$ 1

12. $\frac{2}{3}x - y = 2$ $\frac{2}{3}$

13. $5x + y = 4$ −5

14. $2x - y = 11$ 2

15. $y - \frac{1}{3}x = 0$ $\frac{1}{3}$

Tell whether the equations share *one* solution pair, *no* solution pair, or *all* solution pairs. *(See pp. 240–243.)*

16. $y = 4x$
 $y = 4x + 3$ no

17. $y = x + 4$
 $y + x = -4$ one

18. $y + 5x = 3$
 $y - 3 = -5x$ all

19. $y - 7x = 2$
 $y + 2 = -7x$ one

20. $y - 6x = 2$
 $y = 6x + 5$ no

21. $y + 2x = 6$
 $y = 6 + 2x$ one

Solve by the substitution method. *(See pp. 244–245.)*

22. $y = 3x$
 $x + y = 12$ (3, 9)

23. $y = -4x$
 $x - y = -10$ (−2, 8)

24. $y = 2x + 1$
 $3x + y = 6$ (1, 3)

25. $5x - y = 0$
 $x + 2y = 22$ (2, 10)

26. $3x - y = 1$
 $2x + 3y = 8$ (1, 2)

27. $y + 4x = 5$
 $5x + y = 0$ (−5, 25)

Solve by the addition method. *(See pp. 246–249.)*

28. $2x + y = 0$
 $x - y = 6$ (2, −4)

29. $-3x + 2y = 5$
 $3x + y = 7$ (1, 4)

30. $x + 4y = 16$
 $-x + 3y = -2$ (8, 2)

31. $x + 3y = -1$
 $-2x - 3y = 5$ (−4, 1)

32. $-7x - 3y = -4$
 $6x + 3y = 3$ (1, −1)

33. $3x + 2y = 5$
 $-3x + 2y = -1$ (1, 1)

Solve by the subtraction method. *(See pp. 246–249.)*

34. $x + 2y = 7$
$x - y = 1$ (3, 2)

35. $2x + 3y = 7$
$x + 3y = 2$ (5, −1)

36. $2x - y = 8$
$2x + 2y = 14$ (5, 2)

37. $x - y = 3$
$2x - y = 5$ (2, −1)

38. $5x + 2y = -18$
$3x + 2y = -6$ (−6, 6)

39. $4x - 3y = 1$
$6x - 3y = 3$ (1, 1)

Solve. *(See pp. 244–249, 254–255.)*

40. $2x + 3y = 12$
$4x + y = 14$ (3, 2)

41. $5x - 2y = 11$
$3x - 6y = -3$ (3, 2)

42. $3x + 4y = 1$
$6x + 5y = -1$ (−1, 1)

43. $3x - 2y = 0$
$7x - 8y = 10$ (−2, −3)

44. $3x - 4y = -1$
$5x + 8y = 13$ (1, 1)

45. $9x - 2y = -3$ (1, 6)
$-3x + 5y = 27$

46. $5x + 2y = 9$
$-5x - 6y = 3$ (3, −3)

47. $2x - 3y = 9$
$4x - 5y = 21$ (9, 3)

48. $4x + 5y = 14$ (1, 2)
$-12x + 3y = -6$

Solve. Use two variables and two equations. *(See pp. 250–253, 256–257.)*

49. The sum of two numbers is 14.
Their difference is 16.
Find the numbers. 15, −1

50. One number is 10 more than another.
The sum of the two numbers is 6.
Find the numbers. 8, −2

51. The difference of two numbers is 4.
One number is 3 times the other.
Find the numbers. 6, 2

52. One number is 6 less than another.
The sum of the two numbers is zero.
Find the numbers. 3, −3

53. One pizza and 4 root beers cost $3.40.
Two pizzas and 6 root beers cost $6.10.
How much does each item cost? pizza: $2.00; root beer: $.35

54. Three albums and 2 tapes cost $36.
Four albums and 1 tape cost $33.
How much does each item cost? album: $6.00; tape: $9.00

55. One adult ticket and 3 student tickets cost $6.50.
Three adult tickets and 2 student tickets cost $9.00.
How much does each kind of ticket cost? adult: $2.00; student: $1.50

56. Two milkshakes and 1 banana split cost $2.25.
Three milkshakes and 2 banana splits cost $3.85.
How much does each item cost? milkshake: $.65; banana split: $.95

A permission-to-reproduce chapter test can be found on page T18.

Cumulative Review

Solve.

1. $x + 11 = 27$ 16
2. $m - 9 = 15$ 24
3. $8 - x = 5$ 3
4. $7a = -56$ −8
5. $4b - 9 = 7$ 4
6. $2(n + 3) = -10$ −8
7. $5a - 8a = 18$ −6
8. $7x + 12 = 9x$ 6
9. $11y - 8 = 3y$ 1
10. $3(t - 4) = t$ 6
11. $2x + 3 = 4x - 1$ 2
12. $-x + 4 = 3x + 8$ −1

Simplify.

13. $a(7a)(-2b)$ $-14a^2b$
14. $7(-x^2 + y) - 11y$ $-7x^2 - 4y$
15. $(m + 2) + (3m - 1)$ $4m + 1$
16. $(2x - 3) - (x + 4)$ $x - 7$
17. $(n + 4) - (3n - 5)$ $-2n + 9$
18. $(3a^2 - 4a) + (a^2 + 3)$ $4a^2 - 4a + 3$
19. $(b^2 - 3) - (4b + 7)$ $b^2 - 4b - 10$
20. $3m - n - (m + n)$ $2m - 2n$
21. $2x^2 - 1 - (4x^2 + 2)$ $-2x^2 - 3$
22. $\dfrac{3a^2b}{3b}$ a^2
23. $\dfrac{16m^2n^2}{-4n^2}$ $-4m^2$
24. $\dfrac{x^2 + 5xy}{x}$ $x + 5y$

Multiply.

25. $(m^2n)^2$ m^4n^2
26. $x(4x + 3)$ $4x^2 + 3x$
27. $y(y^2 - 2y + 4)$ $y^3 - 2y^2 + 4y$
28. $(x + 2)(x + 9)$ $x^2 + 11x + 18$
29. $(m + 2)(3m + 1)$ $3m^2 + 7m + 2$
30. $(5y - 7)(y + 1)$ $5y^2 - 2y - 7$
31. $(5x - 8)(5x + 8)$ $25x^2 - 64$
32. $(2x + 4y)(3x + y)$ $6x^2 + 14xy + 4y^2$
33. $(2m - 3n)(5m + n)$ $10m^2 - 13mn - 3n^2$

Factor.

34. $x^2 - 4x$ $x(x - 4)$
35. $2m^2 - 6m$ $2m(m - 3)$
36. $a^3 + 9a^2 - 3a$ $a(a^2 + 9a - 3)$
37. $y^2 + 3y + 2$ $(y + 1)(y + 2)$
38. $x^2 + 6x + 8$ $(x + 4)(x + 2)$
39. $a^2 + 4a + 4$ $(a + 2)^2$
40. $m^2 + m - 12$ $(m - 3)(m + 4)$
41. $x^2 - 6x + 9$ $(x - 3)^2$
42. $y^2 - 2y - 15$ $(y + 3)(y - 5)$
43. $a^2 - a - 6$ $(a + 2)(a - 3)$
44. $x^2 - y^2$ $(x + y)(x - y)$
45. $16a^2 - 9b^2$ $(4a + 3b)(4a - 3b)$

Draw the graph of the equation. Check student's graphs.

46. $y = x + 4$
47. $x + y = -3$
48. $2x + y = 0$
49. $2x - y = 4$

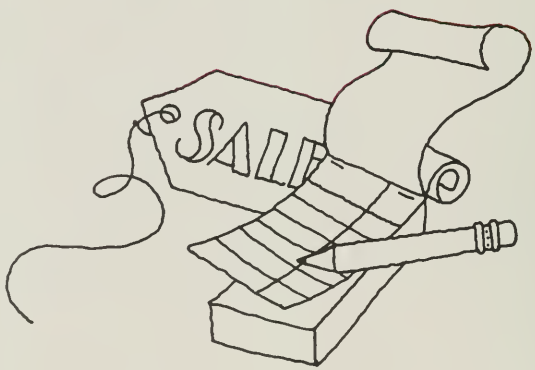
Solve.

50. $\begin{cases} x + 2y = 10 \\ 5x + y = 14 \end{cases}$ (2, 4)
51. $\begin{cases} 3x - y = 13 \\ 6x + y = 23 \end{cases}$ (4, −1)
52. $\begin{cases} 2x + 3y = 0 \\ 3x - 2y = -13 \end{cases}$ (−3, 2)

UNIT E

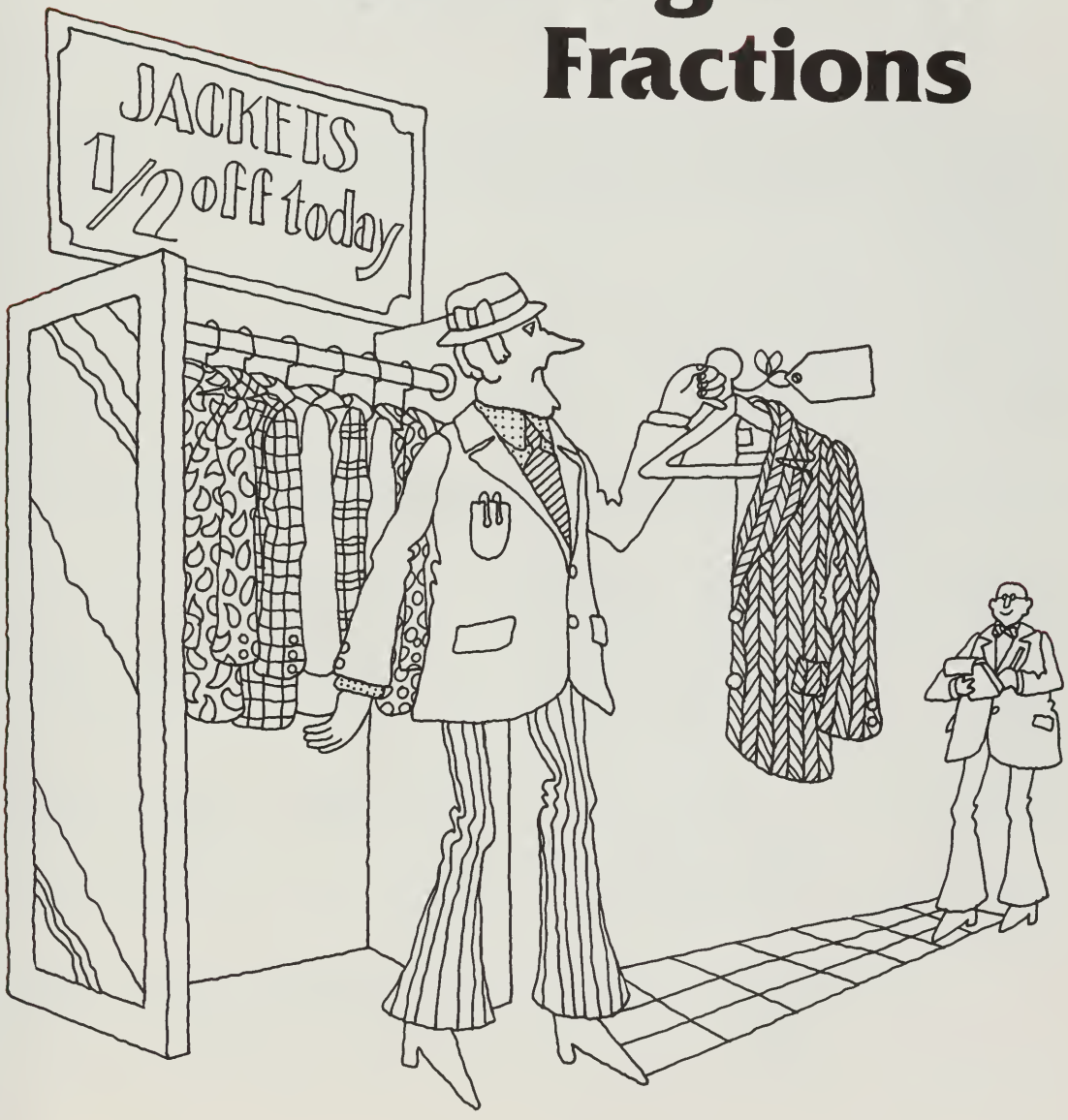
Here's what you'll learn in this chapter:

1. To use factoring to simplify fractions.
2. To solve word problems involving ratios and proportions.
3. To add, subtract, multiply, and divide fractions.
4. To solve equations with fractions.
5. To solve work problems.



Chapter 9

Working With Fractions



1 • Fractions in Algebra

Working with fractions in algebra is like working with fractions in arithmetic. You already know that fractions can be used to express division.

In Arithmetic	*In Algebra*
$\frac{5}{6}$ ← Numerator, Denominator	$\frac{2}{y}$ ← Numerator, Denominator
	The denominator cannot be 0. $y \neq 0$

You can simplify a fraction by dividing both its numerator and denominator by the same number. Keep in mind that when you simplify a fraction, you do not find a new number. You just find a simpler way of writing it.

$$\frac{8}{12} = \frac{2 \cdot \cancel{4}^{1}}{3 \cdot \cancel{4}_{1}} = \frac{2}{3}$$

Both numerator and denominator are divided by 4.

$$\frac{2a}{6a} = \frac{\cancel{2}^{1} \cdot \cancel{a}^{1}}{3 \cdot \cancel{2}_{1} \cdot \cancel{a}_{1}} = \frac{1}{3}$$

Both numerator and denominator are divided by $2a$.

Study a few more examples. Watch the signs.

EXAMPLE 1 $\frac{-6x}{12x} = \frac{\cancel{-6}^{-1} \cdot \cancel{x}^{1}}{\cancel{12}_{2} \cdot \cancel{x}_{1}} = \frac{-1}{2} = -\frac{1}{2}$

negative ÷ positive = negative

$$\frac{-1}{2} = -\frac{1}{2}$$

EXAMPLE 2 $\frac{4a}{-20a} = \frac{\cancel{4}^{1} \cdot \cancel{a}^{1}}{\cancel{-20}_{-5} \cdot \cancel{a}_{1}} = \frac{1}{-5} = -\frac{1}{5}$

positive ÷ negative = negative

$$\frac{1}{-5} = -\frac{1}{5}$$

EXAMPLE 3 $\frac{-3xy}{-6xy^2} = \frac{\cancel{-3}^{-1} \cdot \cancel{x}^{1} \cdot \cancel{y}^{1}}{\cancel{-6}_{-2} \cdot \cancel{x}_{1} \cdot \cancel{y}_{1} \cdot y} = \frac{1}{2y}$

negative ÷ negative = positive

$$\frac{-1}{-2y} = \frac{1}{2y}$$

Classroom Practice

Simplify.

1. $\frac{6}{8}$ $\frac{3}{4}$
2. $\frac{2}{14}$ $\frac{1}{7}$
3. $\frac{-3}{6x}$ $-\frac{1}{2x}$
4. $\frac{4xy}{-8x}$ $\frac{-y}{2}$
5. $\frac{a}{ab}$ $\frac{1}{b}$
6. $\frac{x^3}{x}$ x^2
7. $\frac{abc}{bcd}$ $\frac{a}{d}$
8. $\frac{-9y^2}{-18xy}$ $\frac{y}{2x}$
9. $\frac{8xy^2}{24xy}$ $\frac{y}{3}$
10. $\frac{24r^2s}{24r^2s^3}$ $\frac{1}{s^2}$

Written Exercises

Simplify.

A

1. $\frac{10}{15}$ $\frac{2}{3}$
2. $\frac{9}{12}$ $\frac{3}{4}$
3. $\frac{3}{18a}$ $\frac{1}{6a}$
4. $\frac{7}{21g}$ $\frac{1}{3g}$
5. $\frac{-5x}{10x}$ $-\frac{1}{2}$
6. $\frac{3xy}{9y}$ $\frac{x}{3}$
7. $\frac{7ab}{14ab}$ $\frac{1}{2}$
8. $\frac{4x}{12xy}$ $\frac{1}{3y}$
9. $\frac{-8x^2}{6x}$ $\frac{-4x}{3}$
10. $\frac{12abc}{9bcd}$ $\frac{4a}{3d}$
11. $\frac{28r}{21r^2}$ $\frac{4}{3r}$
12. $\frac{6a}{18a^2}$ $\frac{1}{3a}$
13. $\frac{4ab^2}{12b}$ $\frac{ab}{3}$
14. $\frac{-9st}{-6s^2}$ $\frac{3t}{2s}$
15. $-\frac{3a^2b}{15b}$ $-\frac{a^2}{5}$
16. $\frac{5x^2y}{10xy^2}$ $\frac{x}{2y}$
17. $\frac{8ab}{4a^2b}$ $\frac{2}{a}$
18. $\frac{9x^2y}{15x}$ $\frac{3xy}{5}$
19. $\frac{8a^2x}{4ax^2}$ $\frac{2a}{x}$
20. $\frac{9mn^2}{7n^2}$ $\frac{9m}{7}$
21. $\frac{10x^2y}{25y^2}$ $\frac{2x^2}{5y}$
22. $\frac{-6rs}{15r^2s}$ $-\frac{2}{5r}$
23. $\frac{10m^2k}{8mk}$ $\frac{5m}{4}$
24. $\frac{9n^2r}{24nr^2}$ $\frac{3n}{8r}$
25. $\frac{4pq^2}{32qr}$ $\frac{pq}{8r}$
26. $\frac{7a}{28ab^2c}$ $\frac{1}{4b^2c}$
27. $\frac{6y^2z}{-3y}$ $-2yz$
28. $\frac{12a^2b^4}{4a^3b}$ $\frac{3b^3}{a}$
29. $\frac{10x^2w^3}{35xw^4}$ $\frac{2x}{7w}$
30. $\frac{12y^3t^3}{19y^2t^4}$ $\frac{12y}{19t}$

B

31. $\frac{125a^2b^3c}{-25a^3bc^2}$ $\frac{-5b^2}{ac}$
32. $\frac{-76xy^2z^3}{19x^2z}$ $-\frac{4y^2z^2}{x}$
33. $\frac{12rst^3}{144r^2s^3t}$ $\frac{t^2}{12rs^2}$
34. $\frac{-21x^2yz^3}{-84xy^2z^2}$ $\frac{xz}{4y}$
35. $\frac{121ab^2c^3}{-11a^2b^3}$ $-\frac{11c^3}{ab}$
36. $\frac{-13r^3s^2t}{104rs^3t^2}$ $\frac{-r^2}{8st}$
37. $\frac{102x^3y^2}{-18x^2y^3z}$ $\frac{17x}{3yz}$
38. $\frac{-24a^3b^2c^2}{-168a^2b^3c}$ $\frac{9c}{7b}$

A fraction has no meaning if the denominator is zero. State the value of x for which the fraction has no meaning.

39. $\frac{7}{x}$ 0
40. $\frac{4}{x-1}$ 1
41. $\frac{3}{x-2}$ 2
42. $\frac{2x}{x+1}$ -1
43. $\frac{8}{3x-6}$ 2

2 • Simplifying Fractions

Fractions having several terms in the numerator or denominator can often be simplified too.

Factor the numerator.

$$\frac{4x-8}{8} = \frac{\overset{1}{\cancel{4}}(x-2)}{\underset{2}{\cancel{8}}} = \frac{x-2}{2}$$

WARNING! In examples like these, be sure to write out the factors first.

Correct

$$\frac{4x-8}{8} = \frac{\overset{1}{\cancel{4}}(x-2)}{\underset{2}{\cancel{8}}} = \frac{x-2}{2}$$

Not Correct

$$\frac{\overset{1}{\cancel{4x-8}}}{\underset{1}{\cancel{8}}} = \cancel{\frac{4x-1}{1}} = 4x-1$$

Here are a few more examples to study.

EXAMPLE 1

$$\frac{x^2-3x}{2x-6} = \frac{x(x-3)}{2(x-3)}$$

$$= \frac{x\overset{1}{\cancel{(x-3)}}}{2\underset{1}{\cancel{(x-3)}}}$$

Divide the numerator and denominator by $(x-3)$.

$$= \frac{x}{2}$$

EXAMPLE 2

$$\frac{3t-15}{t^2-25} = \frac{3(t-5)}{(t-5)(t+5)}$$

$$= \frac{3}{t+5}$$

Classroom Practice

Simplify.

1. $\frac{3a + 3b}{3a}$ $\frac{a + b}{a}$
2. $\frac{x + 1}{2x + 2}$ $\frac{1}{2}$
3. $\frac{y - 4}{2y - 8}$ $\frac{1}{2}$
4. $\frac{x^2 - 1}{x - 1}$ $x + 1$
5. $\frac{x^2 + x}{x + 1}$ x
6. $\frac{a - b}{2a - 2b}$ $\frac{1}{2}$
7. $\frac{a^2 - 4}{a + 2}$ $a - 2$
8. $\frac{x^2 + 6x + 9}{x^2 - 9}$ $\frac{x + 3}{x - 3}$
9. For what values of x does the fraction $\frac{5}{x^2 - 4}$ have no meaning? 2, −2

Written Exercises

Simplify.

A

1. $\frac{x + 1}{3x + 3}$ $\frac{1}{3}$
2. $\frac{x - 1}{2x - 2}$ $\frac{1}{2}$
3. $\frac{3p + 3q}{7p + 7q}$ $\frac{3}{7}$
4. $\frac{2a - 2b}{2a + 2b}$ $\frac{a - b}{a + b}$
5. $\frac{x^2 + 2x}{x + 2}$ x
6. $\frac{a + 1}{a^2 + a}$ $\frac{1}{a}$
7. $\frac{3y}{9y^2 - 6y}$ $\frac{1}{3y - 2}$
8. $\frac{6a + 6b}{9a + 9b}$ $\frac{2}{3}$
9. $\frac{2c + 6}{c^2 - 9}$ $\frac{2}{c - 3}$
10. $\frac{a^2 - 4}{a - 2}$ $a + 2$
11. $\frac{2x + 10}{x^2 - 25}$ $\frac{2}{x - 5}$
12. $\frac{x^2 - y^2}{x + y}$ $x - y$
13. $\frac{n^2 - 9}{n^2 + 3n}$ $\frac{n - 3}{n}$
14. $\frac{y^2 - 4y}{2y - 8}$ $\frac{y}{2}$
15. $\frac{2xy - 6x}{y^2 - 3y}$ $\frac{2x}{y}$
16. $\frac{2x - 4xy}{1 - 2y}$ $2x$
17. $\frac{5k + 10}{(k + 2)^2}$ $\frac{5}{k + 2}$
18. $\frac{(m - 4)^2}{3m - 12}$ $\frac{m - 4}{3}$
19. $\frac{6u + 6v}{u^2 - v^2}$ $\frac{6}{u - v}$
20. $\frac{g^2 - h^2}{5g - 5h}$ $\frac{g + h}{5}$
21. $\frac{12x^2y}{3xy^2 + 6x^2y}$ $\frac{4x}{y + 2x}$
22. $\frac{3x - 9xy}{1 - 3y}$ $3x$
23. $\frac{x^2 + 6x + 9}{x^2 - 9}$ $\frac{x + 3}{x - 3}$
24. $\frac{y^2 - 4y + 4}{y^2 - 4}$ $\frac{y - 2}{y + 2}$

B

25. $\frac{2x^2 - 50}{2x + 10}$ $x - 5$
26. $\frac{2ab^2 + 2a^2b}{4a^2 - 4b^2}$ $\frac{ab}{2a - 2b}$
27. $\frac{3x^2y}{6x^2y + 3xy^2}$ $\frac{x}{2x + y}$
28. $\frac{x^2 - 8x + 15}{x^2 - 2x - 15}$ $\frac{x - 3}{x + 3}$
29. $\frac{x^2 - x - 6}{x^2 + 5x + 6}$ $\frac{x - 3}{x + 3}$
30. $\frac{x^2 - 6x + 8}{x^2 - x - 2}$ $\frac{x - 4}{x + 1}$
31. $\frac{y^2 - 4y - 5}{y^2 - 2y - 3}$ $\frac{y - 5}{y - 3}$
32. $\frac{t^2 + 7t + 10}{t^2 - t - 6}$ $\frac{t + 5}{t - 3}$
33. $\frac{z^2 - 12z + 20}{z^2 + 4z - 12}$ $\frac{z - 10}{z + 6}$

3 • The −1 Factor

You know that 1 is a factor of every number, even though the factor 1 is not written very often.

$$2 = 1 \cdot 2 \qquad x = 1 \cdot x \qquad a - 1 = 1(a - 1)$$

You can also write -1 as a factor of every number.

$$2 = -1(-2) \qquad x = -1(-x) \qquad a - 1 = -1(-a + 1)$$
$$\text{or} \qquad a - 1 = -1(1 - a)$$

Now see how useful this -1 factor can be in simplifying fractions.

EXAMPLE 1 $\dfrac{a-1}{1-a} = \dfrac{a-1}{-1(a-1)} = \dfrac{\overset{1}{\cancel{a-1}}}{-1\underset{1}{\cancel{(a-1)}}} = \dfrac{1}{-1} = -1$

EXAMPLE 2 $\dfrac{b-a}{2a-2b} = \dfrac{b-a}{2(a-b)} = \dfrac{-1(a-b)}{2(a-b)} = \dfrac{-1}{2} = -\dfrac{1}{2}$

EXAMPLE 3 $\dfrac{x^2-9}{3-x} = \dfrac{(x-3)(x+3)}{3-x} = \dfrac{(x-3)(x+3)}{-1(x-3)} = \dfrac{x+3}{-1} = -x-3$

Classroom Practice

Complete.

1. $3 = -1(\underline{-3})$
2. $7 = -1(\underline{-7})$
3. $17 = -1(\underline{-17})$
4. $2x = -1(\underline{-2x})$
5. $-x - y = -1(\underline{x + y})$
6. $-x + y = -1(\underline{x - y})$
7. $a + b = (\underline{-1})(-a - b)$
8. $1 - x = (\underline{-1})(x - 1)$
9. $2 - y = -1(\underline{-2 + y \text{ or } y - 2})$
10. $a - b = -1(\underline{b - a})$
11. $3 + x = -1(\underline{-3 - x})$
12. $x - 2 = -1(\underline{2 - x})$
13. $2a - b = -1(\underline{b - 2a})$
14. $-x - 4y = -1(\underline{x + 4y})$
15. $x^2 - 4 = -1(\underline{4 - x^2})$

Written Exercises

Write the expression with a factor of -1.

A **1.** $-2x$ $-1(2x)$ **2.** $-3y$ $-1(3y)$ **3.** $-a-b$ $-1(a+b)$ **4.** $-7-4x$ $-1(7+4x)$

5. $2-3y$ $-1(3y-2)$ **6.** $6-5b$ $-1(5b-6)$ **7.** $3-8m$ $-1(8m-3)$ **8.** $16-r$ $-1(r-16)$

9. $25-v$ $-1(v-25)$ **10.** $3-x^2$ $-1(x^2-3)$ **11.** $-4-y^2$ $-1(4+y^2)$ **12.** a^2-b^2 $-1(b^2-a^2)$

Simplify.

13. $\frac{a-b}{b-a}$ -1 **14.** $\frac{c-2}{2-c}$ -1 **15.** $\frac{x-y}{y-x}$ -1 **16.** $\frac{a-c}{c-a}$ -1

17. $\frac{1-a}{a-1}$ -1 **18.** $\frac{b-1}{1-b}$ -1 **19.** $\frac{m-n}{n-m}$ -1 **20.** $\frac{a-6}{6-a}$ -1

21. $\frac{2x+2y}{-y-x}$ -2 **22.** $\frac{3b+6a}{-b-2a}$ -3 **23.** $\frac{5x-5y}{5y-5x}$ -1 **24.** $\frac{1-b}{2b-2}$ $-\frac{1}{2}$

25. $\frac{a^2-1}{1-a}$ $-a-1$ **26.** $\frac{m^2-1}{1-m}$ $-m-1$ **27.** $\frac{1-r}{r^2-1}$ $-\frac{1}{r+1}$ **28.** $\frac{a^2-9}{3-a}$ $-a-3$

29. $\frac{2-c}{c^2-4}$ $-\frac{1}{c+2}$ **30.** $\frac{3y^2-3}{3-3y}$ $-y-1$ **31.** $\frac{2x^2-50}{10-2x}$ $-x-5$ **32.** $\frac{-a-b}{b^2-a^2}$ $\frac{1}{a-b}$

B **33.** $\frac{8-4t}{t^2+t-6}$ $-\frac{4}{t+3}$ **34.** $\frac{5-5x}{x^2+3x-4}$ $-\frac{5}{x+4}$ **35.** $\frac{p^2-4p-21}{14-2p}$ $-\frac{p+3}{2}$

36. $\frac{z^2-6z+9}{6-2z}$ $-\frac{z-3}{2}$ **37.** $\frac{r^2-7r+12}{9-3r}$ $-\frac{r-4}{3}$ **38.** $\frac{8-4a}{a^2-5a+6}$ $-\frac{4}{a-3}$

For more practice, see page 426.

SELF-TEST

Simplify.

1. $\frac{24}{21x}$ $\frac{8}{7x}$ **2.** $\frac{-6a}{18a}$ $-\frac{1}{3}$ **3.** $\frac{-12r^2s}{-9r^2s^2}$ $\frac{4}{3s}$ **4.** $\frac{5a-5b}{a-b}$ 5

5. $\frac{9-m^2}{3+m}$ $3-m$ **6.** $\frac{6x+6y}{9x+9y}$ $\frac{2}{3}$ **7.** $\frac{x-3y}{3y-x}$ -1 **8.** $\frac{4m-4n}{2n-2m}$ -2

4 • Ratio

One way to compare numbers is by a **ratio.** The ratio of two numbers is their quotient. We often write a ratio as a fraction.

ratio of 12 to 8 | ratio of 6 to 4 | ratio of 3 to 2

$$\frac{12}{8} = \frac{6}{4} = \frac{3}{2}$$

The three ratios mean the same number.

Suppose in your school there are 500 students and 25 teachers. You want to compare the number of students and teachers by a ratio in simplest form.

$$\text{ratio of students to teachers} = \frac{\text{number of students}}{\text{number of teachers}} = \frac{500}{25}$$

$$\frac{500}{25} = \frac{100}{5} = \frac{20}{1}$$

There are 20 students for every 1 teacher. The ratio is 20 to 1.

Classroom Practice

Give the ratios in simplest form.

1. 12 centimeters to 16 centimeters. $\frac{3}{4}$
2. 40 grams to 25 grams $\frac{8}{5}$
3. 6 liters to 18 liters $\frac{1}{3}$
4. 15 minutes to 60 minutes $\frac{1}{4}$
5. A school has 506 students and 23 teachers. Find the ratio of students to teachers. $\frac{506}{23}$ or $\frac{22}{1}$
6. In a small town last year, there were 20 births and 12 deaths. Find the ratio of births to deaths. $\frac{20}{12}$ or $\frac{5}{3}$
7. A recipe for cookies says to mix flour, sugar, and butter in a 3 to 2 to 1 ratio. What does this mean? 3 units flour to 2 units sugar to 1 unit butter

Written Exercises

Give the ratios in simplest form.

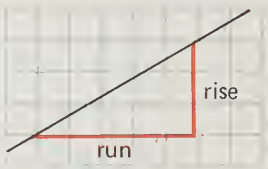
A

1. 25 centimeters to 40 centimeters $\frac{5}{8}$
2. 30 millimeters to 20 millimeters $\frac{3}{2}$
3. 20 minutes to 48 minutes $\frac{5}{12}$
4. 4 months to 12 months $\frac{1}{3}$
5. 12 buckets to 16 buckets $\frac{3}{4}$
6. 6 parts to 4 parts $\frac{3}{2}$
7. One part of cement is mixed with two parts of sand in order to make concrete. What is the ratio of sand to cement? $\frac{2}{1}$



8. A school has 2000 students and 80 teachers. What is the ratio of students to teachers? $\frac{25}{1}$
9. The Rams won 8 games and lost 6 games last season. What is the ratio of wins to losses? $\frac{4}{3}$
10. A school has 2000 students. 1200 students are girls. What is the ratio of boys to girls? $\frac{2}{3}$
11. A basketball team plays 18 games and loses only 4. What is the ratio of wins to losses? $\frac{7}{2}$
12. **a.** The slope of a line is the ratio of its $\frac{\text{rise}}{\text{?}}$ to its $\frac{\text{run}}{\text{?}}$.

 b. The slope of the line shown is ?. $\frac{3}{5}$



Write the ratio using the same units. Give the ratio in simplest form.

Sample 30 minutes to 2 hours $= \frac{30 \text{ min}}{120 \text{ min}} = \frac{1 \text{ min}}{4 \text{ min}}$ *or* $\frac{1}{4}$

2 hours = 120 minutes

B

13. 1 meter to 1 centimeter $\frac{100 \text{ cm}}{1 \text{ cm}} = \frac{100}{1}$
14. 30 minutes to 3 hours $\frac{30 \text{ min}}{180 \text{ min}} = \frac{1}{6}$
15. 3 years to 3 months $\frac{36 \text{ months}}{3 \text{ months}} = \frac{12}{1}$
16. 60 cents to 1 dollar $\frac{60¢}{100¢} = \frac{3}{5}$
17. 50 centimeters to 3 meters $\frac{50 \text{ cm}}{300 \text{ cm}} = \frac{1}{6}$
18. 400 meters to 1 kilometer $\frac{400 \text{ m}}{1000 \text{ m}} = \frac{2}{5}$

5 • Problems Involving Ratio

If you know that the ratio of two numbers is 2 to 3, you can represent these numbers by $2x$ and $3x$.

$$\frac{2x}{3x} = \frac{2}{3} \longleftarrow \text{Ratio of } 2x \text{ to } 3x = \text{Ratio of 2 to 3}$$

Now let's use this idea in solving some problems.

EXAMPLE 1 The ratio of girls to boys in a club is 5 to 4.
There are 45 members in all.
How many are girls? How many are boys?

Let $5x$ = number of girls, $4x$ = number of boys.

number of girls + number of boys = 45

$$\begin{aligned} 5x + 4x &= 45 \\ 9x &= 45 \\ x &= 5 \end{aligned}$$

Answer: Number of girls $= 5x = 5 \cdot 5 = 25$
Number of boys $= 4x = 4 \cdot 5 = 20$

Check: $\frac{25}{20} = \frac{5}{4}$ ✓ $25 + 20 = 45$ ✓

EXAMPLE 2 The ratio of tin to copper in a metal mixture is 3 to 5.
We need to make 400 kg of the mixture.
How much of each metal should be used?

Let $3x$ = kg of tin, $5x$ = kg of copper.

kg of tin + kg of copper = 400

$$\begin{aligned} 3x + 5x &= 400 \\ 8x &= 400 \\ x &= 50 \end{aligned}$$

Answer: Amount of tin $= 3x = 3 \cdot 50 = 150 \longrightarrow 150$ kg
Amount of copper $= 5x = 5 \cdot 50 = 250 \longrightarrow 250$ kg

Check: $\frac{150}{250} = \frac{3}{5}$ ✓ $150 + 250 = 400$ ✓

Written Exercises



A

1. There are 28 people at a party.
The ratio of boys to girls is 4 to 3.
How many boys are there? 16 boys

2. A team played 90 games.
The ratio of wins to losses was 5 to 4.
How many games did they lose?
40 games

3. A baseball pitcher pitched 35 games.
The ratio of wins to losses was 3 to 2.
How many games did he win?
21 games

4. A large bookstore took in $3,000,000 in sales last year.
The ratio of book sales to other sales was 3 to 2.
How much money in book sales did the store take in?
$1,800,000

5. An 80-meter cable is cut into two pieces.
The lengths of the pieces have a ratio of 11 to 5.
How long is the shorter piece? 25 m

6. In a concrete mixture, the ratio of sand to cement is 2 to 1.
How much cement is needed to make 30 cubic meters of concrete?
10 cubic meters

7. Pat pays 20 cents out of every dollar of her income in taxes.
Her yearly income is $10,000.
How much does she have left after taxes? $8,000

8. The ratio of fruit juice to ginger ale in a punch is 2 to 3.
How much ginger ale is needed to make 25 liters of punch? 15 L

9. In a certain salad dressing, the ratio of vinegar to oil is 4 to 3.
How much of each is in 735 milliliters of dressing? 420 mL vinegar; 315 mL oil

B

10. One town had $54,000 to spend on streets, parks, and the library.
The money was divided in the ratio of 4 to 3 to 2.
How much money did each get?
$24,000 for streets; $18,000 for parks; $12,000 for library

11. Three people in an election split the vote in the ratio 5 to 4 to 2.
The number of votes cast was 55,110.
How many votes did each person receive?
25,050 votes; 20,040 votes; 10,020 votes

6 • Proportion

A proportion is a statement that two ratios are equal.

$$\frac{4}{6} = \frac{2}{3}$$ ← 4 to 6 equals 2 to 3.

Note what happens in the proportion if you cross-multiply.

$4 \cdot 3 = 12$ → $\frac{4}{6} = \frac{2}{3}$ ← $6 \cdot 2 = 12$

The products are the same!

If you know three of the four numbers in a proportion, you can use cross-multiplication to find the fourth number.

EXAMPLE 1

$$\frac{5}{4} = \frac{30}{x}$$

$$5x = 4 \cdot 30$$
$$5x = 120$$
$$x = 24$$

Check:

$$\frac{5}{4} = \frac{30}{x}$$

$\frac{5}{4}$	$\frac{30}{24}$
	$\frac{5}{4}$ ✓

EXAMPLE 2

$$\frac{2x}{5} = \frac{x+1}{3}$$

$$2x \cdot 3 = 5(x+1)$$
$$6x = 5x + 5$$
$$6x - 5x = 5x + 5 - 5x$$
$$x = 5$$

Check:

$$\frac{2x}{5} = \frac{x+1}{3}$$

$\frac{10}{5}$	$\frac{6}{3}$
2	2 ✓

Classroom Practice

Solve.

1. $\frac{4}{12} = \frac{x}{9}$ 3
2. $\frac{a}{3} = \frac{16}{12}$ 4
3. $\frac{x}{4} = \frac{9}{12}$ 3
4. $\frac{5}{3} = \frac{a}{6}$ 10
5. $\frac{1}{6x} = \frac{2}{36}$ 3
6. $\frac{3x}{5} = \frac{12}{5}$ 4
7. $\frac{4n}{3} = \frac{8}{6}$ 1
8. $\frac{5n}{3} = \frac{25}{5}$ 3

Written Exercises

Solve.

A

1. $\frac{3}{5} = \frac{x}{15}$ 9
2. $\frac{2}{9} = \frac{6}{x}$ 27
3. $\frac{8}{6} = \frac{x}{9}$ 12
4. $\frac{10}{a} = \frac{5}{10}$ 20
5. $\frac{3x}{4} = \frac{9}{6}$ 2
6. $\frac{2x}{7} = \frac{4}{1}$ 14
7. $\frac{6}{1} = \frac{48}{x}$ 8
8. $\frac{3a}{2} = \frac{15}{5}$ 2
9. $\frac{3a}{5} = \frac{12}{10}$ 2
10. $\frac{15}{5} = \frac{3a}{4}$ 4
11. $\frac{4}{5} = \frac{2x}{25}$ 10
12. $\frac{4r}{8} = \frac{3}{6}$ 1
13. $\frac{x}{4} = \frac{x+2}{5}$ 8
14. $\frac{x}{5-x} = \frac{2}{3}$ 2
15. $\frac{a-2}{a+1} = \frac{1}{2}$ 5
16. $\frac{x+4}{x} = \frac{5}{3}$ 6
17. $\frac{x-1}{3} = \frac{x+1}{9}$ 2
18. $\frac{x+1}{3} = \frac{x}{2}$ 2
19. $\frac{2a+3}{4} = \frac{a-6}{8}$ −4
20. $\frac{y+2}{y-2} = \frac{5}{3}$ 8
21. $\frac{3n-5}{8n} = \frac{1}{4}$ −5
22. $\frac{x+3}{5} = \frac{x+1}{4}$ 7
23. $\frac{y+2}{9} = \frac{y-1}{18}$ −5
24. $\frac{a+4}{6} = \frac{a+6}{4}$ −10
25. $\frac{2x+3}{4x} = \frac{3}{4}$ 3
26. $\frac{3a-4}{1} = \frac{2a}{2}$ 2
27. $\frac{5m+2}{3} = \frac{m-1}{2}$ −1

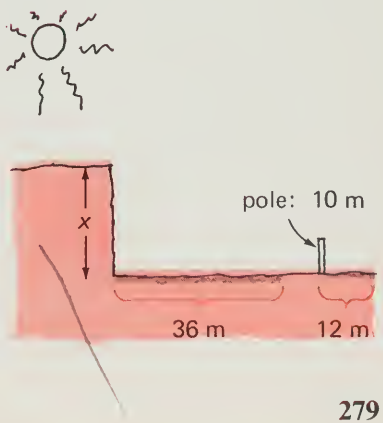
B

28. $\frac{x}{x-2} = \frac{x+2}{x+1}$ −4
29. $\frac{y}{y-2} = \frac{y+2}{y+4}$ −1
30. $\frac{a-1}{a} = \frac{a+1}{a+3}$ 3
31. $\frac{x-3}{x} = \frac{x+3}{x+8}$ 12
32. $\frac{n-3}{n+5} = \frac{n-4}{n+3}$ 11
33. $\frac{c+5}{c-1} = \frac{c-3}{c-5}$ 7

34. You can find the height of the cliff in the drawing by using this proportion.

$$\frac{\text{height of cliff}}{\text{height of pole}} = \frac{\text{length of cliff's shadow}}{\text{length of pole's shadow}}$$

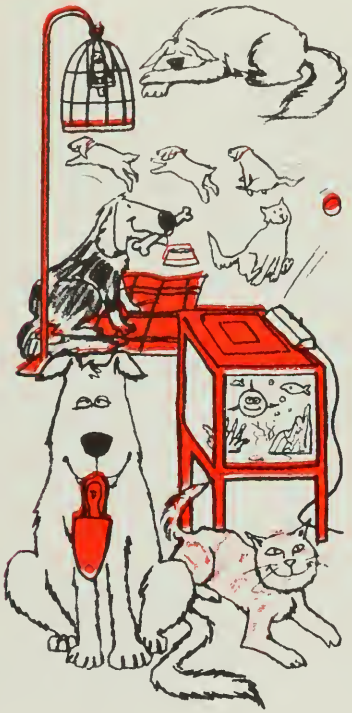
Find the height of the cliff. 30 m



7 • Applying Proportions

Proportions give an easy way to solve many real-life problems.

EXAMPLE 1



One large apartment house has 170 apartments.
Sixty tenants were surveyed.
Forty-two said they had at least one pet.
About how many of the tenants have pets (if the ratio of the survey holds true)?

	Survey	Real
Tenants with pets	42	x
Total	60	170

$$\frac{42}{60} = \frac{x}{170}$$

$$60x = 42 \cdot 170$$

$$60x = 7140$$

$$x = \frac{7140}{60}$$

$$x = 119$$

Answer: About 119 tenants

EXAMPLE 2 One family drinks 14 liters of milk in one week.
They usually drink milk at the same rate.
How many liters of milk does this family drink in 10 days?

Numbers of liters	14	x
Time	7 days	10 days

$$\frac{14}{7} = \frac{x}{10}$$

$$7x = 14 \cdot 10$$

$$7x = 140$$

$$x = 20$$

Answer: 20 liters

EXAMPLE 3 The recipe below serves 8 people.
How much ginger ale will be needed to serve 20 people?
How much sherbet will be needed to serve 20 people?

PARTY PUNCH
6 cups ginger ale
3 cups sherbet
fresh fruit, if desired

Number of People	Ginger Ale	Sherbet
8	6	3
20	x	y

Ginger Ale: $\frac{8}{20} = \frac{6}{x}$

$8x = 120$

$x = 15$

Sherbet: $\frac{8}{20} = \frac{3}{y}$

$8y = 60$

$y = 7\frac{1}{2}$

Answer: 15 cups of ginger ale and $7\frac{1}{2}$ cups of sherbet

Written Exercises

A

1. Three cans of cat food cost 93¢.
Five cans will cost how much? $1.55

Number	3	5
Cost	93¢	? n

2. Six oranges cost 50¢.
How much will 9 oranges cost? $.75

Number	6	? 9
Cost	50¢	? n

3. Four bottles of ginger ale cost 90¢.
How much will 10 bottles cost? $2.25

Number	? 4	? 10
Cost	? 90¢	? n

4. Three loaves of bread cost $1.35.
How much will 5 loaves cost? $2.25

Number	3 ?	5 ?
Cost	$1.35 ?	n ?

5. An airplane can fly 1500 km in 2 hours.
How long would it take to fly 3750 km?
5 hours

Distance	1500 km	? 3750 km
Time	2	n ?

6. A metal pipe 200 cm long is 5 kg.
How much is a piece 80 cm long? 2 kg

Centimeters	200 cm ?	80 ? cm
Kilograms	5 kg ?	n ?

7. A face cream contains 90 g of glycerine in each 240 g of cream.
How many grams of glycerine in 56 g of cream? 21 g

Glycerine	90 g	?	*n*
Cream	240 g	?	56 g

8. A pump can empty a 1500-liter tank in 45 minutes.
How long will it take to empty a 1000-liter tank? 30 minutes

Quantity	1500 L	?	1000 L
Time	45 min	?	*n*

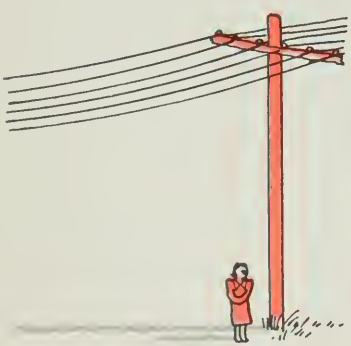
9. A photograph measures 20 cm by 15 cm.
It is enlarged so that the larger measure is 28 cm.
What does the shorter measure become? 21 cm



10. Dr. Denis drove 100 km on 12 L of gasoline.
How many liters will she use to drive 25 km? 3 L

11. A map is drawn so that 5 cm represents 2 km.
On the map, two towns measure 20 cm apart.
Find the actual distance between the towns. 8 km

12. A car worth \$3600 is taxed \$201.
At that rate, what is the tax on a car worth \$2400? \$134



13. A telephone pole casts a shadow of 1500 cm.
A girl who is 150 cm tall is standing nearby.
Her shadow is 125 cm long.
How tall is the telephone pole? 1,800 cm

	Girl	Telephone Pole
Height of object	150 cm	1500 cm
Length of shadow	125 cm	*x*

14. A man 180 cm tall casts a shadow 150 cm long.
The shadow of a flagpole is 1050 cm long.
How tall is the flagpole? 1260 cm

B **15.** The tax on the Reids' apartment is $1200 a year.
Last year they sold the apartment.
They had owned it only 3 months of the year.
How much tax must they pay on the apartment for that year? $300

16. The annual tax on the Carey home is $1452.
On October 1 the Careys sold the house to the Robinsons.
How much tax must the Careys pay for that year? $1089

17. The Addisons want to put an addition onto their house.
Their neighbors' addition cost $1620 for an area of 12 m^2.
The Addisons want to add about 20 m^2.
About how much money should the Addisons expect to pay? $2700



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For more practice, see page 426.

SELF-TEST

Solve.

1. A 65-centimeter string is cut into two pieces.
The lengths of the pieces have a ratio of 3 to 2.
How long is each piece? 39 cm and 26 cm

2. $\frac{7}{9} = \frac{x}{36}$ 28 **3.** $\frac{3}{4} = \frac{6}{x}$ 8 **4.** $\frac{a+2}{a} = \frac{5}{3}$ 3 **5.** $\frac{x}{x+4} = \frac{2}{1}$ −4

6. Daryl drove 75 km on 9 L of gasoline.
How far can he drive on 12 L of gasoline? 100 km

7. A pump can fill a 900-liter tank in 30 minutes.
How long will it take to fill a 1200-liter tank? 40 minutes

8 • Multiplying Fractions

You can multiply fractions in algebra in the same way as arithmetic.

In Arithmetic	*In Algebra*
$\frac{2}{3} \cdot \frac{4}{5} = \frac{2 \cdot 4}{3 \cdot 5} = \frac{8}{15}$	$\frac{3}{4} \cdot \frac{x}{y} = \frac{3 \cdot x}{4 \cdot y} = \frac{3x}{4y}$

Sometimes you can simplify the product of two fractions.

EXAMPLE 1
$$\frac{2}{x^2} \cdot \frac{x}{2} = \frac{2 \cdot x}{x \cdot x \cdot 2} = \frac{1}{x}$$

EXAMPLE 2
$$\frac{r+s}{r-s} \cdot \frac{r-s}{3} = \frac{(r+s)(r-s)}{3(r-s)} = \frac{r+s}{3}$$

EXAMPLE 3
$$\frac{a^2-1}{12} \cdot \frac{6}{a-1} = \frac{6(a^2-1)}{12(a-1)} = \frac{6(a+1)(a-1)}{2 \cdot 6(a-1)} = \frac{a+1}{2}$$

Classroom Practice

Simplify.

1. $\frac{3}{5} \cdot \frac{3}{4}$ $\frac{9}{20}$
2. $\frac{a}{2} \cdot \frac{b}{3}$ $\frac{ab}{6}$
3. $\frac{x}{3} \cdot \frac{2x}{5}$ $\frac{2x^2}{15}$
4. $\frac{2a}{3} \cdot \frac{1}{4a}$ $\frac{1}{6}$
5. $\frac{7}{19} \cdot \frac{19}{20}$ $\frac{7}{20}$
6. $\frac{n}{3} \cdot \frac{6}{2n^2}$ $\frac{1}{n}$
7. $\frac{1}{a-b} \cdot \frac{a-b}{2}$ $\frac{1}{2}$
8. $\frac{a^2-b^2}{a} \cdot \frac{a^2}{a-b}$ $a^2 + a$

Written Exercises

Simplify.

A

1. $\frac{1}{3} \cdot \frac{1}{4}$ $\frac{1}{12}$
2. $\frac{1}{5} \cdot \frac{1}{8}$ $\frac{1}{40}$
3. $\frac{2}{3} \cdot \frac{4}{7}$ $\frac{8}{21}$
4. $\frac{x}{3} \cdot \frac{x}{4}$ $\frac{x^2}{12}$
5. $y \cdot \frac{y}{2}$ $\frac{y^2}{2}$
6. $m \cdot \frac{m}{8}$ $\frac{m^2}{8}$
7. $\frac{a}{b} \cdot \frac{2b}{3a}$ $\frac{2}{3}$
8. $\frac{xy}{5} \cdot \frac{10}{x}$ $2y$
9. $\frac{6x}{5y} \cdot \frac{y^2}{3x}$ $\frac{2y}{5}$
10. $\frac{5a}{3b^2} \cdot \frac{6b^3}{a^2}$ $10ab$
11. $\frac{4x}{3y} \cdot \frac{3}{8xy}$ $\frac{1}{2y^2}$
12. $\frac{16x^2}{5} \cdot \frac{15}{4y^2}$ $\frac{12x^2}{y^2}$
13. $\frac{9r}{7s} \cdot \frac{14rs}{3r}$ $6r$
14. $\frac{x^2}{3y} \cdot \frac{5xy}{x}$ $\frac{5x^2}{3}$
15. $\frac{3a^2}{4} \cdot \frac{20b}{12b^2}$ $\frac{5a^2}{4b}$
16. $\frac{12a}{5a^2} \cdot \frac{a^2}{6a^2}$ $\frac{2}{5a}$
17. $\frac{3}{7} \cdot \frac{7x - 7}{6}$ $\frac{x - 1}{2}$
18. $\frac{5}{4} \cdot \frac{4x - 4}{15}$ $\frac{x - 1}{3}$
19. $\frac{2}{3} \cdot \frac{3x - 12}{4}$ $\frac{x - 4}{2}$
20. $\frac{5}{4x + 16} \cdot \frac{4}{25}$ $\frac{1}{5x + 20}$
21. $\frac{a^2 - a}{5} \cdot \frac{25}{a}$ $5a - 5$
22. $\frac{x}{b} \cdot \frac{b^2 - b}{x^2y}$ $\frac{b - 1}{xy}$
23. $\frac{a^2b}{2a - 2} \cdot \frac{2}{ab^2}$ $\frac{a}{ab - b}$
24. $\frac{9}{x + 2} \cdot \frac{x^2 + 2x}{36}$ $\frac{x}{4}$
25. $\frac{a^2 - b^2}{10} \cdot \frac{5}{a - b}$ $\frac{a + b}{2}$
26. $\frac{x^2 - 1}{4} \cdot \frac{12}{x - 1}$ $3x + 3$
27. $\frac{r^2 - 2r}{4} \cdot \frac{2}{r - 2}$ $\frac{r}{2}$
28. $\frac{3x - 3}{6} \cdot \frac{4}{1 - x}$ -2
29. $\frac{6}{x^2 - 4} \cdot \frac{2 - x}{18}$ $-\frac{1}{3x + 6}$

B

30. $\frac{x^2 - y^2}{x^2 - 49} \cdot \frac{x + 7}{x - y}$ $\frac{x + y}{x - 7}$
31. $\frac{x^2 - x - 6}{14} \cdot \frac{7}{x - 3}$ $\frac{x + 2}{2}$
32. $\frac{2x}{x + 1} \cdot \frac{2x + 2}{6x^2}$ $\frac{2}{3x}$
33. $\frac{x^2 - 3x - 10}{2} \cdot \frac{4}{x^2 + x - 2}$ $\frac{2x - 10}{x - 1}$
34. $\frac{2a - 4}{3a + 6} \cdot \frac{2a + 4}{a - 2}$ $\frac{4}{3}$
35. $\frac{3x - 9}{x - 3} \cdot \frac{x + 2}{3x + 12}$ $\frac{x + 2}{x + 4}$
36. $\frac{4a - 1}{a^2} \cdot \frac{a^3 - a^2}{a - 1}$ $4a - 1$
37. $\frac{n^2 - 1}{3} \cdot \frac{21}{1 - n}$ $-7n - 7$
38. $\frac{y - 2}{y - 1} \cdot \frac{1 - y}{2 - y}$ 1
39. $\frac{4 - 2x}{x^2 - 4} \cdot \frac{1 - x}{x^2 - 1}$ $\frac{2}{x^2 + 3x + 2}$

C

40. $\frac{3 - 3x}{x^2 - 2x - 3} \cdot \frac{x + 1}{x - 1}$ $-\frac{3}{x - 3}$
41. $\frac{m^2 - 1}{m^2 + 4m + 3} \cdot \frac{m^2 + m - 6}{m^2 + m - 2}$ $\frac{m - 2}{m + 2}$

9 • Dividing Fractions

The numbers 2 and $\frac{1}{2}$ are called **reciprocals** because when they are multiplied, the result is 1. The numbers $\frac{3}{4}$ and $\frac{4}{3}$ are reciprocals also. Dividing by a number is the same as multiplying by its reciprocal.

In Arithmetic	*In Algebra*
$6 \div 2 = 6 \cdot \frac{1}{2} = 3$	$4 \div \frac{x}{3} = 4 \cdot \frac{3}{x} = \frac{12}{x}$

EXAMPLE 1
$$\frac{2}{3x} \div \frac{x}{4} = \frac{2}{3x} \cdot \frac{4}{x}$$
$$= \frac{8}{3x^2}$$

EXAMPLE 2
$$\frac{1}{x+2} \div \frac{1}{x+1} = \frac{1}{x+2} \cdot \frac{x+1}{1}$$
$$= \frac{x+1}{x+2}$$

Sometimes you'll notice the same factors in the numerator and in the denominator.

EXAMPLE 3
$$\frac{x^3}{3} \div \frac{x^2}{4} = \frac{x^3}{3} \cdot \frac{4}{x^2}$$
$$= \frac{x \cdot x \cdot x \cdot 4}{3 \cdot x \cdot x}$$
$$= \frac{4x}{3}$$

EXAMPLE 4
$$\frac{t+3}{t^2-25} \div \frac{1}{t-5} = \frac{t+3}{t^2-25} \cdot \frac{t-5}{1}$$
$$= \frac{t+3}{(t+5)(t-5)} \cdot \frac{t-5}{1}$$
$$= \frac{(t+3)(t-5)}{(t+5)(t-5)}$$
$$= \frac{t+3}{t+5}$$

Classroom Practice

Simplify.

1. $\frac{3}{4} \div \frac{9}{8}$ $\frac{2}{3}$
2. $\frac{x}{3} \div \frac{x}{2}$ $\frac{2}{3}$
3. $\frac{a}{b^2} \div \frac{a^2}{b}$ $\frac{1}{ab}$
4. $\frac{c^2}{d^3} \div \frac{c}{d^2}$ $\frac{c}{d}$
5. $\frac{2x}{3y^2} \div \frac{8x^3}{9y^2}$ $\frac{3}{4x^2}$
6. $\frac{21x^2y^3}{5} \div 7x^2y$ $\frac{3y^2}{5}$
7. $\frac{6}{y^2 - 9} \div \frac{3}{y - 3}$ $\frac{2}{y + 3}$
8. $\frac{x^2 - y^2}{8} \div \frac{3x - 3y}{2}$ $\frac{x + y}{12}$

Written Exercises

Simplify.

A

1. $\frac{5}{6} \div \frac{1}{3}$ $\frac{5}{2}$
2. $\frac{x}{2} \div \frac{1}{4}$ $\frac{2x}{1}$
3. $\frac{a}{5} \div \frac{a}{6}$ $\frac{6}{5}$
4. $\frac{a}{4} \div \frac{a^2}{2}$ $\frac{1}{2a}$
5. $\frac{2a^2}{5b^2} \div \frac{4a^3}{10b}$ $\frac{1}{ab}$
6. $\frac{5r}{12t^2} \div \frac{15r^2}{6s^3}$ $\frac{s^3}{6t^2r}$
7. $3m^2 \div \frac{8m}{7}$ $\frac{21m}{8}$
8. $\frac{3km}{8} \div 18k^2$ $\frac{m}{48k}$
9. $\frac{6}{a^2b} \div \frac{a}{a^3b^2}$ $6b$
10. $\frac{(xy)^2}{x} \div \frac{xy}{y}$ y^2
11. $\frac{(ab)^2}{4} \div 2ab$ $\frac{ab}{8}$
12. $\frac{a - b}{7} \div \frac{a - b}{6}$ $\frac{6}{7}$
13. $\frac{h - g}{4} \div \frac{h - g}{8}$ 2
14. $\frac{2x - 2}{4} \div \frac{2}{9}$ $\frac{9x - 9}{4}$
15. $\frac{a^3}{3} \div \frac{a^4}{3a^2 + 3}$ $\frac{a^2 + 1}{a}$
16. $\frac{2x + 2}{5} \div \frac{2}{3}$ $\frac{3x + 3}{5}$
17. $\frac{6}{y^2 - 9} \div \frac{3}{y - 3}$ $\frac{2}{y + 3}$
18. $\frac{a^2 - b^2}{4} \div \frac{a - b}{8}$ $2a + 2b$
19. $\frac{2a - 2}{2} \div (a - 1)$ 1
20. $\frac{x^2 - 1}{2 - x} \div (x - 1)$ $\frac{x + 1}{2 - x}$
21. $\frac{x^2 - 16}{2x} \div (x - 4)$ $\frac{x + 4}{2x}$
22. $\frac{r - 7}{r^2 - 4} \div \frac{1}{r - 2}$ $\frac{r - 7}{r + 2}$

B

23. $\frac{a^2 - a - 6}{a^2 - 9} \div \frac{a + 2}{a - 3}$ $\frac{a - 3}{a + 3}$
24. $\frac{x^2 - 16}{2x} \div \frac{x + 4}{2x^2 - 2x}$ $x^2 - 5x + 4$
25. $\frac{6y + 12}{3y - 9} \div \frac{4y - 8}{3y + 6}$ $2y^2 - 10y + 12$
26. $\frac{a - b}{4ab} \div \frac{b - a}{8a^2}$ $\frac{-2a}{b}$
27. $\frac{y + 2}{y + 1} \div \frac{2y + 4}{y^2 - 2y - 3}$ $y - 3$
28. $\frac{u^2 + u^3}{36 - u^2} \div \frac{u - u^3}{6u + u^2}$ $\frac{u^2}{u^2 - 7u + 6}$

C

29. $\frac{(d - 5)^2}{(d + 3)} \div \frac{d^2 - d - 20}{d^2 + 7d + 12}$ $d - 5$
30. $\frac{h^2 + h - 6}{h^2 - 2h - 15} \div \frac{h^2 + 4h - 5}{h^2 - 25}$ $\frac{h - 2}{h - 1}$
31. $\frac{y^2 - 7y + 12}{y^2 + y - 20} \div \frac{y^2 - 9y + 18}{y^2 + 7y + 10}$ $\frac{y + 2}{y - 6}$
32. $\frac{x^2 + 5x + 6}{x^2 - 4} \div \frac{x^2 + 2x - 3}{x^2 - 3x + 2}$ 1

10 • Add, Subtract—Same Denominators

In arithmetic, when the denominators of two fractions are the same, the fractions can be added or subtracted easily. This is true in algebra also.

In Arithmetic	*In Algebra*
$\frac{1}{7} + \frac{4}{7} = \frac{1+4}{7} = \frac{5}{7}$	$\frac{x}{7} + \frac{4x}{7} = \frac{x+4x}{7} = \frac{5x}{7}$
$\frac{4}{5} - \frac{3}{5} = \frac{4-3}{5} = \frac{1}{5}$	$\frac{4}{y} - \frac{3}{y} = \frac{4-3}{y} = \frac{1}{y}$

EXAMPLE 1 $\frac{5}{2a} + \frac{6}{2a} = \frac{5+6}{2a} = \frac{11}{2a}$

EXAMPLE 2
$$\frac{2r}{r-u} - \frac{2u}{r-u} = \frac{2r-2u}{r-u}$$
$$= \frac{2(r-u)}{r-u}$$
$$= 2$$

EXAMPLE 3
$$\frac{3t}{7} - \frac{7-2t}{7} = \frac{3t-(7-2t)}{7}$$
$$= \frac{3t-7+2t}{7}$$
$$= \frac{5t-7}{7}$$

Be careful. Do not write this: $\frac{3t-7-2t}{7}$

Classroom Practice

Simplify.

1. $\frac{5}{8} + \frac{4}{8}$ $\frac{9}{8}$
2. $\frac{3x}{5} + \frac{x}{5}$ $\frac{4x}{5}$
3. $\frac{5}{a} - \frac{3}{a}$ $\frac{2}{a}$
4. $\frac{2x}{7} + \frac{3x}{7}$ $\frac{5x}{7}$
5. $\frac{y}{11} - \frac{4y}{11}$ $\frac{-3y}{11}$
6. $\frac{2}{a} - \frac{5}{a}$ $-\frac{3}{a}$
7. $\frac{m}{5k} + \frac{9m}{5k}$ $\frac{2m}{k}$
8. $\frac{2r}{3s} + \frac{r}{3s}$ $\frac{r}{s}$
9. $\frac{7y}{y+1} - \frac{y}{y+1}$ $\frac{6y}{y+1}$
10. $\frac{3x}{x+2} + \frac{5x}{x+2}$ $\frac{8x}{x+2}$
11. $\frac{2x}{13} + \frac{x+7}{13}$ $\frac{3x+7}{13}$
12. $\frac{5a}{8} - \frac{a-4}{8}$ $\frac{a+1}{2}$

Written Exercises

Simplify.

A

1. $\frac{4}{5} + \frac{3}{5}$ $\frac{7}{5}$
2. $\frac{4x}{5} + \frac{3x}{5}$ $\frac{7x}{5}$
3. $\frac{6}{2a} + \frac{3}{2a}$ $\frac{9}{2a}$
4. $\frac{8}{9k} + \frac{10}{9k}$ $\frac{2}{k}$
5. $\frac{8n}{9} - \frac{2n}{9}$ $\frac{2n}{3}$
6. $\frac{a}{8} - \frac{4a}{8}$ $-\frac{3a}{8}$
7. $\frac{2x}{14} + \frac{5x}{14}$ $\frac{x}{2}$
8. $\frac{2s}{5} + \frac{8s}{5}$ $2s$
9. $\frac{4y}{3} - \frac{7y}{3}$ $-y$
10. $\frac{8p}{9} - \frac{5p}{9}$ $\frac{p}{3}$
11. $\frac{-3x}{4} - \frac{x}{4}$ $-x$
12. $\frac{-a}{5} - \frac{3a}{5}$ $\frac{-4}{5}$
13. $\frac{x}{y} + \frac{1}{y}$ $\frac{x+1}{y}$
14. $\frac{3}{a} + \frac{b}{a}$ $\frac{3+b}{a}$
15. $\frac{x}{3x} + \frac{1}{3x}$ $\frac{x+1}{3x}$
16. $\frac{a}{2a} - \frac{1}{2a}$ $\frac{a-1}{2a}$
17. $\frac{3}{a+b} + \frac{2}{a+b}$ $\frac{5}{a+b}$
18. $\frac{7}{x-1} + \frac{2}{x-1}$ $\frac{9}{x-1}$
19. $\frac{3}{y-2} - \frac{1}{y-2}$ $\frac{2}{y-2}$
20. $\frac{4}{a+3} - \frac{5}{a+3}$ $-\frac{1}{a+3}$
21. $\frac{2x}{x+1} + \frac{2}{x+1}$ 2
22. $\frac{3a}{a-1} - \frac{3}{a-1}$ 3
23. $\frac{v}{v-4} - \frac{4}{v-4}$ 1
24. $\frac{x+y}{3xy} + \frac{2x-y}{3xy}$ $\frac{1}{y}$
25. $\frac{3a+2b}{4ab} + \frac{a+2b}{4ab}$ $\frac{a+b}{ab}$
26. $\frac{3x}{x+2} - \frac{x+2}{x+2}$ $\frac{2x-2}{x+2}$
27. $\frac{2c+1}{7c} - \frac{4c-3}{7c}$ $\frac{-2c+4}{7c}$
28. $\frac{3w+2}{w+3} - \frac{w-4}{w+3}$ 2

SELF-TEST

Simplify.

1. $\frac{3x}{4} \cdot \frac{2}{x}$ $\frac{3}{2}$
2. $\frac{3x}{2y} \cdot \frac{y}{x}$ $\frac{3}{2}$
3. $\frac{3}{4} \cdot \frac{4a+4}{9}$ $\frac{a+1}{3}$
4. $\frac{x}{4} \div \frac{3}{4}$ $\frac{x}{3}$
5. $4m^2 \div \frac{12m}{5}$ $\frac{5m}{3}$
6. $\frac{3}{a^2b} \div \frac{9a}{ab^2}$ $\frac{b}{a^2}$
7. $\frac{3x-3}{2x^2+2} \div \frac{3}{2}$ $\frac{x-1}{x^2+1}$
8. $\frac{5x}{7} + \frac{4x}{7}$ $\frac{9x}{7}$
9. $\frac{5}{9a} - \frac{3}{9a}$ $\frac{2}{9a}$
10. $\frac{5a}{b} - \frac{6}{b}$ $\frac{5a-6}{b}$
11. $\frac{7x}{5y} + \frac{3x}{5y}$ $\frac{2x}{y}$
12. $\frac{m}{2n} - \frac{5}{2n}$ $\frac{m-5}{2n}$

11 • Renaming Fractions

There are two basic ideas to use when renaming fractions. The first of these we have been using already in this chapter.

1. You can divide both the numerator and denominator by the same number (except 0).

$$\frac{\overset{3}{\cancel{15}}}{\underset{4}{\cancel{20}}} = \frac{3}{4}$$

2. You can multiply the numerator and denominator by the same number (except 0).

$$\frac{3}{8} \overset{\times 2}{=} \frac{6}{16}$$

The second idea is used in the examples below.

EXAMPLE 1

$$\frac{3}{5} = \frac{?}{10}$$

5 was multiplied by 2 to get 10.

$$\frac{3}{5} = \frac{6}{10}$$

So, 3 is multiplied by 2 to get 6.

EXAMPLE 2

$$\frac{3}{a} = \frac{?}{4a}$$

a was multiplied by 4 to get $4a$.

$$\frac{3}{a} = \frac{12}{4a}$$

So, 3 is multiplied by 4 to get 12.

EXAMPLE 3

$$\frac{2}{3x} = \frac{?}{6x^2}$$

$3x$ was multiplied by $2x$ to get $6x^2$.

$$\frac{2}{3x} = \frac{4x}{6x^2}$$

So, 2 is multiplied by $2x$ to get $4x$.

Classroom Practice

Find the missing numerator.

1. $\frac{2}{3} = \frac{?}{12}$ 8
2. $\frac{4}{9} = \frac{?}{18}$ 8
3. $\frac{3a}{7} = \frac{?}{21}$ $9a$
4. $\frac{x}{2y} = \frac{?}{4y}$ $2x$
5. $\frac{7}{3a} = \frac{?}{12ab}$ $28b$
6. $\frac{a}{4a} = \frac{?}{8ab^2}$ $2ab^2$
7. $\frac{5}{a} = \frac{?}{a^2}$ $5a$
8. $\frac{2}{3x} = \frac{?}{3x^2}$ $2x$

Written Exercises

Find the missing numerator.

A

1. $\frac{4}{5} = \frac{?}{10}$ 8
2. $\frac{2}{3} = \frac{?}{18}$ 12
3. $\frac{4}{5} = \frac{?}{20}$ 16
4. $\frac{y}{6} = \frac{?}{18}$ $3y$
5. $\frac{x}{2} = \frac{?}{8}$ $4x$
6. $\frac{x}{3} = \frac{?}{9}$ $3x$
7. $\frac{y}{4} = \frac{?}{12}$ $3y$
8. $\frac{4a}{3} = \frac{?}{9}$ $12a$
9. $\frac{3x}{2} = \frac{?}{4}$ $6x$
10. $\frac{4}{a} = \frac{?}{2a}$ 8
11. $\frac{3}{x} = \frac{?}{3x}$ 9
12. $\frac{2}{b} = \frac{?}{3b}$ 6
13. $\frac{3}{4} = \frac{?}{8a}$ $6a$
14. $\frac{5}{3} = \frac{?}{9x}$ $15x$
15. $\frac{2}{3x} = \frac{?}{6xy}$ $4y$
16. $\frac{4}{5a} = \frac{?}{10ab}$ $8b$
17. $\frac{5}{x} = \frac{?}{3xy}$ $15y$
18. $\frac{3}{a} = \frac{?}{4ab}$ $12b$
19. $\frac{2}{n} = \frac{?}{4nm}$ $8m$
20. $\frac{1}{6r} = \frac{?}{24rs}$ $4s$
21. $\frac{5}{x} = \frac{?}{x^2}$ $5x$
22. $\frac{3a}{b} = \frac{?}{b^2}$ $3ab$
23. $\frac{2}{3b} = \frac{?}{3b^2}$ $2b$
24. $\frac{3}{4x} = \frac{?}{8x^2}$ $6x$

Puzzles & Things

A Stamp Connection

In how many different ways can 4 postage stamps be attached to each other? Here is one way. 19 ways



12 • Add, Subtract—Different Denominators

When you want to add or subtract fractions with different denominators, you will have to rename your fractions so that the denominators are the same. Let's simplify $\frac{n}{4} + \frac{n}{6}$.

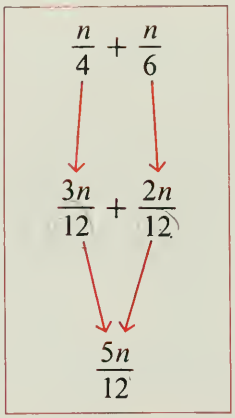
Step 1: Find a new denominator which can be divided by 4 and 6. You can use 12.

Step 2: Now rename your two fractions with the new denominator.

$$\frac{n}{4} = \frac{3n}{12} \qquad \frac{n}{6} = \frac{2n}{12}$$

Step 3: Find the sum.

$$\frac{3n}{12} + \frac{2n}{12} = \frac{5n}{12}$$



In the addition above, you could use 4×6, or 24, for your new denominator, but 12 is a little easier.

EXAMPLE 1 $\frac{3}{2x} + \frac{1}{2}$

Think: The new denominator can be $2x$.

$$\frac{3}{2x} + \frac{x}{2x} = \frac{3 + x}{2x}$$

EXAMPLE 2 $\frac{1}{5} - \frac{2}{y}$

Think: The new denominator can be $5y$.

$$\frac{y}{5y} - \frac{10}{5y} = \frac{y - 10}{5y}$$

EXAMPLE 3 $\frac{2}{ab} + \frac{5}{2a}$

Think: The new denominator can be $2ab$.

$$\frac{4}{2ab} + \frac{5b}{2ab} = \frac{4 + 5b}{2ab}$$

Written Exercises

Simplify.

A

1. $\frac{1}{2} + \frac{1}{3}$ 5/6
2. $\frac{x}{5} + \frac{x}{3}$ 8x/15
3. $\frac{y}{4} + \frac{y}{8}$ 3y/8
4. $\frac{4x}{9} - \frac{x}{6}$ 5x/18
5. $\frac{2x}{3} + \frac{x}{2}$ 7x/6
6. $\frac{3x}{2} + \frac{x}{4}$ 7x/4
7. $\frac{4x}{3} - \frac{x}{6}$ 7x/6
8. $\frac{a}{2} + \frac{2a}{3}$ 7a/6
9. $\frac{3}{2a} + \frac{1}{a}$ 5/2a
10. $\frac{5}{2x} - \frac{1}{4x}$ 9/4x
11. $\frac{4}{3y} - \frac{7}{2y}$ −13/6y
12. $\frac{4}{3a} + \frac{1}{4a}$ 19/12a
13. $\frac{3x}{4} + \frac{x}{3}$ 13x/12
14. $\frac{a}{3} - \frac{2a}{7}$ a/21
15. $\frac{5}{x} + \frac{1}{2x}$ 11/2x
16. $\frac{4}{a} + \frac{3}{2a}$ 11/2a
17. $\frac{5}{n} + \frac{3}{4n}$ 23/4n
18. $\frac{7}{3n} - \frac{4}{9n}$ 17/9n
19. $\frac{3x}{8} - \frac{x}{12}$ 7x/24
20. $\frac{1}{9y} - \frac{1}{15y}$ 2/45y
21. $\frac{1}{5} - \frac{3}{x}$ (x − 15)/5x
22. $\frac{3}{4} + \frac{1}{a}$ (3a + 4)/4a
23. $\frac{4}{x} + \frac{1}{2}$ (8 + x)/2x
24. $\frac{1}{a} + \frac{3}{2ab}$ (2b + 3)/2ab
25. $\frac{2}{3x} + \frac{1}{6xy}$ (4y + 1)/6xy
26. $\frac{1}{2x} - \frac{1}{3xy}$ (3y − 2)/6xy
27. $\frac{2}{x} + \frac{3}{y}$ (2y + 3x)/xy
28. $\frac{3}{b} - \frac{1}{2b^2}$ (6b − 1)/2b²

B

29. $\frac{3x}{4} - \frac{2x}{3} + \frac{x}{6}$ x/4
30. $\frac{a}{3} + \frac{2a}{6} - \frac{5a}{2}$ −11a/6
31. $\frac{2x}{5} - \frac{1}{4} + \frac{3x}{5}$ (4x − 1)/4
32. $\frac{3a}{a} + \frac{a}{a^2} - \frac{2a}{3a}$ (7a + 3)/3a
33. $\frac{x}{2x} + \frac{3x}{x} - \frac{5}{3x}$ (21x − 10)/6x
34. $\frac{6}{y^2} - \frac{5x}{3y} + \frac{4x}{6xy^2}$ (20 − 5xy)/3y²

For more practice, see page 427.

SELF-TEST

Find the missing numerator.

1. $\frac{7}{9} = \frac{?}{36}$ 28
2. $\frac{x}{6} = \frac{?}{24}$ 4x
3. $\frac{3}{m} = \frac{?}{mn}$ 3n
4. $\frac{5}{x} = \frac{?}{x^2}$ 5x

Simplify.

5. $\frac{1}{3} + \frac{1}{5}$ 8/15
6. $\frac{5x}{7} - \frac{x}{3}$ 8x/21
7. $\frac{1}{9a} + \frac{3}{4a}$ 31/36a
8. $\frac{3}{m} - \frac{4}{3m^2}$ (9m − 4)/3m²

13 • More Difficult Fractions (Optional)

When the fractions you add or subtract have more than one term in their numerators, you have a few more steps to do.

EXAMPLE 1

$$\frac{x+2}{3} + \frac{x+1}{5}$$

$$= \frac{5(x+2)}{15} + \frac{3(x+1)}{15}$$

$$= \frac{5(x+2) + 3(x+1)}{15}$$

$$= \frac{5x + 10 + 3x + 3}{15}$$

$$= \frac{8x + 13}{15}$$

Be careful. Don't be tempted to do this:

$$\frac{4a - 2 - a + 3}{8}$$

EXAMPLE 2

$$\frac{2a-1}{4} - \frac{a+3}{8}$$

$$= \frac{2(2a-1)}{8} - \frac{(a+3)}{8}$$

$$= \frac{2(2a-1) - (a+3)}{8}$$

$$= \frac{4a - 2 - a - 3}{8}$$

$$= \frac{3a - 5}{8}$$

Classroom Practice

Complete.

1. $\frac{x+4}{2} + \frac{x-1}{3} = \frac{?}{6} + \frac{?}{6}$ — 3(x + 4), 2(x − 1)

2. $\frac{2x+3}{4} - \frac{x+1}{2} = \frac{?}{4} - \frac{?}{4}$ — 2x + 3, 2(x + 1)

3. $\frac{2a+3}{5} - \frac{a-1}{2} = \frac{?}{10} - \frac{?}{10}$ — 2(2a + 3), 5(a − 1)

4. $\frac{x+1}{3} + \frac{x+2}{5} = \frac{?}{15} + \frac{?}{15}$ — 5(x + 1), 3(x + 2)

5. Find the errors in the work below.

$$\frac{2x+5}{3} - \frac{x-3}{4} = \frac{4(2x+5)}{12} - \frac{3(x-3)}{12}$$

$$= \frac{4(2x+5) - 3(x-3)}{12}$$ (+)

$$= \frac{8x + 20 - 3x - 9}{12}$$

$$= \frac{5x - 11}{12}$$ $\frac{5x + 29}{12}$

Written Exercises

Simplify.

1. $\frac{x+1}{8} + \frac{3}{4}$ $\frac{x+7}{8}$

2. $\frac{2k-1}{6} - \frac{k+1}{9}$ $\frac{4k-5}{18}$

3. $\frac{3t-2}{6} + \frac{2t-1}{4}$ $\frac{12t-7}{12}$

4. $\frac{x+2}{3} + \frac{x}{5}$ $\frac{8x+10}{15}$

5. $\frac{x-5}{4} + \frac{2x}{3}$ $\frac{11x-15}{12}$

6. $\frac{y-9}{4} + \frac{3y}{8}$ $\frac{5y-18}{8}$

7. $\frac{a-2}{4} - \frac{a}{12}$ $\frac{a-3}{6}$

8. $\frac{a+3b}{8} - \frac{a}{2}$ $\frac{3b-3a}{8}$

9. $\frac{x-3}{4} - \frac{5x}{6}$ $-\frac{7x+9}{12}$

10. $\frac{x+2}{3} + \frac{x+1}{2}$ $\frac{5x+7}{6}$

11. $\frac{x-1}{2} + \frac{x+3}{4}$ $\frac{3x+1}{4}$

12. $\frac{a-5}{3} + \frac{a-1}{6}$ $\frac{3a-11}{6}$

13. $\frac{x+5}{3} + 1$ $\frac{x+8}{3}$

14. $\frac{2a+1}{5} - 2$ $\frac{2a-9}{5}$

15. $\frac{4y-4}{3} + 8$ $\frac{4y+20}{3}$

16. $\frac{4y+4}{4} - \frac{y-1}{2}$ $\frac{y+3}{4}$

17. $\frac{4x+2}{2} - \frac{x-2}{3}$ $\frac{5x+5}{3}$

18. $\frac{2b-2}{4} - \frac{b-4}{3}$ $\frac{b+5}{6}$

19. $\frac{r}{4} + \frac{r-2}{8} - \frac{r-4}{16}$ $\frac{5r}{16}$

20. $\frac{4+3a}{6} - \frac{5a}{18} + \frac{3-a}{12}$ $\frac{5a+33}{36}$

21. $\frac{10x+4}{5} - \frac{6x+2}{3} - \frac{2}{15}$ 0

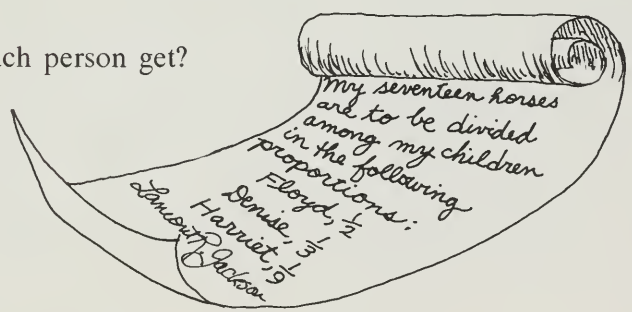
22. $\frac{x+1}{8x} + \frac{3x-1}{12x} - \frac{1}{6x}$ $\frac{3x-1}{8x}$

Puzzles & Things

Horseplay

How many horses will each person get?

Floyd: 9
Denise: 6
Harriet: 2



14 • Equations with Fractions

Sometimes you'll want to solve equations with fractions in them. Let's see how these equations can be solved.

$$\frac{x}{2} + \frac{x}{3} = 10$$

Think about the left side first. Rename the fractions so that they have the same denominator.

$$\frac{3x}{6} + \frac{2x}{6} = 10$$

$$\frac{3x + 2x}{6} = 10$$

$$\frac{5x}{6} = \frac{10}{1}$$

Now you have a proportion. All you have to do is to cross-multiply.

$$5x = 60$$

$$x = 12$$

Check:

$$\frac{12}{2} + \frac{12}{3} \stackrel{?}{=} 10$$

$6 + 4$	10
10	

✓

It is easy to do just one step at a time, and simplify one side of the equation at a time. That way you can get a proportion which is not difficult to solve.

EXAMPLES $\frac{2}{5} + \frac{3}{10} = \frac{7}{n}$

$$\frac{4}{10} + \frac{3}{10} = \frac{7}{n}$$

$$\frac{4 + 3}{10} = \frac{7}{n}$$

$$\frac{7}{10} = \frac{7}{n}$$

$$7n = 70$$

$$n = 10$$

$\frac{a}{4} + 5 = \frac{3a}{2}$

$$\frac{a}{4} + \frac{20}{4} = \frac{3a}{2}$$

$$\frac{a + 20}{4} = \frac{3a}{2}$$

$$2(a + 20) = 4(3a)$$

$$2a + 40 = 12a$$

$$40 = 10a$$

$$4 = a$$

It is always wise to check your answers.

Written Exercises

Solve.

A

1. $\frac{x}{3} - \frac{x}{5} = 4$ 30
2. $\frac{x}{6} + \frac{x}{7} = 13$ 42
3. $\frac{x}{6} + \frac{x}{3} = 6$ 12
4. $\frac{1}{4} + \frac{1}{12} = \frac{x}{3}$ 1
5. $\frac{2}{3} - \frac{1}{6} = \frac{x}{4}$ 2
6. $\frac{4}{5} - \frac{2}{3} = \frac{n}{15}$ 2
7. $\frac{2}{5} + \frac{3}{10} = \frac{7}{n}$ 10
8. $\frac{4}{5} - \frac{2}{15} = \frac{6}{x}$ 9
9. $\frac{3}{4} - \frac{5}{6} = \frac{5}{y}$ −60
10. $\frac{n}{2} + \frac{2n}{3} = 7$ 6
11. $\frac{a}{6} - \frac{2a}{9} = \frac{1}{2}$ −9
12. $\frac{1}{c} + \frac{3}{2c} = \frac{1}{6}$ 15
13. $\frac{3}{4n} + \frac{1}{n} = \frac{7}{8}$ 2
14. $\frac{3n}{7} - \frac{n}{3} = 4$ 42
15. $\frac{2}{x} - \frac{4}{3x} = \frac{2}{9}$ 3

B

16. $\frac{a+2}{3} + \frac{a-1}{6} = 5$ 9
17. $\frac{x-1}{4} - \frac{2x-3}{4} = 5$ −18
18. $\frac{x}{4} - \frac{x+5}{3} = 6$ −92
19. $\frac{x+3}{8} - \frac{x-2}{6} = 1$ −7
20. $\frac{c-1}{2c} + \frac{c+3}{4c} = \frac{5}{8}$ −2
21. $\frac{2-x}{x} - \frac{x+3}{3x} = \frac{-1}{3}$ 1

C

22. $\frac{6x-4}{3} - 2 = \frac{18-4x}{3} + x$ 4
23. $2 - \frac{7x-1}{6} = 3x - \frac{19x+3}{4}$ −5

Puzzles & Things

Diophantus was an ancient Greek mathematician. According to legend, this problem was written on his tombstone:

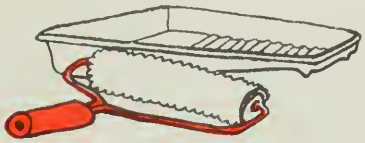
> HERE LIE THE REMAINS OF DIOPHANTUS. HE WAS A CHILD FOR ONE SIXTH OF HIS LIFE. AFTER ONE TWELFTH MORE, HE BECAME A MAN. AFTER ONE SEVENTH MORE, HE MARRIED. FIVE YEARS LATER HIS SON WAS BORN. THE SON LIVED HALF AS LONG AS HIS FATHER AND DIED FOUR YEARS BEFORE HIS FATHER.

How old did Diophantus live to be? 84 years

15 • Work Problems

Suppose an office manager wants to figure how long it will take to get a report typed by two typists who work at different speeds.





Suppose your mother helps you to paint a room and you wonder how long the job will take.

Algebra can help solve both problems.

Let's consider the painting problem.

EXAMPLE Ella and her mother plan to paint the living room. Ella thinks it would take her 10 hours alone. Her mother says she could do it herself in 5 hours. How long would it take them to do the job together?

	Ella	Mother	Together
Hours needed	10	5	n
Part done in one hour	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{n}$

$$\underbrace{\text{Part Ella does in one hour}}_{\frac{1}{10}} + \underbrace{\text{Part Mother does in one hour}}_{\frac{1}{5}} = \underbrace{\text{Part of job done in one hour}}_{\frac{1}{n}}$$

$$\frac{1}{10} + \frac{1}{5} = \frac{1}{n}$$

$$\frac{1}{10} + \frac{2}{10} = \frac{1}{n}$$

$$\frac{1+2}{10} = \frac{1}{n}$$

$$\frac{3}{10} = \frac{1}{n}$$

$$3n = 10$$

$$n = 3\frac{1}{3}$$

Look above in the chart. n represents the number of hours needed if they work together. We've solved the problem.

Answer: It will take them $3\frac{1}{3}$ hours working together.

Written Exercises

A **1.** Mr. Macy's two grandchildren offer to paint his kitchen.
Sandy thinks it would take her 10 hours alone.
Meg says it would take her 15 hours alone.
How long would the job take if they worked together? 6 hours

	Sandy	Meg	Together
Hours needed	10	15	n
Part done in one hour	$\frac{1}{10}$	$\frac{1}{15}$	$\frac{1}{n}$

$$\frac{1}{10} + \frac{1}{15} = \frac{1}{n}$$

2. It would take Fran 3 hours to type the copy for the school paper.
When Luis does it alone it takes 6 hours.
How long would it take if they worked together? 2 hours

	Fran	Luis	Together
Hours needed	3	6	h
Part done in one hour	$\frac{1}{3}$	? $\frac{1}{6}$	? $\frac{1}{h}$

3. A chef at Harbor Restaurant needs 2 hours to make 15 pies.
The assistant chef would need 4 hours.
If they do the job together, how long will it take? $1\frac{1}{3}$ hours

	Chef	Assistant	Together
Hours needed	? 2	? 4	x ?
Part done in one hour	$\frac{1}{2}$?	$\frac{1}{4}$?	? $\frac{1}{x}$

4. A pool can be filled by one pipe in 15 hours.
A larger pipe can do it in 12 hours.
How long will it take if both pipes are used? $6\frac{2}{3}$ hours

	Small Pipe	Large Pipe	Together
Hours needed	15 ?	12 ?	n ?
Part done in one hour	$\frac{1}{15}$?	$\frac{1}{12}$?	$\frac{1}{n}$?

5. Taddy and Rose deliver groceries for a neighborhood store.
Today Taddy needs 4 hours to deliver the groceries alone. $2\frac{2}{9}$ hours
Rose says it would take her 5 hours.
How long would it take to do the job together?

	Taddy	Rose	Together
Hours needed	4 ?	5 ?	x ?
Part done in one hour	$\frac{1}{4}$?	$\frac{1}{5}$?	$\frac{1}{x}$?

6. Fred works on an assembly line in a factory.
It takes him 20 minutes to complete one job alone.
It takes his partner 30 minutes, working alone.
How long would the job take if the two worked together? 12 minutes

	Fred	Partner	Together
Min needed	? 20	? 30	n ?
Part done in one min	$\frac{1}{20}$?	$\frac{1}{30}$?	$\frac{1}{n}$?

7. Pete, Marcia, and Andrea have to type the company annual report.
Pete thinks it would take him 10 hours, working alone.
Andrea says it would take her 10 hours alone also.
Marcia says she could do it in 8 hours.
How long would the job take if they worked together? $3\frac{1}{13}$ hours

	Pete	Marcia	Andrea	Together
Hours needed	10	8	10	x
Part done in one hour	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{10}$	$\frac{1}{x}$

8. Celia, Al, and Julio are going to decorate the gym.
It would take Julio 3 hours, working alone.
It would take Celia and Al each 4 hours, working alone.
How long will the job take if they work together? $1\frac{1}{5}$ hours

	Celia	Al	Julio	Together
Hours needed	4 ?	4 ?	3 ?	n ?
Part done in one hour	$\frac{1}{4}$?	$\frac{1}{4}$?	$\frac{1}{3}$?	$\frac{1}{n}$?

B **9.** Judy Rogers needs 6 hours to correct some papers. When her assistant helps, the job is finished in 4 hours. How long would it take the assistant to work alone? 12 hours

	Judy	Assistant	Together
Hours needed	6 ?	x	4
Part done in one hour	$\frac{1}{6}$?	$\frac{1}{x}$	? $\frac{1}{4}$

10. Frank can do a typing job in 10 hours. When Paula helps him, it takes only 6 hours. If Paula did the job alone, how long would it take her? 15 hours

	Frank	Paula	Together
Hours needed	? 10	? n	? 6
Part done in one hour	? $\frac{1}{10}$	? $\frac{1}{n}$	? $\frac{1}{6}$

11. Working together, Maria and Chris can hang wallpaper in Maria's kitchen in 4 hours. Maria could do the job alone in 6 hours. How long would it take Chris to do the job alone? 12 hours

	Maria	Chris	Together
Hours needed	6 ?	x ?	4 ?
Part done in one hour	$\frac{1}{6}$?	$\frac{1}{x}$?	$\frac{1}{4}$?



C **12.** The boss in the repair shop needs 6 hours to do a special repair job alone. Yesterday her assistant started helping her 2 hours after the job started. They finished the job 3 hours later. How long would it have taken the assistant to do the job alone? 18 hours

For more practice, see page 428.

SELF-TEST

Solve.

1. $\frac{x}{4} - \frac{x}{6} = 11$ 132
2. $\frac{a}{3} + \frac{a}{2} = 5$ 5
3. $\frac{x}{3} - \frac{2x}{9} = 4$ 36
4. $\frac{1}{n} + \frac{3}{2n} = \frac{1}{2}$ 5
5. $\frac{2}{x} + \frac{3}{4x} = \frac{1}{8}$ 22
6. $\frac{1}{2x} + \frac{2}{3x} = \frac{1}{12}$ 14

16 • Binomial Denominators (Optional)

When you want to add or subtract fractions with more than one term in the denominator, there will probably be quite a few steps.

EXAMPLE 1

$$\frac{4}{x} + \frac{3}{x+2}$$

Think: I must find a new denominator which is divisible by both x and $x + 2$. $x(x + 2)$ will work.

$$= \frac{4(x+2)}{x(x+2)} + \frac{3x}{x(x+2)}$$

$$= \frac{4(x+2) + 3x}{x(x+2)}$$

$$= \frac{4x + 8 + 3x}{x(x+2)}$$

$$= \frac{7x + 8}{x(x+2)}$$

EXAMPLE 2

$$\frac{2}{a-4} + \frac{3}{a+2}$$

Think: I must find a new denominator which is divisible by both $a - 4$ and $a + 2$. $(a - 4)(a + 2)$ will work.

$$= \frac{2(a+2)}{(a-4)(a+2)} + \frac{3(a-4)}{(a-4)(a+2)}$$

$$= \frac{2(a+2) + 3(a-4)}{(a-4)(a+2)}$$

$$= \frac{2a + 4 + 3a - 12}{(a-4)(a+2)}$$

$$= \frac{5a - 8}{(a-4)(a+2)}$$

EXAMPLE 3

$$\frac{5}{x-3} - \frac{4}{x+1}$$

Think: I must find a new denominator which is divisible by both $x - 3$ and $x + 1$. $(x - 3)(x + 1)$ will work.

$$= \frac{5(x+1)}{(x-3)(x+1)} - \frac{4(x-3)}{(x-3)(x+1)}$$

$$= \frac{5(x+1) - 4(x-3)}{(x-3)(x+1)}$$

$$= \frac{5x + 5 - 4x + 12}{(x-3)(x+1)}$$

$$= \frac{x + 17}{(x-3)(x+1)}$$

Written Exercises

Simplify. 6. $\frac{-54}{(b+3)(b-3)}$ 9. $\frac{x^2+10x-21}{(x+3)(x-3)}$ 15. $\frac{4(a-b)+2a(a+b)}{(a+b)(a-b)}$

1. $\frac{5}{k}+\frac{3}{k+1}$ $\frac{8k+5}{k(k+1)}$
2. $\frac{2}{t}+\frac{3}{t+5}$ $\frac{5(t+2)}{t(t+5)}$
3. $\frac{2}{x}-\frac{5}{x-3}$ $\frac{-3(x+2)}{x(x+5)}$
4. $\frac{r}{r+5}-\frac{r}{r+2}$ $\frac{-3r}{(r+5)(r+2)}$
5. $\frac{a}{a+4}-\frac{a}{a-4}$ $\frac{-8a}{(a+4)(a-4)}$
6. $\frac{9}{b+3}-\frac{9}{b-3}$
7. $\frac{6}{y+3}-\frac{y}{y-2}$ $\frac{y^2+3y-12}{(y+3)(y-2)}$
8. $\frac{c}{c-4}-\frac{5}{c+5}$ $\frac{c^2+20}{(c-4)(c+5)}$
9. $\frac{7}{x+3}+\frac{x}{x-3}$
10. $\frac{6}{a+4}+\frac{3}{a-1}$ $\frac{3(3a+2)}{(a+4)(a-1)}$
11. $\frac{b}{b+6}+\frac{2b}{b-1}$ $\frac{b(3b+11)}{(b+6)(b-1)}$
12. $\frac{a}{4-a}-\frac{a}{a-4}$ $\frac{2a}{4-a}$
13. $\frac{7}{x+y}-\frac{x}{x-y}$ $\frac{7-x}{x+y}$
14. $\frac{s}{r-s}+\frac{r}{r+2}$ $\frac{r^2+2s}{(r-s)(r+2)}$
15. $\frac{4}{a+b}+\frac{2a}{a-b}$
16. $\frac{3}{x+6}-\frac{11}{x^2-36}$ $\frac{3x-29}{(x+6)(x-6)}$
17. $\frac{8}{(c+d)^2}+\frac{5}{c+d}$ $\frac{8+5c+5d}{(c+d)^2}$
18. $\frac{12}{t+s}-\frac{4}{t^2+ts}$ $\frac{4(3t-1)}{t(t+5)}$
19. $\frac{1}{x^2-y^2}+\frac{3}{x-y}$ $\frac{1+3x+3y}{(x+y)(x-y)}$
20. $\frac{c}{c^2-9}+\frac{1}{c+3}$ $\frac{2c-3}{(c+3)(c-3)}$
21. $\frac{2a+5}{4-a^2}+\frac{5}{a+2}$ $\frac{3(a-5)}{(a+2)(a-2)}$

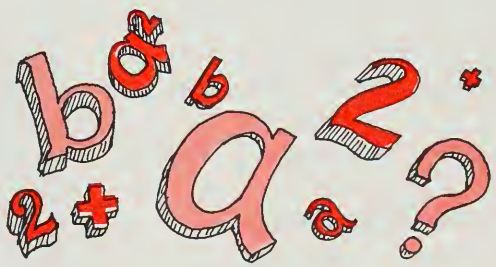
Solve for the variable.

22. $\frac{2}{a+1}-\frac{1}{a+3}=\frac{1}{a}$ 3
23. $\frac{3}{x+1}-\frac{2}{x-1}=\frac{1}{x}$ $\frac{1}{5}$

Puzzles & Things

What Is Wrong?

$$\begin{aligned} ab &= a^2 \\ ab - b^2 &= a^2 - b^2 \\ b(a-b) &= (a+b)(a-b) \\ b &= a+b \\ b &= 2b \\ 1 &= 2 \end{aligned}$$



If $ab = a^2$, then $b = a$ and $a - b = 0$. Thus, we cannot divide by $a - b$, which was done to get from step 3 to step 4.

Reviewing Arithmetic Skills

Recall that decimals are another way of writing fractions.

tens	ones	tenths	hundredths	thousandths
		4		
		4	0	
		5	6	

$\leftarrow \frac{4}{10}$

$\leftarrow \frac{40}{100}$

$\leftarrow \frac{56}{100}$

Note: these are equal.

Let's compare 0.6 and 0.45.

0.60 and 0.45

It's easier to compare numbers with the same number of decimal places.

$\frac{60}{100} > \frac{45}{100}$ *so* $0.60 > 0.45$

Compare. Write $>$ or $<$.

1. 0.2 ? 0.9 (<)

2. 0.72 ? 0.59 (>)

3. 2.4 ? 3.7 (<)

4. 5.8 ? 5.08 (>)

5. 7.41 ? 7.48 (<)

6. 6.05 ? 5.21 (>)

7. 8.36 ? 8.29 (>)

8. 7.4 ? 6.08 (>)

9. 0.09 ? 0.009 (>)

10. 3.07 ? 2.17 (>)

11. 0.03 ? 0.3 (<)

12. 0.042 ? 0.42 (<)

To round decimals, use the same rules as you use to round integers.

Round 2.346 to one decimal place. *Answer:* 2.3

This digit is less than 5. Round *down*.

Round 6.786 to two decimal places. *Answer:* 6.79

This digit is 5 or greater. Round *up*.

Round to one decimal place.

13. 6.73 6.7

14. 5.84 5.8

15. 9.29 9.3

16. 7.68 7.7

17. 4.273 4.3

Round to two decimal places.

18. 0.651 0.65

19. 9.207 9.21

20. 3.415 3.42

21. 0.078 0.08

22. 5.924 5.92

CAREER NOTEBOOK

Environmental Sciences

Society is becoming more and more aware of the need to protect our environment against pollution, land abuse, and destruction of wildlife. Here are some careers in the field of pollution control.



Air pollution engineers study the effects of air pollution, and develop plans for controlling it.

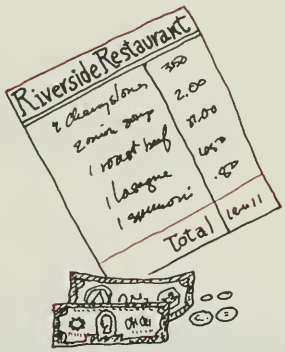


Chemical engineers can recycle waste materials, converting old newspapers into sugar, for example.



Water pollution technicians analyze the chemical and bacterial content of water samples.

CONSUMER CORNER



Tipping

People in a restaurant who wait on tables usually get tips. When you are the customer, do you know how to figure the tip? The amount you leave depends on you, but people usually leave about a 10%, 15%, or 20% tip. All the figuring is done in your head.

First think about percents and what they mean.

100%	50%	25%	10%
the whole	*half*	*one fourth*	*one tenth*

Suppose you want to give a 10% tip. All you have to do is round the amount on your restaurant check, then move the decimal point one place to the left.

Your check	This is about	about 10%
$5.65	$5.70 — *Think:* $5.70 →	$.57

Your check	This is about	about 10%
$2.29	$2.30 — *Think:* $2.30 →	$.23

Try figuring 10% in your head.

1. $4.37 $.43 **2.** $1.98 $.20 **3.** $6.57 $.66 **4.** $9.82 $.98 **5.** $3.65 $.37

6. $7.91 $.79 **7.** $10.85 $1.09 **8.** $12.98 $1.30 **9.** $16.49 $1.65 **10.** $22.35 $2.24

Once you know how to figure 10%, then you can find 20% by doubling your result.

Your check	This is about	about 10%	about 20%
\$3.84	\$3.80	\$.38 *or* \$.40	\$.80

Your check	This is about	about 10%	about 20%
\$9.62	\$9.60	\$.96 *or* \$1.00	\$2.00 (a little less)

Remember, none of your figuring must be exact.

Figure a 20% tip in your head.

11. \$1.48 \$.30 **12.** \$6.97 \$1.40 **13.** \$8.42 \$1.60 **14.** \$3.59 \$.80 **15.** \$4.12 \$.80

16. \$9.35 \$1.80 **17.** \$12.50 \$2.50 **18.** \$15.00 \$3.00 **19.** \$13.25 \$2.60 **20.** \$18.60 \$3.80

Now consider the 15% tip. You can figure the 10% tip, then add to that $\frac{1}{2}$ of the 10% tip. Here's how it works.

Your check	This is about	about 10%	about $\frac{1}{2}$ of 10%	about 15%
\$4.12	\$4.10	\$.41 *or* \$.40	\$.20	\$.60

Figure a 15% tip in your head.

21. \$1.50 \$.25 **22.** \$3.65 \$.60 **23.** \$2.95 \$.45 **24.** \$4.26 \$.60 **25.** \$8.50 \$1.20

26. \$7.35 \$1.05 **27.** \$2.43 \$.35 **28.** \$9.55 \$1.50 **29.** \$4.68 \$.75 **30.** \$10.66 \$1.65

Reviewing the Chapter

Simplify. *(See pp. 268–273.)*

1. $\frac{8}{24}$ $\frac{1}{3}$
2. $\frac{3}{15x}$ $\frac{1}{5x}$
3. $\frac{11x}{22}$ $\frac{x}{2}$
4. $\frac{-4a}{12a}$ $\frac{-1}{3}$
5. $\frac{10n^2}{5n}$ $2n$
6. $\frac{18y}{-27y^2}$ $-\frac{2}{3y}$
7. $\frac{3x^2y}{18xy}$ $\frac{x}{6}$
8. $\frac{-4a^2b}{-ab^2}$ $\frac{4a}{b}$
9. $\frac{16x^2y^3}{8x^2y}$ $2y^2$
10. $\frac{-7ab}{28ab^2}$ $\frac{1}{4b}$
11. $\frac{3a-15}{3}$ $a-5$
12. $\frac{x-4}{3x-12}$ $\frac{1}{3}$
13. $\frac{3x-3y}{3x+3y}$ $\frac{x-y}{x+y}$
14. $\frac{a^2-b^2}{a+b}$ $a-b$
15. $\frac{n^2+4n+4}{n+2}$ $n+2$
16. $\frac{(3+x)^2}{9-x^2}$ $\frac{3+x}{3-x}$
17. $\frac{12m^2n^2}{4mn-8m^2n^2}$ $\frac{3mn}{1-2mn}$
18. $\frac{3x^2-3y^2}{x^2-y^2}$ 3
19. $\frac{3-a}{a-3}$ -1
20. $\frac{x-y}{y-x}$ -1
21. $\frac{1-x}{2x-2}$ $-\frac{1}{2}$
22. $\frac{7-a}{a^2-49}$ $-\frac{1}{a+7}$

Solve. *(See pp. 274–277.)*

23. The ratio of blue to yellow in a paint mixture is 3 to 2.
How much blue paint is needed to make 10 cans of the mixture? 6 cans

24. Central High's soccer team has 22 members, including coaches.
The ratio of coaches to players is 2 to 9.
How many players are there? 18 players

Solve. *(See pp. 278–283.)*

25. $\frac{x}{14}=\frac{2}{7}$ 4
26. $\frac{2n}{6}=\frac{12}{9}$ 4
27. $\frac{5}{x}=\frac{10}{20}$ 10
28. $\frac{3y}{y+1}=\frac{2}{1}$ 2
29. $\frac{n}{3}=\frac{n+1}{6}$ 1
30. $\frac{3a+1}{4}=\frac{a-2}{6}$ -1
31. $\frac{x-2}{x+2}=\frac{1}{3}$ 4
32. $\frac{2b}{b-7}=\frac{1}{4}$ -1

33. 18 oranges cost \$2.70.
How much will 36 oranges cost? \$5.40

34. 3 dozen apples cost \$2.25.
How much will 2 dozen cost? \$1.50

Simplify. *(See pp. 284–285.)*

35. $\frac{9}{21}\cdot\frac{7}{63}$ $\frac{1}{21}$
36. $\frac{x}{4}\cdot\frac{12}{x^2}$ $\frac{3}{x}$
37. $\frac{3a}{7}\cdot\frac{14a}{a^2}$ 6
38. $\frac{10m^2}{3m}\cdot\frac{m}{5}$ $\frac{2m^2}{3}$
39. $\frac{5}{3x-3}\cdot\frac{3}{75}$ $\frac{1}{15x-15}$
40. $\frac{x^2y}{xy^2}\cdot\frac{3z}{4z^2}$ $\frac{3x}{4y^2}$
41. $\frac{x^2-4}{25}\cdot\frac{5}{x+2}$ $\frac{x-2}{5}$
42. $\frac{a+3}{6}\cdot\frac{18}{a^2-9}$ $\frac{3}{a-3}$

Simplify. *(See pp. 286–287.)*

43. $\frac{3}{4} \div \frac{1}{12}$ 9 **44.** $\frac{n}{3} \div \frac{n}{9}$ 3 **45.** $\frac{5}{a^2b^2} \div \frac{25}{ab}$ $\frac{1}{5ab}$ **46.** $\frac{6}{mn^2} \div \frac{3}{m^2n}$ $\frac{2m}{n}$

47. $\frac{x^2 - y^2}{16} \div \frac{x + y}{4}$ $\frac{x - y}{4}$ **48.** $\frac{a + b}{28} \div \frac{a - b}{7}$ $\frac{a + b}{4a - 4b}$ **49.** $\frac{3a - 3}{15} \div \frac{a - 1}{3}$ $\frac{3}{5}$

Simplify. *(See pp. 288–289.)*

50. $\frac{5}{12} + \frac{7}{12}$ 1 **51.** $\frac{5x}{3} + \frac{x}{3}$ $2x$ **52.** $\frac{7}{a + 1} + \frac{8}{a + 1}$ $\frac{15}{a + 1}$ **53.** $\frac{4}{m + n} + \frac{n}{m + n}$ $\frac{4 + n}{m + n}$

54. $\frac{n}{2} - \frac{4}{2}$ $\frac{n - 4}{2}$ **55.** $\frac{12}{x} - \frac{8}{x}$ $\frac{4}{x}$ **56.** $\frac{n + 1}{4b} - \frac{1}{4b}$ $\frac{n}{4b}$ **57.** $\frac{6a}{a + 2} - \frac{3a}{a + 2}$ $\frac{3a}{a + 2}$

Simplify. *(See pp. 290–293.)*

58. $\frac{1}{3} + \frac{1}{5}$ $\frac{8}{15}$ **59.** $\frac{n}{3} - \frac{n}{5}$ $\frac{2n}{15}$ **60.** $\frac{2x}{7} + \frac{x}{2}$ $\frac{11x}{14}$ **61.** $\frac{3a}{4} - \frac{a}{3}$ $\frac{5a}{12}$

62. $\frac{1}{x} + \frac{1}{4}$ $\frac{4 + x}{4x}$ **63.** $\frac{1}{3} - \frac{3}{x}$ $\frac{x - 9}{3x}$ **64.** $\frac{1}{2b} - \frac{4}{3b^2}$ $\frac{3b - 8}{6b^2}$ **65.** $\frac{1}{x} - \frac{1}{y}$ $\frac{y - x}{xy}$

Solve. *(See pp. 296–301.)*

66. $\frac{n}{3} - \frac{n}{4} = 1$ 12 **67.** $\frac{1}{6} + \frac{1}{2} = \frac{x}{3}$ 2 **68.** $\frac{2x}{3} + \frac{x}{2} = 7$ 6

69. $\frac{4x}{5} - \frac{x}{15} = 11$ 15 **70.** $\frac{3}{4x} - \frac{5}{2x} = 7$ $-\frac{1}{4}$ **71.** $\frac{1}{n} + \frac{3}{4n} = \frac{1}{4}$ 7

72. Elton and Paulette are papering their living room.
Elton says he could do it in 6 hours alone.
Paulette says she could do it in 5 hours alone.
If they work together, how long will it take? $2\frac{8}{11}$ hours

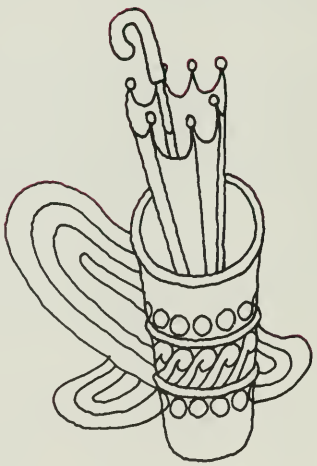
73. John and Marla take turns mowing the lawn each week.
John takes 2 hours for the job. Marla needs 3 hours.
If they work together, how long will it take? $1\frac{1}{5}$ hours

74. Sylvia and Jake are making curtains for the Home Economics kitchen.
Sylvia could do the job in 10 hours alone.
Jake could do it in 7 hours alone.
How long will it take them to do the job together? $4\frac{2}{17}$ hours

A permission-to-reproduce chapter test can be found on page T19.
A cumulative review of Chapters 1–9 can be found on page 442.

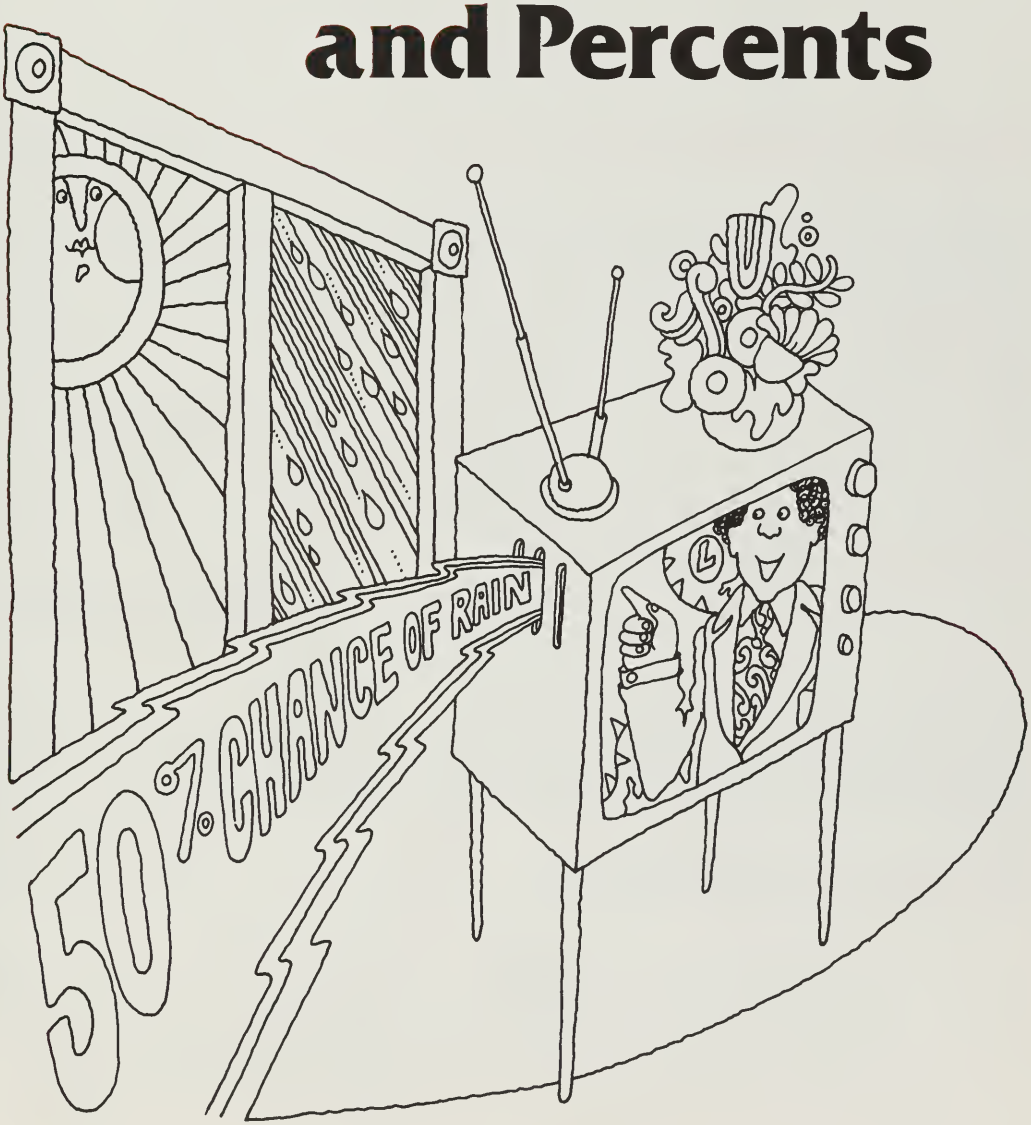
Here's what you'll learn in this chapter:

1. To solve equations with decimals.
2. To use equations with decimals to solve word problems.
3. To solve equations with percents.
4. To solve word problems involving percents.
5. To solve interest, investment, and mixture problems.



Chapter 10

Decimals and Percents



1 • Decimals

Many equations contain decimals. First, take a new look at decimals and how to work with them.

Fraction	Decimal	Read
$\frac{7}{10}$	0.7	seven tenths
$\frac{13}{100}$	0.13	thirteen hundredths
$\frac{27}{1000}$	0.027	twenty-seven thousandths

Working with decimals is often much easier than working with ordinary fractions. For instance, in addition and subtraction all you do is line up the decimal points and compute as you would with whole numbers.

EXAMPLE 1 Add: $2.5 + 0.71 + 26.403$

$$\begin{array}{r} 2.500 \\ 0.710 \\ +26.403 \\ \hline 29.613 \end{array}$$

You may want to fill in zeros.

EXAMPLE 2 Subtract: $311.7 - 2.568$

$$\begin{array}{r} 311.700 \\ -\ \ 2.568 \\ \hline 309.132 \end{array}$$

Fill in zeros here.

To multiply, don't pay attention to the decimal points at first. Multiply as you would with whole numbers, then put the decimal point in the answer.

EXAMPLE 3 Multiply: 0.05×2.7

$$\begin{array}{r} 2.7 \\ \times 0.05 \\ \hline 0.135 \end{array}$$

1 decimal place

2 decimal places

Answer must have 1 + 2, or 3, decimal places.

EXAMPLE 4 Multiply: 0.42×6.65

$$\begin{array}{r} 6.65 \\ \times 0.42 \\ \hline 1330 \\ 26600 \\ \hline 2.7930 \end{array}$$

2 decimal places

2 decimal places

Answer must have 2 + 2, or 4, decimal places.

Written Exercises

Add or subtract.

A **1.** $0.46 + 0.2$ 0.66 **2.** $2.4 + 5.67$ 8.07 **3.** $3.95 - 1.02$ 2.93 **4.** $4.06 - 1.2$ 2.86

5. $2.04 + 0.6$ 2.64 **6.** $0.7 + 3.002$ 3.702 **7.** $6.1 - 0.98$ 5.12 **8.** $3.2 + 5.013$ 8.213

9. $8.4 - 0.635$ 7.765 **10.** $7.2 - 3.963$ 3.237 **11.** $8.81 + 4.007$ 12.817 **12.** $0.4 - 3.567$ −3.167

13. $6 + 2.9 + 5.63$ 14.53 **14.** $8.1 + 0.2 + 3.56$ 11.86 **15.** $7.421 + 5 + 8.09$ 20.511

16. $3.7 + 0.004 + 12$ 15.704 **17.** $6.7 + 0.356 + 4$ 11.056 **18.** $7.9 + 3.0204 + 10$ 20.9204

Multiply.

19. 2×6.3 12.6 **20.** 4.7×5 23.5 **21.** 8.2×0.1 0.82 **22.** 5.6×10 56

23. 2.43×10 24.30 **24.** 6.71×100 671 **25.** 6.1×100 610 **26.** 3.04×0.3 0.912

27. 2.04×2.02 4.1208 **28.** 7.9×0.04 0.316 **29.** 6.25×0.4 2.5 **30.** 3.71×1.02 3.7842

31. 0.421×1.006 0.423526 **32.** 3.07×0.501 1.53807 **33.** 2.006×0.03 0.06018 **34.** 4.06×0.031 0.12586

35. 0.041×0.102 0.004182 **36.** 0.04×0.605 0.02420 **37.** 1.011×0.204 0.206244 **38.** 5.02×3.009 15.10518

39. The Richter Scale measures the greatness of an earthquake. The San Francisco earthquake of 1906 measured 8.3 on the Richter Scale. A much smaller earthquake in 1940 at False Pass, Alaska measured 6.75. How much greater was the San Francisco earthquake? 1.55

2 • Division With Decimals

In dividing by a decimal, the first step is to make the divisor an integer, or a whole number.

EXAMPLE 1

The Problem	*First Step*	*Then Divide*
$1.6\overline{)1.76}$	$1.6.\overline{)1.7.6}$	$16\overline{)17.6}$ with quotient 1.1, 160, 16, 16

The division could be written in fractional form. Moving the decimal points one place to the right in the division is like multiplying the numerator and denominator by 10.

$$\frac{1.76}{1.6} = \frac{17.6}{16}$$ (each fraction: $\times 10$)

EXAMPLE 2

The Problem	*First Step*	*Then Divide*
$0.02\overline{)3.4}$	$0.02.\overline{)3.40.}$	$2\overline{)340}$ with quotient 170

The division could be written in fractional form. Moving the decimal points two places to the right in the division is like multiplying the numerator and denominator by 100.

$$\frac{3.4}{0.02} = \frac{340}{2}$$ (each fraction: $\times 100$)

Often divisions do not come out "even." In those cases, you will have to round the answer.

EXAMPLE 3 Find $3.75 \div 0.07$. Round to one decimal place.

$0.07.\overline{)3.75.00}$ quotient 53.57

Keep dividing until you have *two* decimal places. Round the answer to one decimal place.

Answer: $3.75 \div 0.07 \doteq 53.6$

$\doteq$ is approximately equal to

EXAMPLE 4 Find $14 \div 2.3$. Round to two decimal places.

$$2.3\overline{)14.0000} = 6.086$$

Keep dividing until you have three decimal places. Round the answer to two decimal places.

Answer: $14 \div 2.3 \doteq 6.09$

Classroom Practice

Locate the decimal point in the answer.

1. $0.6\overline{)3.54} = 59$ **2.** $0.8\overline{)67.2} = 84.$ **3.** $0.72\overline{)2.592} = 36$ **4.** $0.25\overline{)41} = 164.$

Round to one decimal place.

5. 0.72 0.7 **6.** 5.63 5.6 **7.** 8.15 8.2 **8.** 0.66 0.7 **9.** 7.89 7.9

Round to two decimal places.

10. 0.235 0.24 **11.** 6.477 6.48 **12.** 7.038 7.04 **13.** 5.214 5.21 **14.** 16.739 16.74

Written Exercises

Divide.

A **1.** $0.4\overline{)2.4} = 6$ **2.** $0.7\overline{)2.17} = 3.1$ **3.** $0.5\overline{)4} = 8$ **4.** $0.9\overline{)1.008} = 1.12$

5. $0.08\overline{)3.68} = 46$ **6.** $2.5\overline{)0.1} = 0.04$ **7.** $0.24\overline{)0.264} = 1.1$ **8.** $0.125\overline{)5} = 40$

Divide. Round to one decimal place.

9. $0.6\overline{)1} = 1.7$ **10.** $0.4\overline{)2.7} = 6.8$ **11.** $0.5\overline{)6.3} = 12.6$ **12.** $1.2\overline{)3.7} = 3.1$

13. $6.2\overline{)93} = 15$ **14.** $0.34\overline{)0.65} = 1.9$ **15.** $0.07\overline{)40} = 571.4$ **16.** $0.51\overline{)4.7} = 9.2$

Divide. Round to two decimal places.

17. $0.3\overline{)6.74} = 22.47$ **18.** $0.9\overline{)0.35} = 0.39$ **19.** $0.4\overline{)10} = 25.00$ **20.** $1.7\overline{)62} = 36.47$

21. $0.08\overline{)2.6} = 32.50$ **22.** $0.34\overline{)3.1} = 9.12$ **23.** $0.26\overline{)62} = 238.46$ **24.** $0.05\overline{)84} = 1680.00$

3 • Decimals For Fractions

Fractions whose numerators and denominators are integers are called **rational numbers.** Here are some examples of rational numbers.

$$\frac{-2}{4} \qquad \frac{1}{2} \qquad \frac{7}{-5} \qquad \frac{5}{1}$$

Since any integer can be written as a fraction, any integer is a rational number.

To find the decimal form for any rational number, carry out the division shown by the fraction.

EXAMPLE 1 Express $\frac{4}{5}$ as a decimal.

$$\begin{array}{r} 0.8 \\ 5\overline{)4.0} \\ 4.0 \\ \hline 0 \end{array}$$

Note that the remainder is 0.

We write: $\frac{4}{5} = 0.8$

We call 0.8 a **terminating decimal** because the division terminates, or comes out "even."

Sometimes the digits in the decimal are repeating digits.

EXAMPLE 2 Express $\frac{1}{3}$ as a decimal.

$$\begin{array}{r} 0.333 \\ 3\overline{)1.000} \\ 900 \\ \hline 100 \\ 90 \\ \hline 10 \\ 9 \\ \hline 1 \end{array}$$

We write: $\frac{1}{3} = 0.33\ldots$

This is a **repeating decimal.** The 3 repeats.

EXAMPLE 3 Express $\frac{3}{11}$ as a decimal.

$$
\begin{array}{r}
0.2727 \\
11\overline{)3.0000} \\
2\,2000 \\
\hline
8000 \\
7700 \\
\hline
300 \\
220 \\
\hline
80 \\
77 \\
\hline
3
\end{array}
$$

We write: $\frac{3}{11} = 0.2727\ldots$

This is a repeating decimal. The 27 repeats.

Written Exercises

Express the fraction as a decimal. Be sure to use three dots with a repeating decimal.

A **1.** $\frac{1}{8}$ 0.125 **2.** $\frac{5}{8}$ 0.625 **3.** $\frac{3}{5}$ 0.6 **4.** $\frac{5}{9}$ 0.55 . . . **5.** $\frac{3}{8}$ 0.375

6. $\frac{2}{3}$ 0.66 . . . **7.** $\frac{2}{7}$ 0.285714 . . . **8.** $\frac{1}{6}$ 0.1666 . . . **9.** $\frac{4}{7}$ 0.571428 . . . **10.** $\frac{4}{9}$ 0.44 . . .

11. $\frac{1}{9}$ 0.11 . . . **12.** $\frac{7}{9}$ 0.77 . . . **13.** $\frac{6}{11}$ 0.5454 . . . **14.** $\frac{5}{6}$ 0.8333 . . . **15.** $\frac{2}{5}$ 0.4

Batting averages are figured as a ratio $\frac{H}{B}$ where H is the number of hits and B is the number of times at bat. Batting averages are given correct to three decimal places. Complete the table.

	Name	H	B	$\frac{H}{B}$
Sample	Ortiz	10	30	0.333
16.	Doggan	15	24	? 0.625
17.	Johnson	18	25	? 0.720
18.	Fisher	9	19	? 0.474
19.	Lykos	17	26	? 0.654
20.	Alfonso	20	28	? 0.714

For more practice, see page 429.

4 • Equations With Decimals

Some equations contain decimals. The examples below show how to handle them. Recall how to multiply by 10, 100, or 1000.

Multiply by 10.	*Multiply by 100.*	*Multiply by 1000.*
0.6.	2.07.	0.423.

Now use this skill in solving equations.

EXAMPLE 1

$$0.4x + 0.8x = 3.6$$
$$10(0.4x + 0.8x) = 10 \times 3.6$$
$$4x + 8x = 36$$
$$12x = 36$$
$$x = 3$$

It's easier to deal with whole numbers. Multiply both sides by 10.

Check:

$0.4x + 0.8x = 3.6$	
$0.4(3) + 0.8(3)$	3.6
$1.2 + 2.4$	
3.6	

EXAMPLE 2

$$0.7 = 0.3 + 0.02a$$
$$100 \times 0.7 = 100(0.3 + 0.02a)$$
$$70 = 30 + 2a$$
$$40 = 2a$$
$$20 = a$$

Multiply both sides by 100.

Check:

$0.7 = 0.3 + 0.02a$	
0.7	$0.3 + 0.02(20)$
	$0.3 + 0.4$
	0.7

EXAMPLE 3

$$0.025x + 0.05x = 0.3$$
$$1000(0.025x + 0.05x) = 1000 \times 0.3$$
$$25x + 50x = 300$$
$$75x = 300$$
$$x = 4$$

Check:

$0.025x + 0.05x = 0.3$	
$0.025(4) + 0.05(4)$	0.3
$0.1 + 0.2$	
0.3	

Classroom Practice

Multiply the expression by 10.

1. $0.4x$ $4x$
2. $2.3y$ $23y$
3. $7.4n$ $74n$
4. $0.3x + 0.5y$ $3x + 5y$
5. $2.5a - 0.4b$ $25a - 4b$
6. $3.7y + 0.6x$ $37y + 6x$
7. $0.8(x + 5)$ $8x + 40$
8. $2.6(a + b)$ $26a + 26b$

Multiply the expression by 100.

9. $0.04x$ $4x$
10. $0.35y$ $35y$
11. $6.02n$ $602n$
12. $0.04x + 0.02y$ $4x + 2y$
13. $0.31x - 0.12y$ $31x - 12y$
14. $0.05x + 0.5y$ $5x + 50y$
15. $2.3x - 0.06y$ $230x - 6y$
16. $0.7a - 0.43b$ $70a - 43b$

Written Exercises

Solve.

A
1. $0.2x + 0.3x = 25$ 50
2. $0.4a + 0.7 = 55$ 135.75
3. $0.6x = 2.4$ 4
4. $0.7y = 4.2$ 6
5. $0.10x + 0.12x = 1.9$ 8.63 . . .
6. $0.75x + 0.10x = 85$ 100
7. $0.35x = 0.91$ 2.6
8. $0.22y = 1.54$ 7
9. $0.2x + 0.04x = 1.2$ 5
10. $0.14a + 0.6a = 5.92$ 8
11. $0.3n - 0.24n = 0.8$ 13.33 . . .
12. $0.05x + 0.5x = 4.4$ 8
13. $0.12n = 0.3 - 0.03n$ 2
14. $0.05d = 2000$ 40,000
15. $0.04x = 264$ 6600
16. $0.03x = 0.15(4 - x)$ 3.33 . . .
17. $90 - x = 0.04(180 - x)$ 86.25
18. $0.2(x - 3) + 0.4x = 0$ 1
19. $0.2x + 0.3(x + 4) = 37$ 71.6
20. $0.07x + 0.03(2x) = 390$ 3000
21. $0.05n + 0.03(n - 20) = 0$ 7.5
22. $0.06x + 0.04(10 - x) = 70$ 3480

B
23. $0.2x + 0.25(9 - x) = 2$ 0.5
24. $0.25(16 - n) + n = 0.4(16)$ 3.2
25. $0.6(15) + s = 0.7(15 + s)$ 5
26. $0.035x + 0.06(600 - x) = 27$ 0.475
27. $0.4x + 0.24(x - 5) = 0.08$ 2
28. $0.025y - 0.05(20 - 2y) = 0.2$ 9.6

5 • Using Decimal Equations

Many problems involve decimals. In order to solve the problems and their equations, you'll put to work the methods you used in the last section.

EXAMPLE 1 One serving of spinach has about 2.6 mg more iron than a piece of chicken. Together they have about 5.4 mg of iron. How much iron does a serving of spinach have? a piece of chicken?

Let x = milligrams of iron in the chicken.
Then $x + 2.6$ = milligrams of iron in the spinach.

$$\underbrace{\text{chicken}} + \underbrace{\text{spinach}} = 5.4$$

$$x + \overbrace{x + 2.6} = 5.4$$

$$2x + 2.6 = 5.4$$

$$10(2x + 2.6) = 10 \times 5.4$$

$$20x + 26 = 54$$

$$20x = 28$$

$$x = 1.4$$

Answer: $x = 1.4 \longrightarrow$ 1.4 mg iron in the chicken
$x + 2.6 \longrightarrow$ 4 mg iron in the spinach

Check: $1.4 + 4 = 5.4$ ✓

EXAMPLE 2 A ball player's batting average is 0.330 this year. This is 1.2 times his average last year. What was his average last year?

Let x = batting average last year.
Then $1.2x$ = batting average this year.

$$1.2x = 0.330$$

$$1000 \times 1.2x = 1000 \times 0.330$$

$$1200x = 330$$

$$x = 0.275 \longleftarrow \begin{array}{r} 0.275 \\ 1200\overline{)330.000} \end{array}$$

Answer: 0.275 batting average

Check: $1.2 \times 0.275 = 0.330$ ✓

Written Exercises

Solve.

A **1.** A number is 0.4 greater than another number.
The sum of the two numbers is 3.8.
What are the two numbers? 1.7, 2.1

2. Book A is 0.7 cm less thick than Book B.
Together the books are 6.3 cm thick.
How thick is each book? A = 2.8 cm, B = 3.5 cm

3. Two boards are arranged end to end.
One board is 2.1 times as long as the other.
Together they are 133.3 cm long.
How long is each board? 43 cm, 90.3 cm

4. One suitcase is 13.2 kg less than another.
Together the two suitcases are 178.2 kg.
How many kilograms is each suitcase? 95.7 kg, 82.5 kg

5. Allegra left her office and drove to a business meeting.
She returned to the office, but later drove 8.3 km more.
Altogether she drove 144.7 km.
How far was it to the business meeting? 68.2 km

6. The Mississippi River is 1.24 times as long as the St. Lawrence.
The Mississippi is about 4352.2 km long.
How long is the St. Lawrence? 3509.8387 km

For more practice, see page 429.

SELF-TEST

Compute. Round Exercise 4 correct to two decimal places.

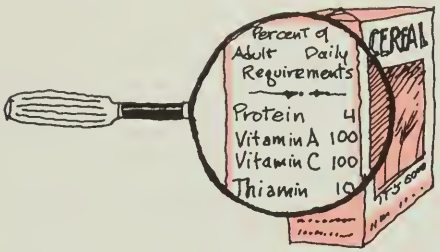
1. $3.6 + 0.05$ 3.65 **2.** $10.735 - 6.25$ 4.485 **3.** 0.6×0.72 0.432 **4.** $4.85 \div 0.72$ 6.74

5. Express $\frac{5}{8}$ as a decimal. 0.625 **6.** Solve: $0.3x + 0.8x = 5.5$ 5

7. The sum of two distances is 7.5 km.
One distance is 1.5 km greater than the other.
Find the two distances. 3 km and 4.5 km

6 • Percents

Percents are used so often in everyday life because they are a convenient way to compare two quantities with one another.



For example, cereal boxes list the percent of the adult daily requirement for several vitamins and minerals in one serving of the cereal. It wouldn't be as easy to compare brands if the percents were listed in fraction form instead.

Percents express the ratio of a number to 100. Fractions and decimals can be rewritten as percents.

Fraction	Fraction with Denominator 100	Decimal	Percent
$\frac{1}{2}$	$\frac{50}{100}$	0.50	50%

Changing Decimals to Percents

Decimals can be written as percents in the following way.

EXAMPLE 1 $0.28 = 28\%$ ← *Think:* 0.28

EXAMPLE 2 $0.275 = 27.5\%$ ← *Think:* 0.275

EXAMPLE 3 $0.04 = 4\%$ ← *Think:* 0.04

Changing Fractions to Percents

Fractions can be written as percents in a couple of ways.

EXAMPLE 4 $\frac{3}{5} = \frac{60}{100}$ *and* $\frac{60}{100} = 60\%$

EXAMPLE 5 $\frac{1}{8} \longrightarrow 8\overline{)1.000}$ $\begin{array}{r}0.125\end{array}$ *and* $0.125 = 12.5\%$

Changing Percents to Decimals or Fractions

To compute with a percent, you will need to change the percent to a decimal or a fraction. It doesn't matter which one you use, but one might be more convenient than the other.

EXAMPLE 6 $46\% = 0.46$ ← *Think:* 46.%

EXAMPLE 7 $5\% = 0.05$ ← *Think:* 0 05.%

EXAMPLE 8 $25\% = \frac{25}{100} = \frac{1}{4}$

Here's a table of some commonly used percents. You might want to copy it for future use.

Percent	Decimal	Fraction
10%	0.10	$\frac{1}{10}$
20%	0.20	$\frac{1}{5}$
25%	0.25	$\frac{1}{4}$
$33\frac{1}{3}\%$	$0.33\frac{1}{3}$ *or* 0.33 . . .	$\frac{1}{3}$
50%	0.50	$\frac{1}{2}$
$66\frac{2}{3}\%$	$0.66\frac{2}{3}$ *or* 0.66 . . .	$\frac{2}{3}$

Classroom Practice

Express as a percent.

1. 0.42 42% **2.** 0.30 30% **3.** 0.05 5% **4.** 0.6 60% **5.** 0.7 70%

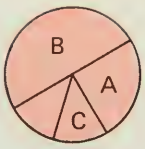
6. $\frac{1}{2}$ 50% **7.** $\frac{1}{4}$ 25% **8.** $\frac{1}{3}$ $33\frac{1}{3}\%$ **9.** $\frac{3}{8}$ $37\frac{1}{2}\%$ **10.** $\frac{2}{5}$ 40%

Express as a decimal.

11. 63% 0.63 **12.** 59% 0.59 **13.** 47% 0.47 **14.** 4% 0.04 **15.** 7% 0.07

Estimate the pieces named.

16. In the pie at the right, about what percent of the whole is A? 25%

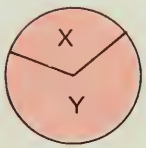


17. About what percent of the whole is B? 50%

18. About what percent of the whole is C? 12% or 13%

19. About what percent of the whole is X? $33\frac{1}{3}\%$

20. About what percent of the whole is Y? $66\frac{2}{3}\%$



Written Exercises

Write as a percent.

A

1. 0.40 40%	**2.** 0.23 23%	**3.** 0.65 65%	**4.** 0.89 89%	**5.** 0.50 50%
6. 0.14 14%	**7.** 0.07 7%	**8.** 0.03 3%	**9.** 0.09 9%	**10.** 0.06 6%
11. 0.045 $4\frac{1}{2}\%$	**12.** 0.036 $3\frac{3}{5}\%$	**13.** 0.214 $21\frac{2}{5}\%$	**14.** 0.525 $52\frac{1}{2}\%$	**15.** 0.125 $12\frac{1}{2}\%$
16. $\frac{2}{5}$ 40%	**17.** $\frac{1}{5}$ 20%	**18.** $\frac{4}{5}$ 80%	**19.** $\frac{3}{4}$ 75%	**20.** $\frac{5}{8}$ $62\frac{1}{2}\%$

Write as a decimal.

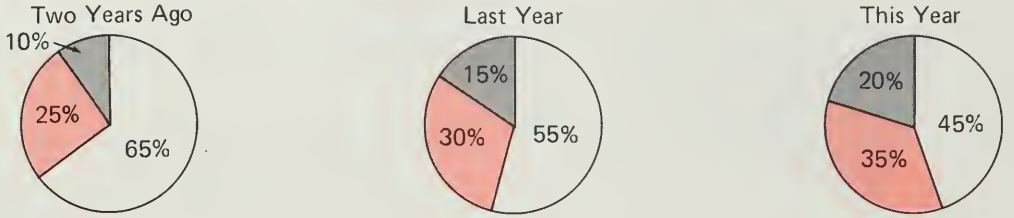
21. 64% 0.64	**22.** 86% 0.86	**23.** 95% 0.95	**24.** 30% 0.30	**25.** 20% 0.20
26. 56% 0.56	**27.** 67% 0.67	**28.** $66\frac{2}{3}\%$ 0.66 . . .	**29.** 47% 0.47	**30.** $33\frac{1}{3}\%$ 0.333 . . .
31. 75% 0.75	**32.** 6% 0.06	**33.** 8% 0.08	**34.** 2% 0.02	**35.** 9% 0.09
36. 3.5% 0.035	**37.** 6.5% 0.065	**38.** 7.5% 0.075	**39.** 5.5% 0.055	**40.** 5.25% 0.0525

Write as a fraction.

41. $33\frac{1}{3}\%$ $\frac{1}{3}$	**42.** $66\frac{2}{3}\%$ $\frac{2}{3}$	**43.** 25% $\frac{1}{4}$	**44.** 75% $\frac{3}{4}$	**45.** 50% $\frac{1}{2}$
46. 20% $\frac{1}{5}$	**47.** 10% $\frac{1}{10}$	**48.** 40% $\frac{2}{5}$	**49.** 60% $\frac{3}{5}$	**50.** 80% $\frac{4}{5}$

The circle graphs below show the kinds of homes built in the last several years. Use the data in the graphs to complete the table.

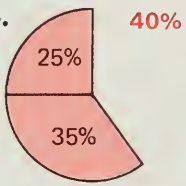
Single family homes Multi-family homes Mobile homes



	Homes	Two Years Ago	Last Year	This Year	Next Year (est.)
51.	Single Family	65%	? 55%	? 45%	? 40%
52.	Multi-family	? 25%	? 30%	? 35%	? 40%
53.	Mobile	? 10%	? 15%	? 20%	? 25%

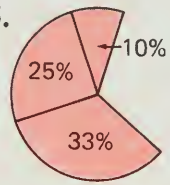
What percent of the whole is the missing piece? Remember, the whole is 100%.

54.



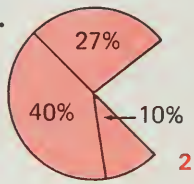
40%

55.



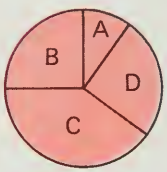
32%

56.



23%

Estimate the percent of the whole for each lettered part.



57. *A* 10% **58.** *B* 25% **59.** *C* 40% **60.** *D* 25%

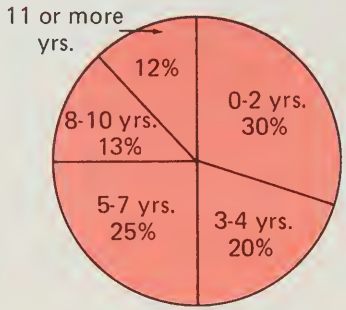
The circle graph at the right shows the ages of cars in the United States.

B **61.** Half of the cars are less than $\frac{?}{4}$ years old.

62. What percent of the cars are 8 years old or older? 25%

63. Draw a bar graph to represent this information. Check student's graphs.

AGES OF CARS IN THE UNITED STATES



7 • Using Percents



Look in any newspaper. You'll probably see some advertisements for sales at local stores. Often the discount (amount you save) is given as a percent of the price the item originally cost.

EXAMPLE 1 Luggage was on sale in one store at a 30% discount. Find the discount on a \$56 suitcase.

$$30\% \times \$56 = 0.30 \times \$56$$
$$= \$16.80$$

EXAMPLE 2 During a sale shoes are reduced 20%. One kind of shoes usually sell for \$30. What is the sale price of the shoes?

Compute discount.

$$20\% \times \$30 = 0.20 \times \$30$$
$$= \$6$$

Compute sale price.

$$\$30 - \$6 = \$24$$

Figuring percents in your head can be a very useful skill. Follow these hints. You can extend these ideas to figure other percents.

	Hints	of \$120	of \$4.80
To find 10%	Move the decimal point one place to the left.	\$12	\$.48
To find 20%	Double what you get for 10%.	\$24	\$.96
To find 25%	Find one fourth (or divide by 4).	\$30	\$1.20
To find $33\frac{1}{3}\%$	Find one third (or divide by 3).	\$40	\$1.60
To find 50%	Find one half (or divide by 2).	\$60	\$2.40
To find $66\frac{2}{3}\%$	Double what you find for $33\frac{1}{3}\%$.	\$80	\$3.20

Classroom Practice

Figure the work in your head.

1. 50% of 80 40
2. 10% of 62 6.2
3. 25% of 60 15
4. 10% of 79 32
5. $33\frac{1}{3}\%$ of 96 7.9
6. $66\frac{2}{3}\%$ of 45 30
7. 20% of 28 5.6
8. 25% of 76 19
9. 50% of \$2.50 \$1.25
10. 30% of \$12 \$3.60
11. 40% of \$25 \$10.00
12. 75% of \$36 \$27.00

Compute.

13. 36% of 400 144
14. 24% of 86 20.64
15. 5% of 92 4.6
16. 1% of 67 0.67

Written Exercises

Compute.

A
1. 30% of 120 36
2. 60% of 46 27.6
3. 25% of 18 4.50
4. 75% of 50 37.50
5. 15% of 56 8.40
6. 40% of 65 26
7. 35% of 80 28
8. 45% of 25 11.25
9. 60% of 150 90
10. 80% of 210 168
11. 15% of 250 37.5
12. 45% of 500 225

Complete the table.

SALE! 20% off			
	Item	Original price	Discount
13.	shoes	\$27	\$5.40 ?
14.	shirt	\$18	\$3.60 ?
15.	jeans	\$12	\$2.40 ?

SALE! 15% off			
	Item	Original price	Sale price
16.	shovel	\$7	? \$1.05
17.	rake	\$5.60	? \$.84
18.	paint	\$11.90	? \$1.79

19. Camping tents are on sale at a $33\frac{1}{3}\%$ discount.

Find the sale price on a \$75 tent. \$50.02

20. A buyer for a men's store buys ties at \$3.30 each. To sell them in the store, the price is increased 40%. For how much do those ties sell in the store? \$4.62

For more practice, see page 430.

8 • Percents in Equations



Sales taxes are figured as a percent of the cost. Suppose a car costs \$4000, and the sales tax is 7%. The tax is figured this way.

7% of \$4000 is what number?

$$\begin{aligned} 7\% \times \$4000 &= n \\ 0.07 \times 4000 &= n \\ 280 &= n \end{aligned}$$

The tax is \$280.

You can solve all types of percent problems if you can set up your equation using the pattern shown above.

EXAMPLE 1 Only 40% of the seats in a theater are occupied. The theater has 320 seats. How many seats are occupied?

40% of 320 is what number?

$$\begin{aligned} 40\% \times 320 &= x \\ 0.40 \times 320 &= x \\ 128 &= x \end{aligned}$$

Answer: 128 seats are occupied.

EXAMPLE 2 In one game, a football quarterback threw 20 passes and completed 12 of them. What percent of the passes were completed?

What percent of 20 is 12?

$$n \times 20 = 12$$

$$n = \frac{12}{20}$$

$$\begin{array}{r} 0.60 \\ 20\overline{)\,12.00} \end{array}$$

$$n = 0.60$$

Answer: 60% of the passes

EXAMPLE 3 In a nationwide poll about two shampoos, 2000 people were surveyed. 648 people said they preferred Brand A to Brand B. What percent of the people surveyed preferred Brand A?

What percent of 2000 is 648?

$$x \times 2000 = 648$$

$$x = \frac{648}{2000}$$

$$2000\overline{)648.0000} = 0.3240$$

Answer: 32.4% of the people surveyed

EXAMPLE 4 In the six o'clock news, it was reported that 60% of the registered voters voted in the election. This was 48,000 people. How many registered voters were there?

60% of what number is 48,000?

$$60\% \times x = 48{,}000$$
$$0.60x = 48{,}000$$
$$100 \times 0.60x = 100 \times 48{,}000$$
$$60x = 4{,}800{,}000$$
$$x = 80{,}000$$

Answer: 80,000 registered voters

Classroom Practice

Solve.

1. 20% of 52 is what number? 10.4
2. 4% of 96 is what number? 3.84
3. What percent of 80 is 20? 25%
4. What percent of 75 is 15? 20%
5. 60% of what number is 90? 150
6. 30% of what number is 13.5? 45
7. Candidate Brown received 72% of the 4200 votes in the election. How many votes did she get? 3024 votes
8. James answered 90% of the 60 questions correctly. How many did he answer correctly? 54 questions

9. Fifty out of 60 members attended the club meeting.
What percent is this? $83\frac{1}{3}\%$

10. A highway is to be 240 km long.
So far 180 km are complete.
What percent of the road is complete? 75%

Written Exercises

Solve.

A
1. 25% of 88 is what number? 22
2. 80% of 62 is what number? 49.6
3. What percent of 35 is 14? 40%
4. What percent of 56 is 14? 25%
5. 5% of 53 is what number? 2.65
6. What percent of 85 is 17? 20%
7. 40% of what number is 22? 55
8. 65% of what number is 84.5? 130
9. 95% of 1000 is what number? 950
10. What percent of 280 is 42? 15%

11. The sales tax rate in one area is 4%.
How much tax is due on an item costing \$3.75? \$.15

12. About 60% of the human body is water.
Jerry is 53 kg in all.
How many kilograms of water is she? 31.8 kg

13. About 1 out of 700 blood cells in humans is a white blood cell.
What percent of the blood cells are white cells?
(Express your percent correct to one decimal place.) 0.1%

14. A large company had a blood donation drive.
68% of the 1200 workers donated blood.
How many people donated? 816 people



15. A book has 420 pages.
Tom has read 105 pages.
What percent of the book is this? 25%

16. Consider the same book again.
Sylvia has read 84 pages.
What percent of the book is this? 20%

17. One day 8% of the students in a school were absent.
194 students were absent.
How many students are enrolled in the school? 2425 students

18. In one vacation spot, hotels add a 4% tax to the room price.
What is the tax on a room priced at $28 per day? $1.12

19. The human body needs about 50 grams of protein daily.
A hamburger contains about 20 grams of protein.
What percent of the daily need is the protein in the hamburger? 40%

20. A person needs about 5000 units of Vitamin A daily.
A cup of tomato soup provides 1200 units.
What percent of the daily need is this? 24%



21. Last year the rent on an apartment was $200 per month.
This year the rent was increased $20.
What percent of last year's rent was the increase? 10%

For more practice, see page 430.

SELF-TEST

Write as a percent.

1. 0.52 52% **2.** 0.04 4% **3.** $\frac{3}{5}$ 60% **4.** $\frac{3}{4}$ 75% **5.** $\frac{3}{8}$ $37\frac{1}{2}\%$

Write as a decimal.

6. 35% 0.35 **7.** 14% 0.14 **8.** 7% 0.07 **9.** 65% 0.65 **10.** 1.25% 0.0125

Compute.

11. 30% of $65 is what number? 19.5

12. 15% of $65 is what number? 9.75

13. What percent of 35 is 7? 20%

14. 60% of what number is 15? 25

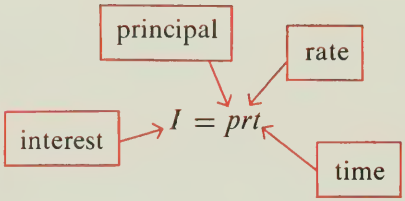
15. A bill in a restaurant comes to $5.50.
An 8% meal tax is to be added to the bill.
How much money would the tax be? 44¢

16. Thirty out of 45 tickets are sold.
What percent of the tickets are sold? $66\frac{2}{3}\%$

9 • Interest



When you put money into a savings account at a bank, you expect to earn **interest** for letting the bank use your money. Here's a formula for computing interest.



The formula can be used to solve many types of problems.

EXAMPLE 1 Lana put \$450 into a savings account.

In that bank, the money earns $5\frac{1}{2}\%$ interest per year.

How much interest will the money earn in a year?

$$I = prt$$
$$I = 450 \times 5\tfrac{1}{2}\% \times 1$$

$5\frac{1}{2}\% = 5.5\% = 0.055$

$$= 450 \times 0.055 \times 1$$
$$= 24.75$$

Answer: \$24.75 interest

EXAMPLE 2 James put \$200 into a savings account in January.
Six months later he put \$50 into the account.
The interest rate is 6% per year.
How much interest will be earned at the end of the year?

$$I = prt$$
$$I = 200 \times 6\% \times 1$$
$$= 200 \times 0.06 \times 1$$
$$= 12$$

$$I = prt$$
$$I = 50 \times 6\% \times \tfrac{1}{2}$$
$$= 50 \times 0.06 \times \tfrac{1}{2}$$
$$= 1.50$$

The \$50 earns interest for only $\frac{1}{2}$ year.

Answer: \$12 + \$1.50 = \$13.50 ⟶ \$13.50 interest

EXAMPLE 3 How long will it take \$100 to double to \$200 when the interest rate is $6\frac{1}{2}\%$ per year?

When the principal doubles, the interest will be \$100.

$$I = prt$$
$$100 = 100 \times 6\tfrac{1}{2}\% \times t$$

$6\frac{1}{2}\% = 6.5\% = 0.065$

$$100 = 100 \times 0.065 \times t$$
$$100 = 6.5t$$
$$10 \times 100 = 10 \times 6.5t$$
$$1000 = 65t$$
$$15.3 \doteq t$$

Answer: about 15 years

Written Exercises

Solve.

A **1.** How much interest can be earned in one year on \$500 at 5%? \$25

2. How much interest does \$275 earn each year at $5\frac{1}{2}\%$? \$15.13

3. Lavette invested \$1000 at 6.25% interest for 1 year.
How much interest did the money earn? \$46.88

4. In January Sam put \$785 into an account.
Six months later he put \$400 more into the account.
The account earns 6% interest per year.
How much interest did the money earn at the end of the year? \$59.10

5. How long will it take \$500 to double at 7% interest? about 14 years

6. How long will it take \$1000 to double at 7% interest? about 14 years

7. Fran's bank offers 5% interest per year.
The bank across the street pays 6% interest.
Fran has \$500 to put into an account.
How much more money will be earned in one year at the second bank? \$7.50

8. Ted wants his \$1000 to earn \$500 interest in 10 years.
He puts it into an account earning 7% interest per year.
Will the money earn enough interest?
How much more or less than Ted's goal? yes; \$150 more

10 • Investment

The tables you used to solve motion and cost problems are helpful for investment problems too.

EXAMPLE 1 Mark invested some money at 6% yearly interest and \$800 at 7%. The money earned \$128 interest in a year. How much was invested at 6% interest?

	$p \times r \times t = I$			
Amount at 6%	x	0.06	1	$0.06x$
Amount at 7%	800	0.07	1	56
				128 ← total

Multiply each side by 100 to get this step.

$$0.06x + 56 = 128$$
$$6x + 5600 = 12800$$
$$6x = 7200$$
$$x = 1200$$

Answer: \$1200 at 6% interest

EXAMPLE 2 A club invested \$1000, part at 5% and the rest at 7%. The interest for one year came to \$62. Find how much was invested at each rate.

Let x = principal invested at 5%.
Then $1000 - x$ = principal invested at 7%.

	$p \times r \times t = I$			
Amount at 5%	x	0.05	1	$0.05x$
Amount at 7%	$1000 - x$	0.07	1	$0.07(1000 - x)$
				62 ← total

Multiply each side by 100 to get this step.

$$0.05x + 0.07(1000 - x) = 62$$
$$5x + 7(1000 - x) = 6200$$
$$5x + 7000 - 7x = 6200$$
$$-2x = -800$$
$$x = 400$$

Answer: $x = 400 \longrightarrow$ \$400 at 5% interest
$1000 - x = 600 \longrightarrow$ \$600 at 7% interest

Written Exercises

A **1.** Dr. Sanchez invested \$600 at 7% interest.
She also invested some more money at 6.5% interest.
At the end of a year her money had earned \$68 interest.
How much money was invested at 6.5%. \$400

	p	$\times$ r	$\times$ t	$= I$
Amount at 7%	600	0.07	1	? 42.00
Amount at 6.5%	x	? 0.065	1	? 26.00
				68

2. Hank invested \$1200, part at 5% interest, and the rest at 8%.
The money earned \$84 in interest in one year.
How much was invested at each rate? \$400 at 5%, \$800 at 8%

	p	$\times$ r	$\times$ t	$= I$
Amount at 5%	x	? 0.05	1	? $0.05x$
Amount at 8%	$1200 - x$	? 0.08	1	? $0.08(1200 - x)$
				? 84

3. A high school raised \$12,000.
The interest each year, \$920, is used for scholarships.
Part of the \$12,000 was invested at 7%, and the rest at 9%.
How much was invested at each rate? \$8000 at 7%, \$4000 at 9%

4. A union invested \$100,000 in two unemployment funds.
One fund paid 7% interest, and the other 6%.
The interest earned each year was \$6,700.
How much was invested at each rate?
\$70,000 at 7%, \$30,000 at 6%



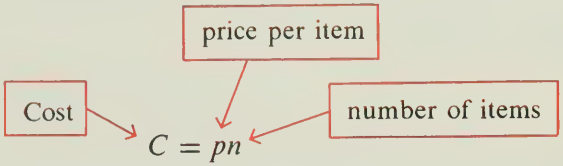
5. An organization invested \$8000 in stocks and bonds.
The stocks pay 4% and the bonds pay 7%.
The amount earned each year from the two is \$410.
How much is invested in the stocks? in the bonds?
\$5000 in stocks, \$3000 in bonds

6. The Pattersons invested \$7000 in two small businesses.
The printing shop pays 8%, while the grocery shop pays 5%.
The annual income from the two is \$431.
How much is invested in each business? \$2700 in printing shop, \$4300 in grocery

For more practice, see page 431.

11 • Mixture Problems

Problems about mixtures have many applications in business and industry. Recall the cost formula you used in earlier chapters.



You'll use this formula in Example 1.

EXAMPLE 1



A coffee supply house mixes two kinds of coffee to get the blend they sell wholesale at 80¢ a can. One of the kinds is worth 50¢ a can, the other, 90¢. How much of each kind should be mixed to get 100 cans of the blend?

Let x = number of cans of 50¢ coffee.
Then $100 - x$ = number of cans of 90¢ coffee.

	$p \times$	n =	C
50¢ kind	50	x	$50x$
90¢ kind	90	$100 - x$	$90(100 - x)$
Mixture	80	100	8000

50¢ kind + 90¢ kind = Mixture

$$50x + 90(100 - x) = 8000$$
$$50x + 9000 - 90x = 8000$$
$$-40x = -1000$$
$$x = \frac{-1000}{-40}$$
$$x = 25$$

Answer: $x = 25 \longrightarrow$ 25 cans of 50¢ coffee
$100 - x = 75 \longrightarrow$ 75 cans of 90¢ coffee

Check:
$$25 \times \$.50 = \$12.50$$
$$75 \times \$.90 = \underline{\ 67.50}$$
$$\$80.00$$

$100 \times \$.80 = \$80$ ✓

Labels on bottles or cans often tell you that a mixture contains a certain percent of the substance named. Sometimes mixtures are made to reduce or to increase that percent.



EXAMPLE 2 A salt-water solution contains 6% salt. How much water must be added to 10 liters of solution to get a mixture which is only 4% salt?

Let x = number of liters of water to be added.

	Amount of Solution	× % of salt	= Amount of salt
Original solution	10	6% *or* 0.06	0.6
Water added	x	0%	0
New solution	$10 + x$	4% *or* 0.04	$0.04(10 + x)$

$$\begin{aligned} 0.6 + 0 &= 0.04(10 + x) \\ 0.6 &= 0.04(10 + x) \\ 100 \times 0.6 &= 100 \times 0.04(10 + x) \\ 60 &= 4(10 + x) \\ 60 &= 40 + 4x \\ 20 &= 4x \\ 5 &= x \end{aligned}$$

Answer: 5 liters of water

Check: Original solution: $0.06 \times 10 = 0.6 \Longrightarrow$ 0.6 L salt
New solution: $0.04 \times 15 = 0.6 \Longrightarrow$ 0.6 L salt ✓

Written Exercises

A **1.** Two kinds of candy are mixed to sell at 85¢ a kilogram. One kind sells at \$.70 per kilogram, the other at \$.95 per kilogram. How much of each kind are used in 100 kilograms of the mixture?

40 kg at \$.70 per kg, 60 kg at \$.95 per kg

	p	× n	= C
First kind	70	x	$70x$
Second kind	95	$100 - x$	$95(100 - x)$
Mixture	85	100	8500

2. At the grand opening of a bank, 2000 souvenirs were given away. The bank paid \$.50 for each notebook and \$.60 for each calendar. The total cost was \$1080.
How many notebooks and calendars did the bank buy?

1200 notebooks, 800 calendars

	$p \times n = C$			
Notebooks	? 0.5	x	?	$0.5x$
Calendars	? 0.6	? $2000 - x$	?	$0.6(2000 - x)$
			1080	

3. At a concert, student tickets cost \$1.25, while others cost \$1.75. The receipts for 600 tickets totaled \$850.
How many student tickets were sold? 400 student tickets

	$p \times n = C$			
Student	? 1.25	? x	?	$1.25x$
Others	? 1.75	? $600 - x$	?	$1.75(600 - x)$
			850	

4. A pancake syrup made of maple and corn syrups sells for \$3.05 per liter. The maple syrup is worth \$5.25 per liter; the corn syrup, \$2.56.
How much maple syrup is used to make 100 liters of the mixture?

18.2 liters maple syrup

	$p \times n = C$			
Maple syrup	? 5.25	? x	?	$5.25x$
Corn syrup	? 2.56	? $100 - x$	?	$2.56(100 - x)$
			?	305

5. A company has 1000 liters of lemonade. The lemonade is 30% lemon juice.
How much water should be added to make the mixture 25% lemon juice?

200 liters water

	Amount of Lemonade $\times$	% of Lemon Juice $=$	Amount of Lemon Juice
Old Mixture	1000	30%	? 300
Water	x	0%	? 0
New Mixture	? $1000 + x$	25%	? $0.25(1000 + x)$

6. A canning factory has 1000 kg of a syrup of 5% sugar, 95% water.
The company plans to boil away the water to thicken the syrup.
The new syrup should be 8% sugar.
How much water must be boiled away? 375 kg water

	Amount of Syrup ×	% of Sugar =	Amount of Sugar
Old syrup	1000	? 5%	? 50
Boiled-away water	x	0%	? 0
New syrup	$1000 - x$	? 8%	? $0.08(1000 - x)$

7. The "lead" in pencils is a mixture of clay and graphite.
In a pencil factory, 500 liters of the mixture is 45% clay.
To make softer pencils, the clay is reduced to 40%.
How much graphite must be added to make the new mixture?
62.5 liters graphite

	Amount of Mixture ×	% of Clay =	Amount of Clay
Old mixture	500	? 45%	? 225
Graphite added	x	0%	? 0
New mixture	$500 + x$?	? 40%	? $0.40(500 + x)$



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For more practice, see page 431.

SELF-TEST

1. Joe has $700 in one bank paying 5% interest.
He has some more money in a bank paying 6%.
His interest on the two accounts for one year is $65.
How much money is earning 6% interest? $500

2. Some $5 per kilogram candy is mixed with $4 per kilogram candy.
The mixture is to be worth $4.50 per kilogram.
How much of each kind will make 100 kilograms of the mixture?
50 kilograms of each kind

CONSUMER CORNER

In Section 9 of this chapter you worked some problems involving simple interest. Actually, savings banks compute interest so that your money earns more interest. They pay **compound interest.** Let's see what this means.

Simple Interest

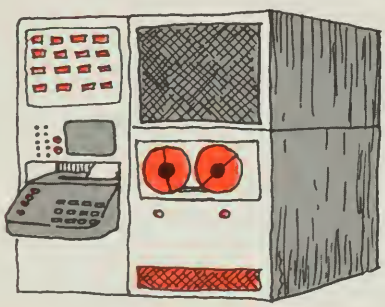
Suppose \$1000 is deposited at 5% simple interest. In one year the money earns \$50 interest. In four years the money earns \$200.

Compound Interest

Suppose that \$1000 is deposited at 5% interest compounded yearly. At the end of the first year the interest is added to the principal. Here's how \$1000 grows.

Principal	\$1000	
Interest at 5%	50	← 5% of \$1000
New principal	\$1050	
Interest at 5%	52.50	← 5% of \$1050
New principal	\$1102.50	
Interest at 5%	55.125	← 5% of \$1102.50
New principal	\$1157.625	
Interest at 5%	57.881	← 5% of \$1157.625
Amount after 4 years	\$1215.506	

Instead of \$200, the money earned \$215.51 interest!



Most banks today compound interest several times each year—every 6 months, every 3 months, or even every day. In each case, computers do the calculations needed for figuring interest.

Here's a statement from a bank advertisement.

> ***About Our Regular Savings Accounts . . .***
>
> Deposits earn 5% interest from day of deposit until day of withdrawal if \$10 remains in the account until the next dividend date. Interest is compounded daily and dividends are paid on or before the tenth day of March, June, September, and December on balances of \$10 or more.

The dividend date is the date when you can collect interest.

1. How often does this bank add interest to the principal to form a new principal? daily

2. How many times a year does this bank pay dividends? 4 times

3. Why should at least \$10 always remain in the account? Any amount less than \$10 will not earn interest; also, the payment of interest from day of deposit to day of withdrawal depends on this \$10 reserve.

The table below shows how \$100 grows at 6% interest. Notice how your money grows faster when the interest is compounded more often.

Interest Period	Years of Interest		
	1	2	10
Yearly	\$106	\$112.36	\$179.08
Quarterly	106.14	112.65	181.40
Monthly	106.17	112.72	181.94
Daily	106.27	112.94	183.79

4. If you have \$100 on deposit at 6% compounded daily, how much interest do you earn in ten years? \$183.79

5. How much more is this than interest compounded annually? \$4.71

6. How much more can you earn by depositing \$1000 for a 10-year period if the interest is compounded daily rather than yearly? \$47.10

Reviewing the Chapter

Add or subtract. *(See pp. 312–313.)*

1. $0.34 + 5.36$ 5.70 **2.** $14.55 + 2.605$ 17.155 **3.** $0.8 + 50.35$ 51.15 **4.** $16 + 50.5 + 0.62$ 67.12

5. $34.4 - 29.5$ 4.9 **6.** $15 - 3.25$ 11.75 **7.** $103.6 - 25.52$ 78.08 **8.** $5.95 - 3.605$ 2.345

Multiply. *(See pp. 312–313.)*

9. 3×4.08 12.24 **10.** 10×3.50 3.5 **11.** 5×4.6 23.0 **12.** 1.5×0.3 0.45

13. 4.06×0.2 0.812 **14.** 0.025×0.6 0.015 **15.** 11.5×1.4 16.10 **16.** 100×9.38 938

Divide. Round the answer to one decimal place. *(See pp. 314–315.)*

17. $3\overline{)15.65}$ 5.2 **18.** $0.3\overline{)1.565}$ 5.2 **19.** $0.5\overline{)26.2}$ 52.4 **20.** $0.05\overline{)50}$ 1000.0

21. $0.1\overline{)31.6}$ 316.0 **22.** $0.02\overline{)42.85}$ 2142.5 **23.** $0.6\overline{)0.034}$ 0.1 **24.** $1.9\overline{)3.139}$ 1.7

Write as a decimal. Use three dots with a repeating decimal. *(See pp. 316–317.)*

25. $\frac{3}{8}$ 0.375 **26.** $\frac{1}{8}$ 0.125 **27.** $\frac{5}{6}$ 0.833 . . . **28.** $\frac{1}{11}$ 0.0909 . . . **29.** $\frac{4}{9}$ 0.44 . . .

Solve for the variable. *(See pp. 318–319.)*

30. $0.3n + 0.9n = 4.8$ 4 **31.** $1.2x - 0.9x = 33$ 110 **32.** $0.7y = 4.9$ 7

33. $0.12x = 3.6$ 30 **34.** $0.12n = 45 - 0.03n$ 300 **35.** $5.5(2x - 3) = 10x$ 16.5

Solve. *(See pp. 320–321.)*

36. A jacket and a pair of shoes cost \$30.50 in all.
The jacket cost \$21 more than the shoes.
How much did the jacket cost? \$25.75

37. Lou grew 8.3 cm in two years.
He grew 4.5 cm more the second year than the first.
How much did he grow the first year? 1.9 cm

Write as a percent. *(See pp. 322–325.)*

38. 0.04 4% **39.** 0.32 32% **40.** 0.136 13.6% **41.** 0.025 2.5% **42.** 0.9 90%

43. 0.3 30% **44.** $\frac{1}{4}$ 25% **45.** $\frac{1}{5}$ 20% **46.** $\frac{1}{8}$ 12.5% **47.** $\frac{1}{3}$ $33\frac{1}{3}\%$

Write as a decimal. *(See pp. 322–325.)*

48. 36% 0.36 **49.** 85% 0.85 **50.** 40% 0.40 **51.** 70% 0.70 **52.** 29% 0.29

53. 6.5% 0.065 **54.** 2.5% 0.025 **55.** 3.15% 0.0315 **56.** $6\frac{1}{2}\%$ 0.065 **57.** $7\frac{1}{2}\%$ 0.075

Write as a fraction. *(See pp. 322–325.)*

58. 25% $\frac{1}{4}$ **59.** 10% $\frac{1}{10}$ **60.** $66\frac{2}{3}\%$ $\frac{2}{3}$ **61.** 50% $\frac{1}{2}$ **62.** 75% $\frac{3}{4}$

63. $33\frac{1}{3}\%$ $\frac{1}{3}$ **64.** 40% $\frac{2}{5}$ **65.** 80% $\frac{4}{5}$ **66.** 60% $\frac{3}{5}$ **67.** 90% $\frac{9}{10}$

Write an equation and solve. *(See pp. 326–331.)*

68. 40% of 50 is what number? 20

69. $33\frac{1}{3}\%$ of 15 is what number? 5

70. What percent of 34 is 17? 50%

71. What percent of \$100 is \$30? 30%

72. What percent of 16 is 2? 12.5%

73. What percent of 12 is 4? $33\frac{1}{3}\%$

74. 3% of what number is 3? 100

75. 32% of what number is 48? 150

Solve. *(See pp. 332–339.)*

76. A bank pays 6% interest per year.
How much interest will \$500 earn in a year? \$30

77. Connie invested \$2000, part at 4%, and the rest at 9%.
Her money earned \$120 in one year.
How much was invested at each interest rate? \$1,200 at 4%; \$800 at 9%

78. Two fund-raising dinners were attended by a total of 550 people.
People attending one dinner paid \$15 each.
People attending the other dinner paid \$100 each.
Together the dinners raised \$21,000.
How many attended each of the dinners? 400 people attended the \$15 dinner.
150 people attended the \$100 dinner.

79. Ten liters of a salt-water solution has 3% salt.
The salt is to be reduced to 2% of the solution.
How much water must be added? 5 liters

A permission-to-reproduce chapter test can be found on page T20.

Cumulative Review

Simplify.

1. $5x(6x)(-x)$ $-30x^3$
2. $(3m + 7) - (6m - 5)$ $-3m + 12$
3. $4x - y - (x + y)$ $3x - 2y$
4. $-b + a - (a - 2b)$ b
5. $0.05m + 0.4m$ $0.45m$
6. $0.6a - 0.08a$ $0.52a$
7. $\frac{5m^2n}{35mn^3}$ $\frac{m}{7n^2}$
8. $\frac{6 - a}{a^2 - 36}$ $-\frac{1}{a + 6}$
9. $\frac{x^2 - 5x + 6}{x - 3}$ $x - 2$

Solve.

10. $5a + 13 = 58$ 9
11. $9(2y - 5) = 135$ 10
12. $6b - 10 = 32$ 7
13. $7x + 3 = 5x - 9$ -6
14. $3x - 4 = 2(3x + 1)$ -2
15. $9y - 4 = 6y + 11$ 5
16. $4x + 3 = 11 + 2x$ 4
17. $-5m + 7 = 5m - 13$ 2
18. $-2(a + 1) = 3a + 8$ -2
19. $0.5x + 0.6x = 5.5$ 5
20. $0.16m - 1 = 0.14m$ 50
21. $0.2a + 0.4(a - 1) = 8$ 14
22. $\frac{m}{35} = \frac{2}{7}$ 10
23. $\frac{x}{4} = \frac{x - 2}{3}$ 8
24. $\frac{3x}{4} - \frac{x}{3} = 5$ 12
25. $\begin{cases} x + 3y = 14 \\ 4x - y = 4 \end{cases}$ $x = 2$, $y = 4$
26. $\begin{cases} 3x + y = -6 \\ 2x + 3y = 3 \end{cases}$ $x = -3$, $y = 3$
27. $\begin{cases} 5a - b = 46 \\ 3a + 2b = 25 \end{cases}$ $a = 9, b = -1$
28. What percent of 93 is 31? $33\frac{1}{3}\%$
29. 16 is 40% of what number? 40

Draw the graph of the equation. Check student's graphs.

30. $y = x - 5$
31. $x + y = -4$
32. $x - 2y = 0$

Factor.

33. $b^2 + 2b$ $b(b + 2)$
34. $5x^3 - 10x$ $5x(x^2 - 2)$
35. $2n^2 + 6n$ $2n(n + 3)$
36. $x^2 + 6x - 27$ $(x + 9)(x - 3)$
37. $y^2 + 12y + 20$ $(y + 10)(y + 2)$
38. $a^2 + 2a - 8$ $(a + 4)(a - 2)$
39. $x^2 - 3x - 10$ $(x - 5)(x + 2)$
40. $m^2 - 6m - 7$ $(m - 7)(m + 1)$
41. $b^2 + 7b + 6$ $(b + 6)(b + 1)$
42. $y^2 - 4y - 21$ $(y - 7)(y + 3)$
43. $x^2 - y^2$ $(x - y)(x + y)$
44. $a^2 + 4a - 12$ $(a + 6)(a - 2)$
45. $a^2 + 6a + 9$ $(a + 3)(a + 3)$
46. $b^2 - 5b + 6$ $(b - 3)(b - 2)$
47. $4a^2 - b^2$ $(2a - b)(2a + b)$
48. $n^2 - 8n - 33$ $(n - 11)(n + 3)$
49. $9x^2 - 25y^2$ $(3x - 5y)(3x + 5y)$
50. $a^2 + 4a + 4$ $(a + 2)(a + 2)$

UNIT F

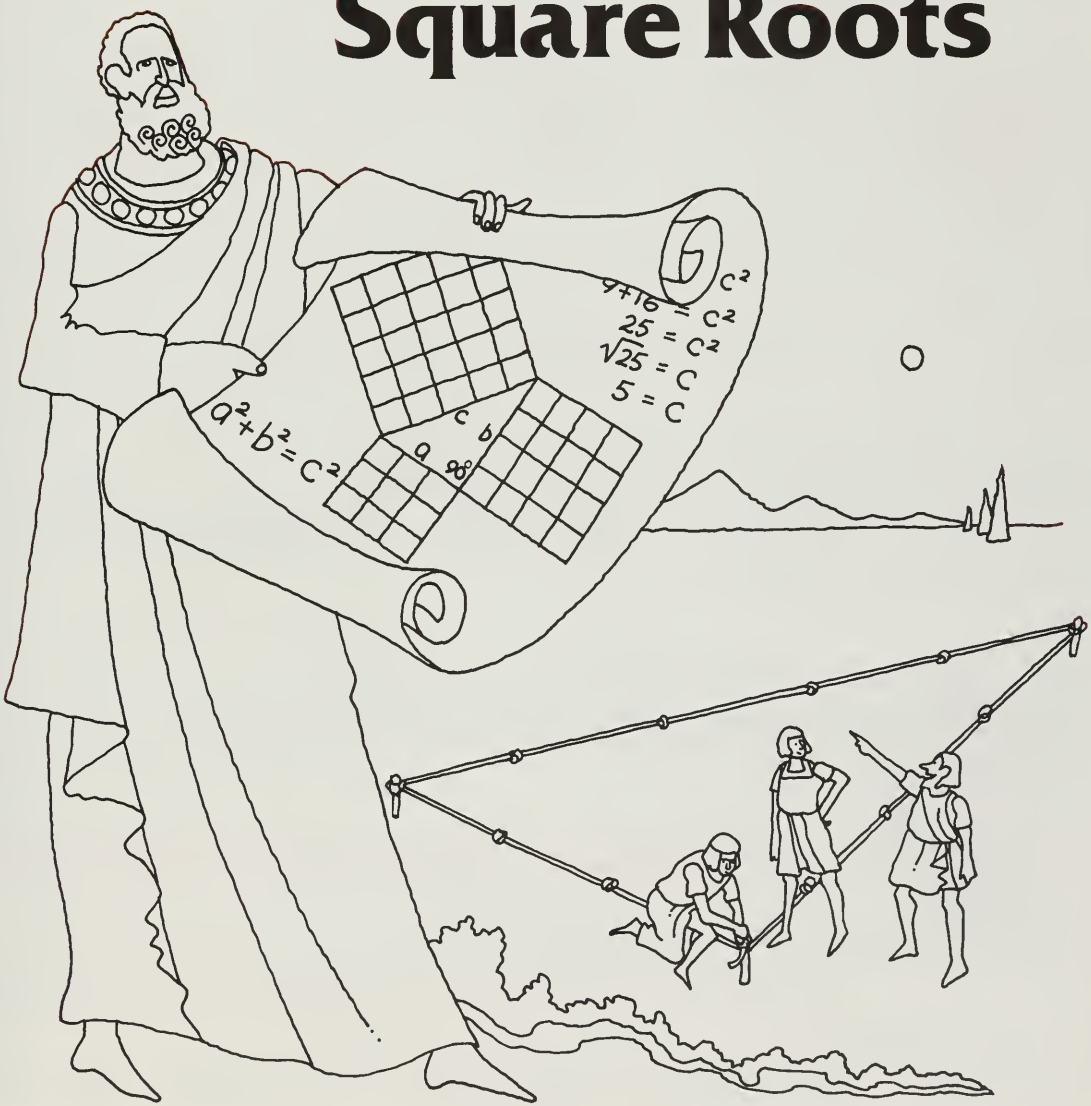
Here's what you'll learn in this chapter:

1. To find the positive and negative square roots of a number.
2. To use a square root table.
3. To identify rational and irrational numbers.
4. To simplify square roots.
5. To solve equations by using square roots.
6. To multiply and divide with radicals.
7. To add and subtract expressions with radicals.



Chapter 11

Squares and Square Roots



1 • Square Roots

When you square 3 or -3 you get 9. We say that each of the numbers 3 and -3 is a **square root** of 9.

Read: The positive, or principal, square root of 9 is 3. → $\sqrt{9} = 3$

$-\sqrt{9} = -3$ ← Read: The negative square root of 9 is -3.

The symbol $\sqrt{\ }$ is called the **radical sign.** It means the square root. $\sqrt{9}$ is a **radical number,** or **radical.**

Number	Positive Square Root	Negative Square Root
9	$\sqrt{9}$, or 3	$-\sqrt{9}$, or -3
36	$\sqrt{36}$, or 6	$-\sqrt{36}$, or -6
0	$\sqrt{0}$, or 0	

Note that 0 has only one square root, 0.

Sometimes we want to speak of *both* the positive and negative square roots of a number. In that case, the symbol $\pm$ is used.

Read: positive or negative.

$\pm\sqrt{25} = \pm 5$

$\sqrt{9}$, $\sqrt{36}$, $\sqrt{64}$, and $\sqrt{25}$ are easy to find because 9, 36, 64, and 25 are **perfect squares.** Any number which is the square of an integer is called a perfect square.

Classroom Practice

Which numbers are perfect squares?

1. 49 yes **2.** 50 no **3.** 1 yes **4.** 2 no **5.** 0 yes

Find the value.

6. $\sqrt{81}$ 9 **7.** $-\sqrt{81}$ -9 **8.** $\sqrt{100}$ 10 **9.** $-\sqrt{7^2}$ -7 **10.** $\sqrt{13^2}$ 13

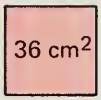
Written Exercises

Find the value.

A

1. $\sqrt{4}$ 2	**2.** $-\sqrt{9}$ −3	**3.** $\sqrt{25}$ 5	**4.** $-\sqrt{25}$ −5	**5.** $\sqrt{16}$ 4
6. $-\sqrt{16}$ −4	**7.** $-\sqrt{4}$ −2	**8.** $\sqrt{0}$ 0	**9.** $\sqrt{1}$ 1	**10.** $\sqrt{49}$ 7
11. $-\sqrt{49}$ −7	**12.** $-\sqrt{1}$ −1	**13.** $-\sqrt{36}$ −6	**14.** $\sqrt{9}$ 3	**15.** $\sqrt{64}$ 8
16. $\sqrt{100}$ 10	**17.** $-\sqrt{64}$ −8	**18.** $\sqrt{81}$ 9	**19.** $\sqrt{36}$ 6	**20.** $-\sqrt{81}$ −9
21. $\sqrt{8^2}$ 8	**22.** $\sqrt{9^2}$ 9	**23.** $\sqrt{3^2}$ 3	**24.** $-\sqrt{5^2}$ −5	**25.** $-\sqrt{10^2}$ −10

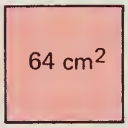
Find the perimeter of the square.

26. 

24 cm

27. 

20 cm

28. 

32 cm

B **29.** Find the value of $\sqrt{3^2 + 4^2}$. The answer is *not* $3 + 4$. 5

30. Find the value of $\sqrt{6^2 + 8^2}$. The answer is *not* $6 + 8$. 10

31. Find the value of $\sqrt{12^2 + 5^2}$. 13

Puzzles & Things

The Four 4's Puzzle

Can you group four 4's to make whole numbers from 1 to 16? Of course, there might be several ways to write some numbers, but you need to write only one. The first four are done for you.



$1 = (4 + 4) \div (4 + 4)$
$2 = 4 - (4 + 4) \div 4$
$3 = (4 + 4 + 4) \div 4$
$4 = (\sqrt{4} \cdot 4) \div (4 - \sqrt{4})$

See page 17 of Answers to Odd-Numbered Exercises in the back of the text.

2 • Using a Square Root Table

The table below gives the square roots of integers from 1 to 100. Notice that most of the values in the Square Root column are not integers. These values are not exact, but are rounded to three decimal places. They are *approximate* values.

Number	Positive Square Root	Number	Positive Square Root	Number	Positive Square Root	Number	Positive Square Root
n	$\sqrt{n}$	n	$\sqrt{n}$	n	$\sqrt{n}$	n	$\sqrt{n}$
1	1	**26**	5.099	**51**	7.141	**76**	8.718
2	1.414	**27**	5.196	**52**	7.211	**77**	8.775
3	1.732	**28**	5.292	**53**	7.280	**78**	8.832
4	2	**29**	5.385	**54**	7.348	**79**	8.888
5	2.236	**30**	5.477	**55**	7.416	**80**	8.944
6	2.449	**31**	5.568	**56**	7.483	**81**	9
7	2.646	**32**	5.657	**57**	7.550	**82**	9.055
8	2.828	**33**	5.745	**58**	7.616	**83**	9.110
9	3	**34**	5.831	**59**	7.681	**84**	9.165
10	3.162	**35**	5.916	**60**	7.746	**85**	9.220
11	3.317	**36**	6	**61**	7.810	**86**	9.274
12	3.464	**37**	6.083	**62**	7.874	**87**	9.327
13	3.606	**38**	6.164	**63**	7.937	**88**	9.381
14	3.742	**39**	6.245	**64**	8	**89**	9.434
15	3.873	**40**	6.325	**65**	8.062	**90**	9.487
16	4	**41**	6.403	**66**	8.124	**91**	9.539
17	4.123	**42**	6.481	**67**	8.185	**92**	9.592
18	4.243	**43**	6.557	**68**	8.246	**93**	9.644
19	4.359	**44**	6.633	**69**	8.307	**94**	9.695
20	4.472	**45**	6.708	**70**	8.367	**95**	9.747
21	4.583	**46**	6.782	**71**	8.426	**96**	9.798
22	4.690	**47**	6.856	**72**	8.485	**97**	9.849
23	4.796	**48**	6.928	**73**	8.544	**98**	9.899
24	4.899	**49**	7	**74**	8.602	**99**	9.950
25	5	**50**	7.071	**75**	8.660	**100**	10

$\sqrt{13} \doteq 3.606$

$\sqrt{74} \doteq 8.602$

Written Exercises

Use the table to find the value.

A

1. $\sqrt{8}$ 2.828
2. $\sqrt{15}$ 3.873
3. $\sqrt{40}$ 6.325
4. $\sqrt{30}$ 5.477
5. $\sqrt{18}$ 4.243
6. $\sqrt{65}$ 8.062
7. $\sqrt{20}$ 4.472
8. $\sqrt{51}$ 7.141
9. $\sqrt{79}$ 8.888
10. $\sqrt{19}$ 4.359
11. $\sqrt{89}$ 9.434
12. $\sqrt{10}$ 3.162
13. $\sqrt{35}$ 5.916
14. $\sqrt{62}$ 7.874
15. $\sqrt{75}$ 8.660
16. $-\sqrt{53}$ −7.280
17. $-\sqrt{38}$ −6.164
18. $-\sqrt{29}$ −5.385
19. $-\sqrt{50}$ −7.071
20. $-\sqrt{74}$ −8.602
21. $\sqrt{57}$ 7.550
22. $\pm\sqrt{81}$ ±9
23. $\pm\sqrt{16}$ ±4
24. $\pm\sqrt{35}$ ±5.916
25. $\pm\sqrt{67}$ ±8.185

Round to one decimal place.

26. $\sqrt{7}$ 2.6
27. $\sqrt{14}$ 3.7
28. $\sqrt{10}$ 3.2
29. $\sqrt{57}$ 7.6
30. $\sqrt{83}$ 9.1
31. $\sqrt{51}$ 7.1
32. $\sqrt{21}$ 4.6
33. $\sqrt{19}$ 4.4
34. $\sqrt{27}$ 5.2
35. $\sqrt{37}$ 6.1
36. $\sqrt{56}$ 7.5
37. $-\sqrt{18}$ −4.2
38. $-\sqrt{43}$ −6.6
39. $-\sqrt{71}$ −8.4
40. $-\sqrt{48}$ −6.9

Find the length of a side of the square correct to one decimal place.

B

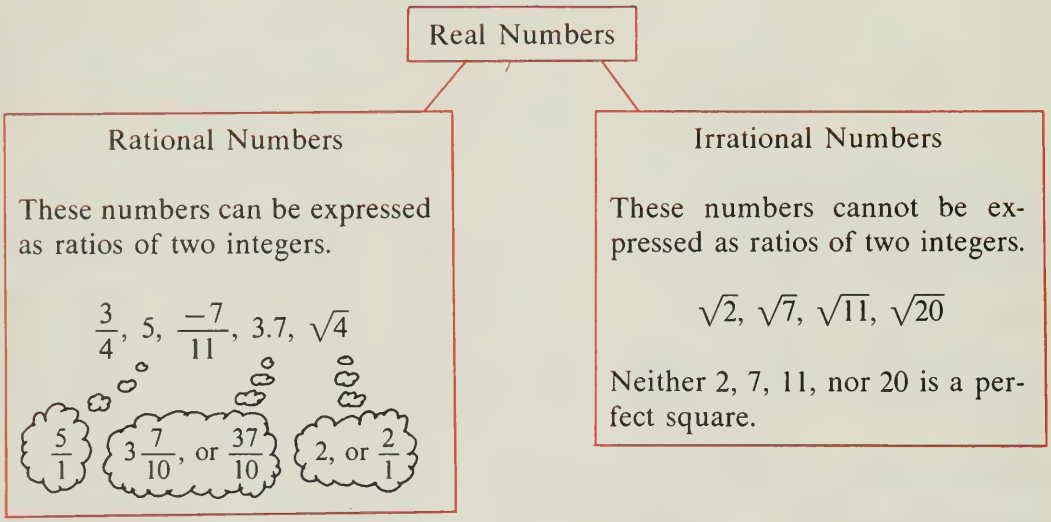
41. 70 cm^2 — 8.4 cm
42. 56 cm^2 — 7.5 cm
43. 20 cm^2 — 4.5 cm
44. 35 cm^2 — 5.9 cm

Find the perimeter of the square correct to one decimal place.

45. 48 cm^2 — 27.7 cm
46. 26 cm^2 — 20.4 cm
47. 38 cm^2 — 24.7 cm
48. 65 cm^2 — 32.2 cm
49. 24 cm^2 — 19.6 cm
50. 60 cm^2 — 31.0 cm
51. 40 cm^2 — 25.3 cm
52. 72 cm^2 — 33.9 cm

3 • Irrational Numbers

All the numbers you've worked with in this course are real numbers. Real numbers are the positive numbers, negative numbers, and zero. Real numbers can be separated into two kinds of numbers, the rational numbers and the irrational numbers.



All irrational numbers have decimal forms which go on and on, but never repeat.

$$\sqrt{2} = 1.4142\square\square\square\square\square\square\square\square\square\ldots$$
$$\sqrt{7} = 2.6457\square\square\square\square\square\square\square\square\square\ldots$$
$$\sqrt{11} = 3.3166\square\square\square\square\square\square\square\square\square\ldots$$
$$\sqrt{20} = 4.4721\square\square\square\square\square\square\square\square\square\ldots$$

One famous irrational number is π (pi). A common place for it to occur is in formulas like $C = \pi d$ and $A = \pi r^2$ to calculate the circumference and area of a circle. π is the ratio of C to d in any circle.

$$\pi = 3.1415926535\ldots$$

but we often write → $\pi \doteq 3.14$

or → $\pi \doteq 3\frac{1}{7}$

Using a computer, people have carried the value of π to thousands of decimal places.

Written Exercises

Is the number rational or is it irrational?

A **1.** 4 r **2.** $\frac{1}{5}$ r **3.** $\sqrt{4}$ r **4.** $\frac{1}{8}$ r **5.** -8 r

6. $\sqrt{7}$ i **7.** $\sqrt{9}$ r **8.** $\frac{3}{4}$ r **9.** $\sqrt{2}$ i **10.** $\frac{1}{3}$ r

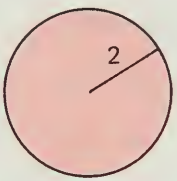
11. $\sqrt{6}$ i **12.** -7 r **13.** $\frac{4}{5}$ r **14.** 11 r **15.** $4\frac{3}{5}$ r

16. $\sqrt{16}$ r **17.** 6.72 r **18.** 3.715 r **19.** $-\sqrt{25}$ r **20.** 2π i

21. $\frac{2}{3}$ r **22.** $-\sqrt{10}$ i **23.** $\frac{5}{8}$ r **24.** -17 r **25.** $-\sqrt{49}$ r

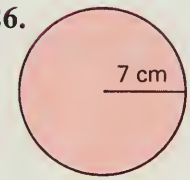
Find the approximate area. Use the formula $A = \pi r^2$. Use 3.14 for π.

Sample



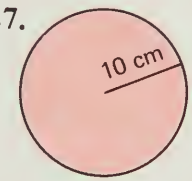
$A = \pi r^2$
$A \doteq 3.14 \cdot 2^2$
$A \doteq 3.14 \cdot 4$
$A \doteq 12.56$

26.



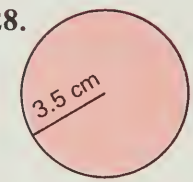
153.86 cm²

27.



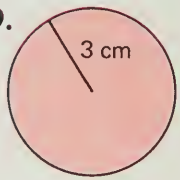
314 cm²

28.



38.465 cm²

29.



28.26 cm²

30. If $\sqrt{n}$ is a rational number, n must be a ___?___ ___?___. perfect square

Add or subtract.

B **31.** $\frac{2}{3} + \frac{5}{6}$ $\frac{3}{2}$ or $1\frac{1}{2}$ **32.** $\frac{4}{5} - \frac{3}{10}$ $\frac{1}{2}$ **33.** $\frac{4}{7} + \frac{3}{5}$ $\frac{41}{35}$ or $1\frac{6}{35}$ **34.** $\frac{2}{9} - \frac{5}{6}$ $-\frac{11}{18}$

35. Look at your answers to Exercises 31–34. Do you think that the sum or difference of two rational numbers is always a rational number? yes

36. Try to find an example that shows that the sum or difference of two irrational numbers is *not* always an irrational number. Answers may vary. $\sqrt{2} - \sqrt{2} = 0$, which is rational.

For more practice, see page 432.

4 • Simplifying Square Roots

You know → $\sqrt{36} = 6$

but also →
$$\begin{aligned}\sqrt{36} &= \sqrt{9 \cdot 4} \\ &= \sqrt{9} \cdot \sqrt{4} \\ &= 3 \cdot 2 \\ &= 6\end{aligned}$$



You can see that the following rule has been used.

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$

The rule above can be used to find square roots of numbers not found in the table on page 350.

EXAMPLE 1
$$\begin{aligned}\sqrt{2500} &= \sqrt{25 \cdot 100} \\ &= \sqrt{25} \cdot \sqrt{100} \\ &= 5 \cdot 10 \\ &= 50\end{aligned}$$

We choose the factors 25 and 100 because they are perfect squares.

EXAMPLE 2
$$\begin{aligned}\sqrt{400} &= \sqrt{4 \cdot 100} \\ &= \sqrt{4} \cdot \sqrt{100} \\ &= 2 \cdot 10 \\ &= 20\end{aligned}$$

EXAMPLE 3
$$\begin{aligned}\sqrt{125} &= \sqrt{5 \cdot 25} \\ &= \sqrt{5} \cdot \sqrt{25} \\ &= \sqrt{5} \cdot 5 \\ &= 5\sqrt{5}\end{aligned}$$

You won't always be able to find two factors which are perfect squares.

Product of a rational and an irrational number

Often it is convenient to leave your answer with a radical sign in it, as $5\sqrt{5}$. If your answer expresses a measurement, then you could use the square root table to find $\sqrt{5}$.

$$5\sqrt{5}\text{ cm} \doteq 5 \times 2.236\text{ cm} \doteq 11.180\text{ cm}$$

In working with variables, use the same rules. In this chapter, when you see a variable under a radical sign, assume that it represents a number greater than zero.

EXAMPLE 4

$$\begin{aligned}\sqrt{8x^2} &= \sqrt{4 \cdot 2 \cdot x^2} \\ &= \sqrt{4} \cdot \sqrt{2} \cdot \sqrt{x^2} \\ &= 2x\sqrt{2}\end{aligned}$$

$\sqrt{x^2} = x$

Written Exercises

Find the value. Leave your answer in simplest radical form if the number is not a perfect square.

A

1. $\sqrt{144}$ 12
2. $\sqrt{243}$ $9\sqrt{3}$
3. $\sqrt{225}$ 15
4. $\sqrt{196}$ 14
5. $\sqrt{1000}$ $10\sqrt{10}$
6. $\sqrt{12}$ $2\sqrt{3}$
7. $\sqrt{2000}$ $20\sqrt{5}$
8. $\sqrt{324}$ 18
9. $\sqrt{128}$ $8\sqrt{2}$
10. $\sqrt{288}$ $12\sqrt{2}$
11. $\sqrt{180}$ $6\sqrt{5}$
12. $\sqrt{338}$ $13\sqrt{2}$
13. $\sqrt{108}$ $6\sqrt{3}$
14. $\sqrt{343}$ $7\sqrt{7}$
15. $\sqrt{192}$ $8\sqrt{3}$
16. $\sqrt{392}$ $14\sqrt{2}$
17. $\sqrt{1089}$ 33
18. $\sqrt{1156}$ 34
19. $\sqrt{1764}$ 42
20. $\sqrt{1920}$ $8\sqrt{30}$

Find the value. Leave your answer in simplest radical form.

B

21. $\sqrt{16x^2}$ $4x$
22. $\sqrt{12y^2}$ $2y\sqrt{3}$
23. $\sqrt{64x^2y}$ $8x\sqrt{y}$
24. $\sqrt{15x^2y^2}$ $xy\sqrt{15}$
25. $\sqrt{75a^4}$ $5a^2\sqrt{3}$
26. $\sqrt{36xy^2}$ $6y\sqrt{x}$
27. $\sqrt{125x^3}$ $5x\sqrt{5x}$
28. $\sqrt{3a^2b^2}$ $ab\sqrt{3}$

C

29. Show by an example that $\sqrt{x^2} \neq x$ when x is negative. Answers may vary.
Let $x = -2$; $\sqrt{(-2)^2} = \sqrt{4} = 2 \neq -2$

For more practice, see page 432.

SELF-TEST

Find the square root. Leave your answer in simplest radical form if it is not a perfect square.

1. $\sqrt{49}$ 7
2. $\sqrt{1}$ 1
3. $-\sqrt{9}$ -3
4. $-\sqrt{100}$ -10
5. $\sqrt{10000}$ 100
6. $\sqrt{75}$ $5\sqrt{3}$
7. $\sqrt{90}$ $3\sqrt{10}$
8. $\sqrt{3600}$ 60

Is the number rational or irrational?

9. $\sqrt{16}$ r
10. 4.25 r
11. -3 r
12. $\sqrt{18}$ i

5 • Solving Equations

Some equations can be solved by using square roots.

$x^2 = 36$

Take the square root. → $x = 6$ ← Take the square root.

or

$x = -6$

So, the equation $x^2 = 36$ has two solutions, 6 and -6.

Check:

$x^2 = 36$	
6^2	36
36	

✓

$x^2 = 36$	
$(-6)^2$	36
36	

✓

EXAMPLE 1 $a^2 = 25$

Take the square root of both sides.

$a = \pm 5$

So, $a = 5$ or $a = -5$.

EXAMPLE 2 $\frac{x}{9} = \frac{4}{x}$

Cross multiply first.

$x^2 = 4 \cdot 9$

$x^2 = 36$

$x = \pm 6$

EXAMPLE 3 $y^2 - 7 = 57$

$y^2 = 57 + 7$

$y^2 = 64$

Take the square root of both sides.

$y = \pm 8$

EXAMPLE 4 $x^2 + 3 = 27$

$x^2 = 27 - 3$

$x^2 = 24$

Take the square root of both sides.

$x = \pm\sqrt{24}$

$x = \pm\sqrt{4 \cdot 6}$

$x = \pm 2\sqrt{6}$

Written Exercises

Find the two solutions to the equation.

A

1. $x^2 = 64$ ± 8
2. $x^2 = 81$ ± 9
3. $m^2 = 16$ ± 4
4. $n^2 = 100$ ± 10
5. $y^2 = 9$ ± 3
6. $n^2 = 49$ ± 7
7. $n^2 = 121$ ± 11
8. $x^2 = 144$ ± 12
9. $\frac{x}{3} = \frac{3}{x}$ ± 3
10. $\frac{x}{4} = \frac{25}{x}$ ± 10
11. $\frac{4}{x} = \frac{x}{9}$ ± 6
12. $\frac{y}{2} = \frac{8}{y}$ ± 4
13. $y^2 - 1 = 35$ ± 6
14. $y^2 - 5 = 20$ ± 5
15. $x^2 + 6 = 31$ ± 5
16. $n^2 + 4 = 53$ ± 7
17. $x^2 = 12$ $\pm 2\sqrt{3}$
18. $n^2 = 10$ $\pm \sqrt{10}$
19. $y^2 = 6$ $\pm \sqrt{6}$
20. $x^2 = 27$ $\pm 3\sqrt{3}$
21. $x^2 + 2 = 8$ $\pm \sqrt{6}$
22. $y^2 - 7 = 20$ $\pm 3\sqrt{3}$
23. $5 + y^2 = 12$ $\pm \sqrt{7}$
24. $8 + y^2 = 30$ $\pm \sqrt{22}$
25. $x^2 + 5 = 23$ $\pm 3\sqrt{2}$
26. $n^2 - 2 = 40$ $\pm \sqrt{42}$
27. $x^2 - 5 = 11$ ± 4
28. $7 + a^2 = 9$ $\pm \sqrt{2}$
29. $x^2 - 1 = 3$ ± 2
30. $y^2 + 5 = 21$ ± 4
31. $x^2 - 5 = 31$ ± 6
32. $a^2 + 1 = 82$ ± 9

Find the two solutions correct to one decimal place.

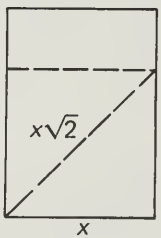
33. $n^2 = 40$ ± 6.3
34. $x^2 = 56$ ± 7.5
35. $y^2 = 38$ ± 6.2
36. $x^2 = 91$ ± 9.5
37. $x^2 - 1 = 75$ ± 8.7
38. $y^2 + 3 = 80$ ± 8.8
39. $x^2 + 7 = 52$ ± 6.7
40. $n^2 - 4 = 75$ ± 8.9

For more practice, see page 432.

Puzzles & Things

Many great artists used a special rectangle as a basis for their drawings. It is special because the ratio of the length to the width is $\sqrt{2}$.

Look at the figure at the right. Do you see why the length of the diagonal of the square is $x\sqrt{2}$?



Now measure the longer side in the large rectangle. Compare its length to the length of the diagonal of the square. What do you notice?

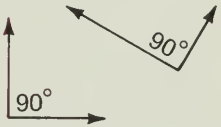
In Germany standard sizes of paper are made with this same ratio of length to width.

See page 17 of Answers to Odd-Numbered Exercises in the back of the text.

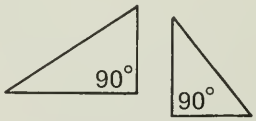
6 • Law of Pythagoras



Centuries ago there was a brilliant Greek mathematician named Pythagoras. He is often credited with the discovery of one of the most useful laws in all mathematics. It deals with right angles and right triangles.

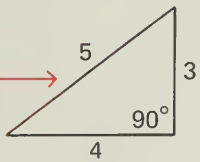


right angles



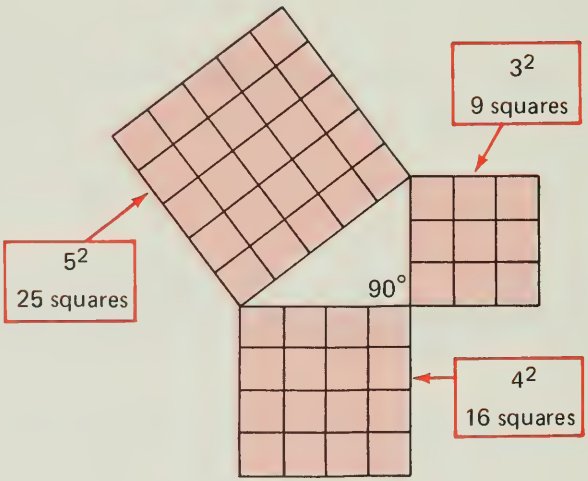
right triangles

1. Consider this triangle.



Notice that the side opposite the right angle is the longest. This is true in all right triangles. This side is called the **hypotenuse.**

2. Build squares on its sides.



$9 + 16 = 25$

$3^2 + 4^2 = 5^2$

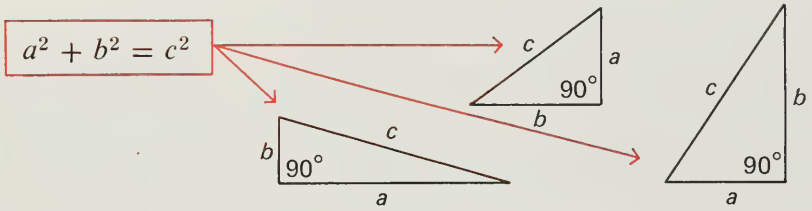
The relationship between the lengths of the sides of the triangle shown above is true in *all* right triangles.



LAW OF PYTHAGORAS

The square of the length of the hypotenuse of a right triangle equals the sum of the squares of the lengths of the other two sides.

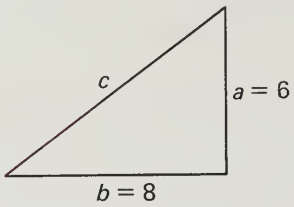
Let's consider what this Law of Pythagoras means. Take any right triangle. Call the length of its hypotenuse c. Call the lengths of the other two sides a and b. Then the following formula is true.



EXAMPLE 1 Find the length of the hypotenuse in this triangle.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 6^2 + 8^2 &= c^2 \\ 36 + 64 &= c^2 \\ 100 &= c^2 \\ \sqrt{100} &= c \\ 10 &= c \end{aligned}$$

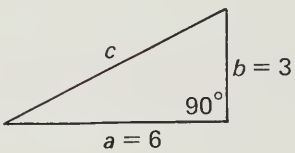
We use only the positive square root of 100 because the negative root makes no sense here.



EXAMPLE 2 Find the length of the hypotenuse in this triangle.

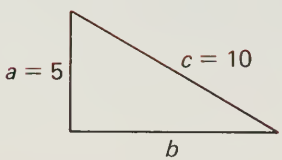
$$\begin{aligned} a^2 + b^2 &= c^2 \\ 6^2 + 3^2 &= c^2 \\ 36 + 9 &= c^2 \\ 45 &= c^2 \\ \sqrt{45} &= c \\ 3\sqrt{5} &= c \end{aligned}$$

Simplify: $\sqrt{45} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$



EXAMPLE 3 Find b in the following triangle.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 5^2 + b^2 &= 10^2 \\ 25 + b^2 &= 100 \\ b^2 &= 75 \\ b &= \sqrt{75} \\ b &= 5\sqrt{3} \end{aligned}$$



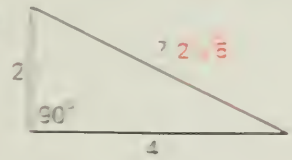
Classroom Practice

Complete.

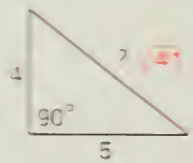
1. In a right triangle there is always one ___right___ angle.
2. The hypotenuse of a right triangle is the side ___opposite___ the right angle.
3. If you know the lengths of ___two___ sides of a ___right___ triangle, you can use the Law of ___Pythagoras___ to find the ___third___ side.
4. In the formula $a^2 + b^2 = c^2$, c stands for the length of the ___hypotenuse___ of a ___right___ triangle.
5. The Law of Pythagoras is true for ___all___ right triangles.

Written Exercises

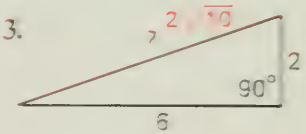
Find the length of the third side of the triangle.

1. 

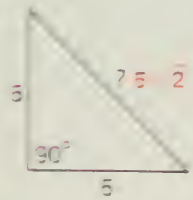
≈ 2.8

2. 

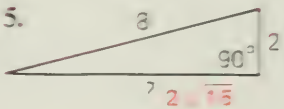
≈ 6.4

3. 

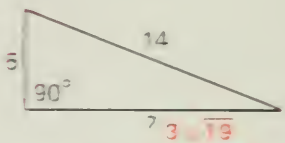
≈ 10

4. 

≈ 7.1

5. 

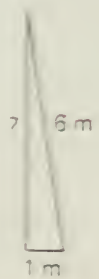
≈ 15

6. 

≈ 19

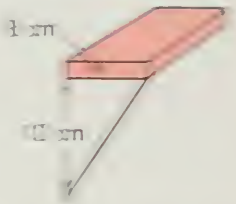
Solve. Give your results in radical form, and then correct to one decimal place.

7. A ladder 6 meters long leans against a wall. It rests on the ground 1 meter from the wall. How high on the wall does the ladder rest? ≈ 5.9 m



8. A city park is a square 1 km on each side.
How far do you walk from one corner to the opposite one?
Express your answer to a decimal correct to one decimal place. [illegible] km

9. A shelf support must be how long to meet the conditions shown in the picture? [illegible] cm

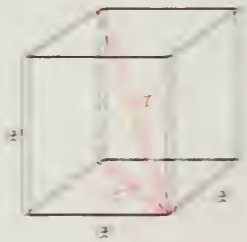


10. What is the longest line you can draw with a ruler on a piece of paper 20 cm by 30 cm? [illegible] cm

11. How long an umbrella can you carry in the bottom of a suitcase 45 cm by 18 cm? [illegible] cm



B 12. The drawing at the right shows the diagonal of a cube.
Find a formula for the length of the diagonal d. (Hint: first find a, the diagonal of the base.) $d = s\sqrt{3}$



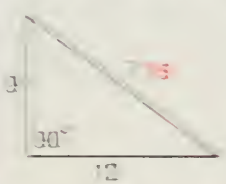
For more practice, see page 420.

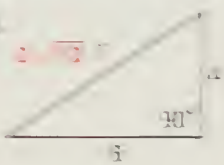
SELF-TEST

Find the two solutions to the equation.

1. $x^2 = 49$ [illegible]
2. $y^2 = 100$ [illegible]
3. $\frac{5}{10} = \frac{x}{8}$ [illegible]
4. $c^2 + 2 = 18$ [illegible]
5. $y^2 - 3 = 22$ [illegible]
6. $x^2 + 0 = 10$ [illegible]

Find the length of the third side of the triangle.

7. 

8. 

9. 

7 • Quotients of Square Roots

You may sometimes need to find the square root of a fraction. Here's a method to use when the denominator is a perfect square.

EXAMPLE 1 $\sqrt{\frac{144}{9}} = \frac{\sqrt{144}}{\sqrt{9}} = \frac{12}{3} = 4$

EXAMPLE 2 $\sqrt{\frac{6}{16}} = \frac{\sqrt{6}}{\sqrt{16}} = \frac{\sqrt{6}}{4} = \frac{1}{4}\sqrt{6}$

EXAMPLE 3 $\sqrt{\frac{y^2}{4}} = \frac{\sqrt{y^2}}{\sqrt{4}} = \frac{y}{2}$

EXAMPLE 4 $\sqrt{\frac{3x^2}{16}} = \frac{\sqrt{3x^2}}{\sqrt{16}} = \frac{x\sqrt{3}}{4}$



You can see that the following rule has been used.

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Classroom Practice

Find the square root. Results should be in simplest form.

1. $\sqrt{\frac{4}{9}}$ $\frac{2}{3}$
2. $\sqrt{\frac{3}{4}}$ $\frac{\sqrt{3}}{2}$
3. $\sqrt{\frac{5}{9}}$ $\frac{\sqrt{5}}{3}$
4. $\sqrt{\frac{3}{16}}$ $\frac{\sqrt{3}}{4}$
5. $\sqrt{\frac{7}{25}}$ $\frac{\sqrt{7}}{5}$
6. $\sqrt{\frac{5}{36}}$ $\frac{\sqrt{5}}{6}$
7. $\sqrt{\frac{7}{49}}$ $\frac{\sqrt{7}}{7}$
8. $\sqrt{\frac{15}{36}}$ $\frac{\sqrt{15}}{6}$
9. $\sqrt{\frac{a^2}{4}}$ $\frac{a}{2}$
10. $\sqrt{\frac{x^2}{9}}$ $\frac{x}{3}$
11. $\sqrt{\frac{2x^2}{25}}$ $\frac{x\sqrt{2}}{5}$
12. $\sqrt{\frac{4n^2}{49}}$ $\frac{2n}{7}$

Find the square root. Express the result correct to two decimal places.

13. $\sqrt{\frac{2}{4}}$ 0.71
14. $\sqrt{\frac{1}{9}}$ 0.33
15. $\sqrt{\frac{1}{4}}$ 0.50
16. $\sqrt{\frac{10}{16}}$ 0.79

Written Exercises

Find the square root. Results should be in simplest form.

A **1.** $\sqrt{\frac{4}{25}}$ $\frac{2}{5}$ **2.** $\sqrt{\frac{16}{64}}$ $\frac{1}{2}$ **3.** $\sqrt{\frac{121}{4}}$ $\frac{11}{2}$ **4.** $\sqrt{\frac{9}{144}}$ $\frac{1}{4}$

5. $\sqrt{\frac{1}{16}}$ $\frac{1}{4}$ **6.** $\sqrt{\frac{16}{49}}$ $\frac{4}{7}$ **7.** $\sqrt{\frac{36}{64}}$ $\frac{3}{4}$ **8.** $\sqrt{\frac{225}{625}}$ $\frac{3}{5}$

9. $\sqrt{\frac{9}{196}}$ $\frac{3}{14}$ **10.** $\sqrt{\frac{169}{16}}$ $\frac{13}{4}$ **11.** $\sqrt{\frac{1}{144}}$ $\frac{1}{12}$ **12.** $\sqrt{\frac{1}{400}}$ $\frac{1}{20}$

Simplify.

13. $\sqrt{\frac{x^2}{36}}$ $\frac{x}{6}$ **14.** $\sqrt{\frac{y^2}{81}}$ $\frac{y}{9}$ **15.** $\sqrt{\frac{4y^2}{9}}$ $\frac{2y}{3}$ **16.** $\sqrt{\frac{16x^2}{25}}$ $\frac{4x}{5}$

17. $\sqrt{\frac{3x^2}{4}}$ $\frac{x\sqrt{3}}{2}$ **18.** $\sqrt{\frac{x^2y^2}{100}}$ $\frac{xy}{10}$ **19.** $\sqrt{\frac{7a^2}{64}}$ $\frac{a\sqrt{7}}{8}$ **20.** $\frac{1}{2}\sqrt{\frac{4n^2}{9}}$ $\frac{n}{3}$

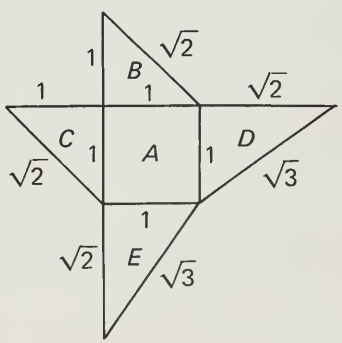
Find the square root. Express the result correct to two decimal places.

21. $\sqrt{\frac{3}{4}}$ 0.87 **22.** $\sqrt{\frac{10}{16}}$ 0.79 **23.** $\sqrt{\frac{12}{9}}$ 1.54 **24.** $\sqrt{\frac{22}{49}}$ 0.67

25. $\sqrt{\frac{20}{25}}$ 0.89 **26.** $\sqrt{\frac{50}{4}}$ 3.54 **27.** $\sqrt{\frac{21}{9}}$ 1.53 **28.** $\sqrt{\frac{15}{36}}$ 0.65

Puzzles & Things

If you copy this diagram carefully, only larger, making it in the order of the letters, you can fold it to form a pyramid. If you make three of these you can fit them together to form a cube.



8 • Multiplication and Division

We can multiply and divide radical expressions. Just turn around the rules you've learned.



$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ can be written $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$.

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ can be written $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$.

These two new rules can be helpful when you do not see a perfect square in an expression.

EXAMPLE 1 $\sqrt{2} \cdot \sqrt{8} = \sqrt{2 \cdot 8}$
$= \sqrt{16}$
$= 4$

EXAMPLE 2 $3\sqrt{2} \cdot 4\sqrt{3} = 3 \cdot 4 \cdot \sqrt{2} \cdot \sqrt{3}$
$= 12\sqrt{6}$

EXAMPLE 3 $3\sqrt{a} \cdot 2\sqrt{a} = 3 \cdot 2 \cdot \sqrt{a} \cdot \sqrt{a}$
$= 6\sqrt{a^2}$
$= 6a$

Examine divisions with radicals closely too. If you do not see perfect squares in the expressions you are dividing, you can simplify in the following way.

EXAMPLE 4 $\frac{\sqrt{18}}{\sqrt{2}} = \sqrt{\frac{18}{2}} = \sqrt{9} = 3$

EXAMPLE 5 $\frac{\sqrt{12}}{\sqrt{6}} = \sqrt{\frac{12}{6}} = \sqrt{2}$

EXAMPLE 6 $\frac{\sqrt{x^3}}{\sqrt{x}} = \sqrt{\frac{x^3}{x}} = \sqrt{x^2} = x$

Written Exercises

Simplify. Leave no perfect-square factor under the radical sign.

A

1. $\sqrt{3} \cdot \sqrt{3}$ 3
2. $\sqrt{2} \cdot \sqrt{2}$ 2
3. $\sqrt{7} \cdot \sqrt{7}$ 7
4. $\sqrt{5} \cdot \sqrt{5}$ 5
5. $\sqrt{3} \cdot \sqrt{6}$ $3\sqrt{2}$
6. $\sqrt{2} \cdot \sqrt{6}$ $2\sqrt{3}$
7. $\sqrt{2} \cdot \sqrt{8}$ 4
8. $\sqrt{10} \cdot \sqrt{2}$ $2\sqrt{5}$
9. $2 \cdot 2\sqrt{3}$ $4\sqrt{3}$
10. $2\sqrt{2} \cdot 2\sqrt{3}$ $4\sqrt{6}$
11. $4\sqrt{3} \cdot \sqrt{3}$ 12
12. $4\sqrt{5} \cdot 2\sqrt{5}$ 40
13. $\sqrt{18} \cdot 5$ $15\sqrt{2}$
14. $2\sqrt{18} \cdot 3$ $18\sqrt{2}$
15. $\sqrt{2} \cdot \sqrt{18}$ 6
16. $2\sqrt{6} \cdot \sqrt{12}$ $12\sqrt{2}$
17. $\sqrt{x} \cdot \sqrt{x}$ x
18. $\sqrt{a} \cdot \sqrt{a}$ a
19. $2\sqrt{a} \cdot 4\sqrt{a}$ $8a$
20. $3x\sqrt{2} \cdot x\sqrt{2}$ $6x^2$
21. $4\sqrt{x} \cdot 2\sqrt{x}$ $8x$
22. $2\sqrt{n} \cdot \sqrt{4n}$ $4n$
23. $\dfrac{\sqrt{10}}{\sqrt{2}}$ $\sqrt{5}$
24. $\dfrac{\sqrt{12}}{\sqrt{3}}$ 2
25. $\dfrac{\sqrt{14}}{\sqrt{2}}$ $\sqrt{7}$
26. $\dfrac{\sqrt{18}}{\sqrt{3}}$ $\sqrt{6}$
27. $\dfrac{\sqrt{20}}{\sqrt{5}}$ 2
28. $\dfrac{\sqrt{21}}{\sqrt{7}}$ $\sqrt{3}$
29. $\dfrac{10\sqrt{6}}{\sqrt{3}}$ $10\sqrt{2}$
30. $\dfrac{6\sqrt{10}}{\sqrt{2}}$ $6\sqrt{5}$
31. $\dfrac{8\sqrt{12}}{2\sqrt{3}}$ 8
32. $\dfrac{14\sqrt{14}}{7\sqrt{2}}$ $2\sqrt{7}$
33. $\dfrac{\sqrt{a^3}}{\sqrt{a}}$ a
34. $\dfrac{2\sqrt{x}}{\sqrt{x}}$ 2
35. $\dfrac{\sqrt{2n^2}}{\sqrt{n}}$ $\sqrt{2n}$
36. $\dfrac{\sqrt{8x}}{\sqrt{2x}}$ 2
37. $\dfrac{\sqrt{27a^2}}{\sqrt{3b^2}}$ $\dfrac{3a}{b}$
38. $\dfrac{\sqrt{3xy^3}}{\sqrt{2xy}}$ $y\sqrt{\dfrac{3}{2}}$
39. $\sqrt{\dfrac{1}{9}} \cdot \sqrt{\dfrac{1}{4}}$ $\dfrac{1}{6}$
40. $\sqrt{\dfrac{2}{3}} \cdot \sqrt{\dfrac{2}{27}}$ $\dfrac{2}{9}$
41. $\sqrt{\dfrac{2}{3}} \cdot \sqrt{\dfrac{3}{2}}$ 1
42. $\sqrt{\dfrac{4}{5}} \cdot \sqrt{\dfrac{5}{4}}$ 1
43. $\sqrt{\dfrac{10}{4}} \cdot \sqrt{\dfrac{4}{2}}$ $\sqrt{5}$
44. $\sqrt{\dfrac{12}{5}} \cdot \sqrt{\dfrac{5}{3}}$ 2

B

45. $3\sqrt{x^3} \cdot 2\sqrt{x}$ $6x^2$
46. $5\sqrt{n} \cdot \dfrac{1}{5}\sqrt{\dfrac{1}{n}}$ 1
47. $-3x\sqrt{2} \cdot x\sqrt{8}$ $-12x^2$
48. $\sqrt{\dfrac{1}{7}} \cdot -x\sqrt{14}$ $-x\sqrt{2}$
49. $-x^3 \cdot -\sqrt{\dfrac{1}{4x^4}}$ $\dfrac{x}{2}$
50. $\sqrt{2} \cdot \sqrt{3} \cdot \sqrt{6}$ 6
51. $\sqrt{3x} \cdot \sqrt{2x} \cdot \sqrt{3}$ $3x\sqrt{2}$
52. $3x\sqrt{9x^2} \cdot \sqrt{6x^4}$ $9x^4\sqrt{6}$
53. $2a^2\sqrt{3a^2} \cdot 5\sqrt{5a^2}$ $10a^4\sqrt{15}$

C

54. $\sqrt{n} \cdot \dfrac{\sqrt{n^4}}{\sqrt{n}}$ n^2
55. $x \cdot \dfrac{-2\sqrt{x^3}}{\sqrt{4x}}$ $-x^2$
56. $\dfrac{\sqrt{n^4}}{\sqrt{n^2}} \cdot 3n\sqrt{\dfrac{9n^3}{n}}$ $9n^3$

9 • Rationalizing the Denominator

Can you find the decimal value of $\frac{\sqrt{2}}{\sqrt{5}}$? You might look in the table of square roots and write $\frac{\sqrt{2}}{\sqrt{5}} \doteq \frac{1.414}{2.236}$. Unless you had a calculator, you'd have troublesome division to do. Here is an easier way:

$$\frac{\sqrt{2}}{\sqrt{5}} = \frac{\sqrt{2} \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}}$$
$$= \frac{\sqrt{10}}{5}$$
$$= \frac{1}{5}\sqrt{10}$$

You can multiply the numerator and denominator by the same number. You choose $\sqrt{5}$ because you'll then have a whole number for the denominator.

Now if you need to find the decimal value, you can find $\sqrt{10}$ in the table and calculate.

$$\frac{1}{5}\sqrt{10} \doteq \frac{1}{5} \times 3.162 \doteq 0.632$$



When a fraction has a radical in the denominator, you can simplify it by multiplying both numerator and denominator by a number that will make the denominator a whole number. This process is called **rationalizing the denominator.**

EXAMPLE 1

$$\frac{\sqrt{5}}{\sqrt{3}} = \frac{\sqrt{5} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}}$$
$$= \frac{\sqrt{15}}{3} \text{ or } \frac{1}{3}\sqrt{15}$$

Either form is correct.

EXAMPLE 2

$$\sqrt{\frac{3}{7}} = \frac{\sqrt{3}}{\sqrt{7}}$$
$$= \frac{\sqrt{3} \cdot \sqrt{7}}{\sqrt{7} \cdot \sqrt{7}}$$
$$= \frac{\sqrt{21}}{7} \text{ or } \frac{1}{7}\sqrt{21}$$

EXAMPLE 3

$$\sqrt{\frac{1}{2}} \cdot \sqrt{3} = \sqrt{\frac{3}{2}}$$
$$= \frac{\sqrt{3}}{\sqrt{2}}$$
$$= \frac{\sqrt{3} \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}}$$
$$= \frac{\sqrt{6}}{2} \text{ or } \frac{1}{2}\sqrt{6}$$

Keep in mind these suggestions for simplifying radical expressions.

a. Leave no factor under the radical sign if the number is a perfect square.

b. Leave no radical expression in the denominator.

c. Leave no fraction under the radical sign.

Written Exercises

Express in simplest form.

A

1. $\sqrt{\frac{1}{2}}$ $\frac{\sqrt{2}}{2}$
2. $\sqrt{\frac{3}{2}}$ $\frac{\sqrt{6}}{2}$
3. $\sqrt{\frac{3}{4}}$ $\frac{\sqrt{3}}{2}$
4. $\sqrt{\frac{4}{5}}$ $\frac{2\sqrt{5}}{5}$
5. $\sqrt{\frac{1}{3}}$ $\frac{\sqrt{3}}{3}$
6. $\sqrt{\frac{3}{5}}$ $\frac{\sqrt{15}}{5}$
7. $\sqrt{\frac{5}{8}}$ $\frac{\sqrt{10}}{4}$
8. $\sqrt{\frac{6}{7}}$ $\frac{\sqrt{42}}{7}$
9. $\sqrt{\frac{7}{10}}$ $\frac{\sqrt{70}}{10}$
10. $\sqrt{\frac{5}{6}}$ $\frac{\sqrt{30}}{6}$
11. $\sqrt{\frac{4}{7}}$ $\frac{2\sqrt{7}}{7}$
12. $\sqrt{\frac{5}{12}}$ $\frac{\sqrt{15}}{6}$
13. $\frac{3x}{\sqrt{5}}$ $\frac{3x\sqrt{5}}{5}$
14. $\frac{a^2}{\sqrt{a}}$ $a\sqrt{a}$
15. $\sqrt{\frac{a^2}{2}}$ $\frac{a\sqrt{2}}{2}$
16. $\sqrt{\frac{x^2}{3}}$ $\frac{x\sqrt{3}}{3}$
17. $\frac{-x^2}{\sqrt{y}}$ $\frac{-x^2\sqrt{y}}{y}$
18. $\frac{3\sqrt{18}}{\sqrt{2}}$ 9
19. $\frac{2\sqrt{8}}{\sqrt{2}}$ 4
20. $3\sqrt{\frac{n^2}{5}}$ $\frac{3n\sqrt{5}}{5}$

Find the decimal value correct to two decimal places.

B

21. $5\sqrt{\frac{1}{2}}$ 3.50
22. $4\sqrt{\frac{1}{3}}$ 2.31
23. $6\sqrt{\frac{2}{5}}$ 3.79
24. $2\sqrt{\frac{3}{8}}$ 1.22
25. $\sqrt{\frac{1}{3}} \cdot \sqrt{\frac{1}{3}}$ 0.33
26. $\sqrt{\frac{1}{3}} \cdot \sqrt{\frac{1}{2}}$ 0.41
27. $\sqrt{\frac{3}{4}} \cdot \sqrt{\frac{1}{2}}$ 0.61
28. $\sqrt{\frac{5}{2}} \cdot \sqrt{\frac{2}{5}}$ 1.00

For more practice, see page 433.

Some radicals can be combined. Others cannot. You know that you can combine terms in an expression like $2a + 3a$.

like terms

$$2a + 3a = 5a$$

You can combine terms with radicals, but *only* if you have like terms.

like terms

$$2\sqrt{6} + 3\sqrt{6} = 5\sqrt{6}$$

like terms

$$2\sqrt{7} - 3\sqrt{7} = -\sqrt{7}$$

You cannot combine terms in the expressions below. All three expressions are in their simplest form.

unlike terms

$$\sqrt{2} + \sqrt{3}$$

unlike terms

$$\sqrt{a} + \sqrt{b}$$

unlike terms

$$2\sqrt{x} - 3\sqrt{y}$$

EXAMPLE 1 $3\sqrt{2} + 10\sqrt{2} = 13\sqrt{2}$

EXAMPLE 2 $7\sqrt{3} - \sqrt{3} = 6\sqrt{3}$

Be on the lookout for radicals which you could change in order to make like terms.

EXAMPLE 3
$$\begin{aligned}\sqrt{27} + \sqrt{3} &= \sqrt{9 \cdot 3} + \sqrt{3} \\ &= 3\sqrt{3} + \sqrt{3} \\ &= 4\sqrt{3}\end{aligned}$$

EXAMPLE 4
$$\begin{aligned}2\sqrt{\frac{1}{2}} + \sqrt{2} &= 2\sqrt{\frac{2}{4}} + \sqrt{2} \\ &= 2 \cdot \frac{1}{2}\sqrt{2} + \sqrt{2} \\ &= \sqrt{2} + \sqrt{2} \\ &= 2\sqrt{2}\end{aligned}$$

Written Exercises

Express in simplest form.

A

1. $2\sqrt{2} + 6\sqrt{2}$ $8\sqrt{2}$
2. $6\sqrt{2} - 4\sqrt{2}$ $2\sqrt{2}$
3. $8\sqrt{5} + 5\sqrt{5}$ $13\sqrt{5}$
4. $-\sqrt{3} + 2\sqrt{3}$ $\sqrt{3}$
5. $-\sqrt{5} + 5\sqrt{5}$ $4\sqrt{5}$
6. $10\sqrt{7} - \sqrt{7}$ $9\sqrt{7}$
7. $6\sqrt{3} - 5\sqrt{3}$ $\sqrt{3}$
8. $\sqrt{2} - \sqrt{2}$ 0
9. $\sqrt{18} + \sqrt{2}$ $4\sqrt{2}$
10. $4\sqrt{12} - 2\sqrt{3}$ $6\sqrt{3}$
11. $2\sqrt{8} - \sqrt{2}$ $3\sqrt{2}$
12. $2\sqrt{3} + 3\sqrt{12}$ $8\sqrt{3}$
13. $6\sqrt{3} - 4\sqrt{27}$ $-6\sqrt{3}$
14. $\sqrt{2} + \sqrt{3} + 2\sqrt{2}$ $3\sqrt{2} + \sqrt{3}$
15. $\sqrt{5} - \sqrt{3} + 3\sqrt{5}$ $4\sqrt{5} - \sqrt{3}$
16. $\sqrt{6} + \sqrt{3} + 2\sqrt{6}$ $3\sqrt{6} + \sqrt{3}$
17. $4\sqrt{5} - 2\sqrt{2} + 6\sqrt{5}$ $10\sqrt{5} - 2\sqrt{2}$
18. $\sqrt{2} - 6\sqrt{5} + 5\sqrt{2}$ $6\sqrt{2} - 6\sqrt{5}$
19. $\sqrt{12} - 4\sqrt{3} + \sqrt{6}$ $-2\sqrt{3} + \sqrt{6}$
20. $\sqrt{24} + 5\sqrt{6} - \sqrt{2}$ $7\sqrt{6} - \sqrt{2}$
21. $\sqrt{45} + 2\sqrt{5} - 4\sqrt{7}$ $5\sqrt{5} - 4\sqrt{7}$

B

22. $\sqrt{2a} + \sqrt{8a}$ $3\sqrt{2a}$
23. $3\sqrt{b} + 3\sqrt{b^3}$ $(3 + 3b)\sqrt{b}$
24. $\sqrt{49a^2} + 6\sqrt{b} - \sqrt{4b^3}$ $7a + (6 - 2b)\sqrt{b}$
25. $\sqrt{27a} - \sqrt{3a} - \sqrt{12a}$ 0
26. $\sqrt{5} - (2\sqrt{5} - \sqrt{125})$ $4\sqrt{5}$
27. $\sqrt{6} - (\sqrt{24} + 6\sqrt{6})$ $-7\sqrt{6}$
28. $\sqrt{8} - 2\sqrt{\frac{1}{8}} + 6\sqrt{24}$ $\frac{3\sqrt{2}}{2} + 12\sqrt{6}$
29. $\sqrt{27} - \sqrt{\frac{2}{3}}$ $3\sqrt{3} - \frac{\sqrt{6}}{3}$
30. $\sqrt{\frac{1}{4}} - \sqrt{2} + 2\sqrt{8}$ $\frac{1}{2} + 3\sqrt{2}$
31. $\sqrt{5} + 2\sqrt{\frac{1}{5}} + \sqrt{75}$ $\frac{7\sqrt{5}}{5} + 5\sqrt{3}$

For more practice, see page 433.

SELF-TEST

Find the square root.

1. $\sqrt{\frac{1}{4}}$ $\frac{1}{2}$
2. $\sqrt{\frac{5}{16}}$ $\frac{\sqrt{5}}{4}$
3. $\sqrt{\frac{8}{25}}$ $\frac{2\sqrt{2}}{5}$
4. $\sqrt{\frac{20}{49}}$ $\frac{2\sqrt{5}}{7}$

Simplify.

5. $\sqrt{2} \cdot 4\sqrt{5}$ $4\sqrt{10}$
6. $\frac{\sqrt{21}}{\sqrt{3}}$ $\sqrt{7}$
7. $3\sqrt{6} \cdot 4\sqrt{3}$ $36\sqrt{2}$
8. $\sqrt{3} + 9\sqrt{3}$ $10\sqrt{3}$
9. $\sqrt{18} + \sqrt{2}$ $4\sqrt{2}$
10. $10\sqrt{3} - 4\sqrt{27}$ $-2\sqrt{3}$

11 • Radicals and Binomials (Optional)

You will recall that $(a + b)(a - b) = a^2 - b^2$. You can use the same idea to find a product of binomials with radicals.

EXAMPLE 1 $(2 + \sqrt{3})(2 - \sqrt{3}) = 2^2 - (\sqrt{3})^2$
$= 4 - 3$
$= 1$

← This follows the form $(a + b)(a - b) = a^2 - b^2$.

This idea is often used to rationalize a denominator.

EXAMPLE 2
$$\frac{14}{3 - \sqrt{2}} = \frac{14(3 + \sqrt{2})}{(3 - \sqrt{2})(3 + \sqrt{2})}$$
$$= \frac{14(3 + \sqrt{2})}{3^2 - (\sqrt{2})^2}$$
$$= \frac{14(3 + \sqrt{2})}{7}$$
$$= 2(3 + \sqrt{2})$$
$$= 6 + 2\sqrt{2}$$

EXAMPLE 3
$$\frac{3}{4 + \sqrt{5}} = \frac{3(4 - \sqrt{5})}{(4 + \sqrt{5})(4 - \sqrt{5})}$$
$$= \frac{3(4 - \sqrt{5})}{4^2 - (\sqrt{5})^2}$$
$$= \frac{3(4 - \sqrt{5})}{16 - 5}$$
$$= \frac{3(4 - \sqrt{5})}{11}$$
$$= \frac{12 - 3\sqrt{5}}{11}$$

Examples 4 and 5 show you how to square a binomial containing a radical.

EXAMPLE 4 $(5 + \sqrt{3})^2 = (5 + \sqrt{3})(5 + \sqrt{3})$
$= 5^2 + 5\sqrt{3} + 5\sqrt{3} + (\sqrt{3})^2$
$= 25 + 10\sqrt{3} + 3$
$= 28 + 10\sqrt{3}$

← This is one way you can square a binomial. Just write the two factors and use the FOIL method.

EXAMPLE 5 $(3 - \sqrt{6})^2 = 3^2 - 2 \cdot 3\sqrt{6} + (\sqrt{6})^2$
$= 9 - 6\sqrt{6} + 6$
$= 15 - 6\sqrt{6}$

Another way to square a binomial is to use the rule
$(a - b)^2 = a^2 - 2ab + b^2$

Written Exercises

Express in simplest form.

A **1.** $(1 - \sqrt{2})(1 + \sqrt{2})$ -1 **2.** $(5 - \sqrt{5})(5 + \sqrt{5})$ 20 **3.** $(\sqrt{3} - 4)(\sqrt{3} + 4)$ -13

4. $(2 - \sqrt{7})(2 + \sqrt{7})$ -3 **5.** $(1 - \sqrt{6})(1 + \sqrt{6})$ -5 **6.** $(7 + \sqrt{5})(7 - \sqrt{5})$ 44

7. $(\sqrt{6} - 2)(\sqrt{6} + 2)$ 2 **8.** $(\sqrt{8} + 4)(\sqrt{8} - 4)$ -8 **9.** $(2\sqrt{2} + 1)(2\sqrt{2} - 1)$ 7

10. $(4\sqrt{5} + 2)(4\sqrt{5} - 2)$ 76 **11.** $(3\sqrt{5} - 1)(3\sqrt{5} + 1)$ 44 **12.** $(3\sqrt{3} - 2)(3\sqrt{3} + 2)$ 23

Rationalize the denominator.

13. $\dfrac{1}{\sqrt{3} + 1}$ $\dfrac{\sqrt{3} - 1}{2}$ **14.** $\dfrac{5}{2 + \sqrt{3}}$ $10 - 5\sqrt{3}$ **15.** $\dfrac{2}{\sqrt{2} + 3}$ $\dfrac{2\sqrt{2} - 6}{-7}$ **16.** $\dfrac{\sqrt{5}}{2 + \sqrt{6}}$ $\dfrac{2\sqrt{5} - \sqrt{30}}{-2}$

17. $\dfrac{\sqrt{6}}{1 - \sqrt{6}}$ $\dfrac{\sqrt{6} + 6}{-5}$ **18.** $\dfrac{\sqrt{8}}{4 - \sqrt{2}}$ $\dfrac{4\sqrt{2} + 2}{7}$ **19.** $\dfrac{\sqrt{10}}{3 + \sqrt{6}}$ $\dfrac{3\sqrt{10} - 2\sqrt{15}}{3}$ **20.** $\dfrac{\sqrt{7}}{\sqrt{3} + 5}$ $\dfrac{\sqrt{21} - 5\sqrt{7}}{-22}$

21. $\dfrac{2\sqrt{7}}{\sqrt{7} - 1}$ $\dfrac{7 + \sqrt{7}}{3}$ **22.** $\dfrac{4\sqrt{3}}{5 - \sqrt{2}}$ $\dfrac{20\sqrt{3} + 4\sqrt{6}}{23}$ **23.** $\dfrac{6\sqrt{2}}{2 - \sqrt{5}}$ $-12\sqrt{2} - 6\sqrt{10}$ **24.** $\dfrac{4\sqrt{3}}{6 - \sqrt{6}}$ $\dfrac{4\sqrt{3} + 2\sqrt{2}}{5}$

25. $\dfrac{\sqrt{3} - 1}{\sqrt{3} + 1}$ $2 - \sqrt{3}$ **26.** $\dfrac{1 - \sqrt{3}}{\sqrt{3} + 1}$ $\sqrt{3} - 2$ **27.** $\dfrac{2 - \sqrt{7}}{\sqrt{7} - 4}$ $\dfrac{1 - 2\sqrt{7}}{-9}$ **28.** $\dfrac{2 - 3\sqrt{3}}{3\sqrt{2} + 2}$ $\dfrac{6\sqrt{2} - 4 - 9\sqrt{6} + 6\sqrt{3}}{14}$

Simplify.

29. $(2 - \sqrt{3})^2$ $7 - 4\sqrt{3}$ **30.** $(5 + \sqrt{2})^2$ $27 + 10\sqrt{2}$ **31.** $(1 - \sqrt{5})^2$ $6 - 2\sqrt{5}$ **32.** $(3 + \sqrt{8})^2$ $17 + 6\sqrt{8}$

33. $(7 + \sqrt{5})^2$ $54 + 14\sqrt{5}$ **34.** $(6 - \sqrt{6})^2$ $42 - 12\sqrt{6}$ **35.** $(4 - \sqrt{10})^2$ $26 - 8\sqrt{10}$ **36.** $(2\sqrt{2} + 5)^2$ $33 + 20\sqrt{2}$

37. $(3\sqrt{5} - 2\sqrt{3})^2$ $57 - 12\sqrt{15}$ **38.** $(4\sqrt{8} - 2\sqrt{7})^2$ $156 - 16\sqrt{56}$ **39.** $(2\sqrt{5} - 3\sqrt{8})^2$ $92 - 24\sqrt{10}$ **40.** $(4\sqrt{2} - 3\sqrt{3})^2$ $59 - 24\sqrt{6}$

41. What is the area of a square whose sides measure $7 + 2\sqrt{3}$ on each side? $61 + 28\sqrt{3}$

Reviewing Arithmetic Skills

Find the increase or decrease in price.

1. from \$2.98 to \$3.50 \$.52
2. from \$6.50 to \$9.00 \$2.50
3. from \$4.25 to \$3.50 \$.75
4. from \$25 to \$16.80 \$8.20
5. from \$10.35 to \$12.59 \$2.24
6. from \$230 to \$255 \$25

Find the percent of increase in price.

Sample from \$3.00 to \$4.50 ⟶ *increase:* \$1.50

What percent of *the original price* is *the increase?*

$$\begin{aligned} x \times \$3.00 &= \$1.50 \\ 3.00x &= 1.50 \\ 100 \times 3.00x &= 100 \times 1.50 \\ 300x &= 150 \\ x &= 0.5 \end{aligned}$$

Answer: $0.5 = 0.50 = 50\%$

7. from \$2.00 to \$3.00 50%
8. from \$5.00 to \$8.00 60%
9. from \$4.00 to \$6.00 50%
10. from \$2.00 to \$3.40 70%
11. from \$8.00 to \$9.60 20%
12. from \$6.00 to \$7.80 30%
13. from \$20 to \$25 25%
14. from \$40 to \$50 25%
15. from \$60 to \$84 40%
16. from \$100 to \$150 50%

Find the percent of decrease in price.
Use the guide: ***What percent of the original price is the decrease?***

17. from \$20 to \$10 50%
18. from \$18 to \$9 50%
19. from \$30 to \$24 20%
20. from \$50 to \$30 40%
21. from \$80 to \$56 30%
22. from \$24 to \$18 25%
23. from \$200 to \$150 25%
24. from \$125 to \$100 20%

CAREER NOTEBOOK

Manufacturing

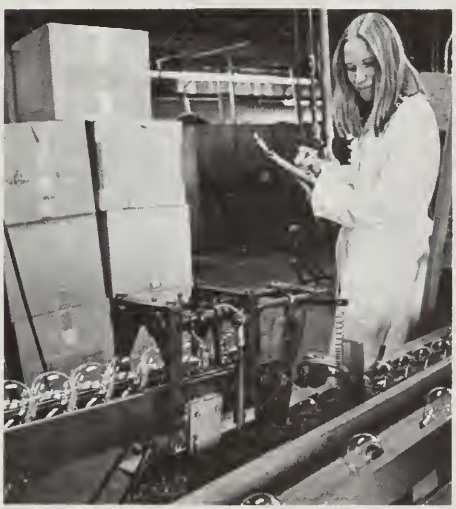
Products of the manufacturing industry range from a child's toy to a large jetliner. Manufacturing involves many diverse processes, therefore offering a vast variety of career opportunities.



Traffic managers must determine the quickest and most economical means of shipping goods to and from the company.

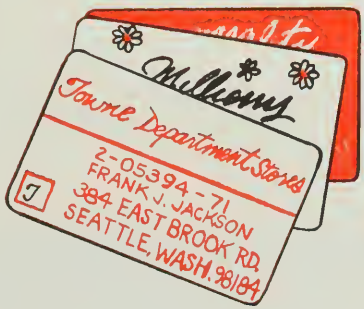


Inventory controllers record data for ordering, receiving, storing, and shipping goods.



Efficiency experts study procedures to help make better use of people, time, and equipment.

CONSUMER CORNER



Credit Cards

For many people, credit cards are a very convenient way to purchase items and pay for services. Instead of carrying cash, people can have a purchase charged to their accounts. Let's consider a few credit card statements (or bills) to understand how charge accounts work.

Take a look at this typical charge account statement for a department store.

Towne Department Store		Customer's Statement	Revolving Account		Billing Date: June 7 Account No.: 2-05394-71
Date	Reference No.	Dept. No.	Transaction Description	Purchases, Other Charges	Payments, Other Credits
5/9	0113572	5	COSMETICS	4.50	
5/10			PAYMENT, THANK YOU		47.54
5/17	274967	7	ACCESSORIES	12.95	
5/20	634592	11	NOTIONS	5.65	
5/23	647920	12	MENS CLOTHING	24.00	
5/30	762954	10	WOMENS SPORTSWEAR	34.95	
5/31	359270	11	RETURN, NOTIONS		5.65

Previous Balance	Purchases, Other Charges	Payments, Other Credits	FINANCE CHARGE	New Balance	MINIMUM PAYMENT
47.54	82.05	53.19	NONE	76.40	10.00

Finance charge is computed at a monthly rate of 1 1/2%.
Equivalent annual percentage rate is 18%. Minimum is $.50.

1. This is a statement for a revolving charge account. (This means that the new balance of $76.40 does not have to be paid at this time.) How much must be paid now? $10.00

2. If the minimum payment is made at this time, how much money will be owed to the store? $66.40

3. The finance charge for not paying the balance all at this time is 1 1/2% per month. What is the yearly rate? 18%

4. If the minimum payment is made at this time, how much finance charge will be owed next month? $1.00

5. Suppose $25 is paid at this time. How much finance charge will be owed next month? $.77

Here's the statement on the same account for the next month. Complete the missing information.

Towne Department Store	Customer's Statement	Revolving Account	Billing Date: July 7 Account No.: 2-05394-71

Date	Reference	Dept. No.	Transaction Description	Purchases, Other Charges	Payments, Other Credits
6/7	6354003	7	ACCESSORIES	8.95	
6/7	3079240	12	MENS CLOTHING	18.35	
6/11	3871235		PAYMENT, THANK YOU		20.00
6/15	2400740	9	SMALL APPLIANCES	24.95	
6/21	3592560	15	FURNITURE	103.75	
6/21	7113509	5	COSMETICS	6.99	
6/21	5619205	18	LUGGAGE	40.00	
6/28	63711242	21	AUTOMOTIVE	45.00	

Previous Balance	Purchases, Other Charges	Payments, Other Credits	FINANCE CHARGE	New Balance	MINIMUM PAYMENT
? $76.40	? $247.99	? $20.00	? $.85	? $305.24	10.00

Finance charge is computed at a monthly rate of 1 1/2%.
Equivalent annual percentage rate is 18%. Minimum is $.50.

1. How much of a payment did the customer make last time? $20.00

2. Complete to find the missing information.

Reviewing the Chapter

Find the value. *(See pp. 348–349.)*

1. $\sqrt{25}$ 5
2. $-\sqrt{36}$ −6
3. $\sqrt{81}$ 9
4. $-\sqrt{100}$ −10
5. $\sqrt{9}$ 3
6. $\sqrt{8^2}$ 8
7. $-\sqrt{49}$ −7
8. $\sqrt{64}$ 8
9. $\sqrt{12^2}$ 12
10. $-\sqrt{15^2}$ −15

Use the table on page 350 to find the value. Round to one decimal place. *(See pp. 350–351.)*

11. $\sqrt{19}$ 4.4
12. $\sqrt{29}$ 5.4
13. $\sqrt{60}$ 7.7
14. $\sqrt{42}$ 6.5
15. $\sqrt{7}$ 2.6
16. $-\sqrt{5}$ −2.2
17. $\sqrt{21}$ 4.6
18. $-\sqrt{11}$ −3.3
19. $\sqrt{85}$ 9.2
20. $\sqrt{72}$ 8.5

Is the number rational or irrational? *(See pp. 352–353.)*

21. 6 r
22. $\frac{1}{6}$ r
23. $\sqrt{6}$ i
24. 1.6 r
25. $\sqrt{81}$ r
26. $\sqrt{9}$ r
27. $-\sqrt{81}$ r
28. $4\frac{1}{4}$ r
29. $6\frac{1}{3}$ r
30. π i

Find the value in simplest radical form. *(See pp. 354–355.)*

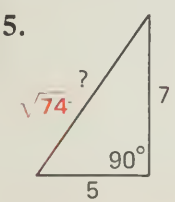
31. $\sqrt{18}$ $3\sqrt{2}$
32. $\sqrt{32}$ $4\sqrt{2}$
33. $\sqrt{28}$ $2\sqrt{7}$
34. $\sqrt{150}$ $5\sqrt{6}$
35. $\sqrt{45}$ $3\sqrt{5}$
36. $\sqrt{175}$ $5\sqrt{7}$
37. $\sqrt{140}$ $2\sqrt{35}$
38. $\sqrt{200}$ $10\sqrt{2}$

Find the solutions to the equation. *(See pp. 356–357.)*

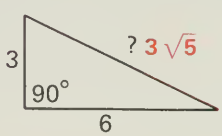
39. $n^2 = 9$ ±3
40. $x^2 = 81$ ±9
41. $y^2 = 36$ ±6
42. $\frac{n}{3} = \frac{3}{n}$ ±3
43. $y^2 = 48$ $\pm 4\sqrt{3}$
44. $a^2 - 6 = 43$ ±7

Find the length of the third side of the triangle. *(See pp. 358–361.)*

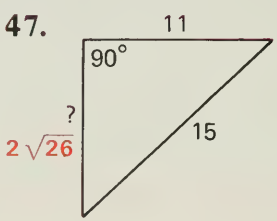
45.



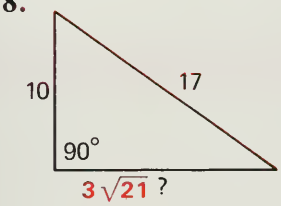
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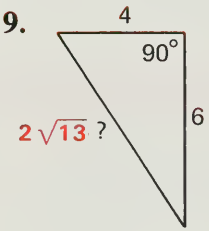
47.



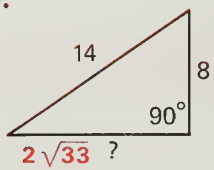
48.



49.



50.



Find the square root. Results should be in simplest form. *(See pp. 362-363.)*

51. $\sqrt{\frac{4}{144}}$ $\frac{1}{6}$

52. $\sqrt{\frac{121}{64}}$ $\frac{11}{18}$

53. $\sqrt{\frac{3}{81}}$ $\frac{\sqrt{3}}{9}$

54. $\sqrt{\frac{13}{36}}$ $\frac{\sqrt{13}}{6}$

55. $\sqrt{\frac{25}{100}}$ $\frac{1}{2}$

56. $\sqrt{\frac{4n^2}{49}}$ $\frac{2n}{7}$

57. $\sqrt{\frac{14x^2}{64}}$ $\frac{x\sqrt{14}}{8}$

58. $\sqrt{\frac{x^2y^2}{9}}$ $\frac{xy}{3}$

Simplify. Leave no perfect-square factor under the radical sign. *(See pp. 364-367.)*

59. $\sqrt{10} \cdot \sqrt{10}$ 10

60. $3\sqrt{3} \cdot 2\sqrt{3}$ 18

61. $2\sqrt{5} \cdot \sqrt{25}$ $10\sqrt{5}$

62. $3\sqrt{n} \cdot 6\sqrt{n}$ $18n$

63. $\frac{\sqrt{15}}{\sqrt{3}}$ $\sqrt{5}$

64. $\frac{\sqrt{21}}{\sqrt{7}}$ $\sqrt{3}$

65. $\frac{\sqrt{2r^2}}{\sqrt{r}}$ $\sqrt{2r}$

66. $\frac{\sqrt{24x}}{\sqrt{6x}}$ 2

67. $\sqrt{\frac{1}{5}}$ $\frac{\sqrt{5}}{5}$

68. $\sqrt{\frac{7}{8}}$ $\frac{\sqrt{14}}{4}$

69. $\sqrt{\frac{a^2}{5}}$ $\frac{a\sqrt{5}}{5}$

70. $-\sqrt{\frac{n^2}{6n}}$ $-\frac{\sqrt{6n}}{6}$

71. $\sqrt{\frac{2x}{5}}$ $\frac{\sqrt{10x}}{5}$

72. $4\sqrt{\frac{x^2}{4}}$ $2x$

73. $4\sqrt{\frac{2}{\pi}}$ $\frac{4\sqrt{2\pi}}{\pi}$

74. $\frac{2n}{\sqrt{8}}$ $\frac{n\sqrt{2}}{2}$

Express in simplest form. *(See pp. 368-369.)*

75. $3\sqrt{3} - \sqrt{3}$ $2\sqrt{3}$

76. $\sqrt{7} + 10\sqrt{7}$ $11\sqrt{7}$

77. $2\sqrt{5} - 4\sqrt{5}$ $-2\sqrt{5}$

78. $2\sqrt{27} - \sqrt{3} + 6\sqrt{3}$ $11\sqrt{3}$

79. $\sqrt{2} + \sqrt{8} - \sqrt{12}$ $3\sqrt{2} - 2\sqrt{3}$

80. $2\sqrt{45} - \sqrt{5} + \sqrt{75}$ $5\sqrt{5} + 5\sqrt{3}$

81. $5\sqrt{3} + \sqrt{48} - \sqrt{4}$ $9\sqrt{3} - 2$

82. $3\sqrt{24} + \sqrt{8} - 6\sqrt{20}$ $6\sqrt{6} + 2\sqrt{2} - 12\sqrt{5}$

83. $\sqrt{7} - 3\sqrt{21} + \sqrt{49}$ $\sqrt{7} - 3\sqrt{21} + 7$

84. $\sqrt{35} - 3\sqrt{3} + 4\sqrt{7}$ $\sqrt{35} - 3\sqrt{3} + 4\sqrt{7}$

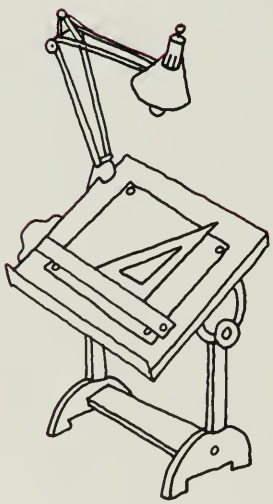
85. $\sqrt{2} - 9\sqrt{18} + 3\sqrt{3}$ $-26\sqrt{2} + 3\sqrt{3}$

86. $2\sqrt{11} + 4\sqrt{22} + \sqrt{2}$ $2\sqrt{11} + 4\sqrt{22} + \sqrt{2}$

A permission-to-reproduce chapter test can be found on page T21.

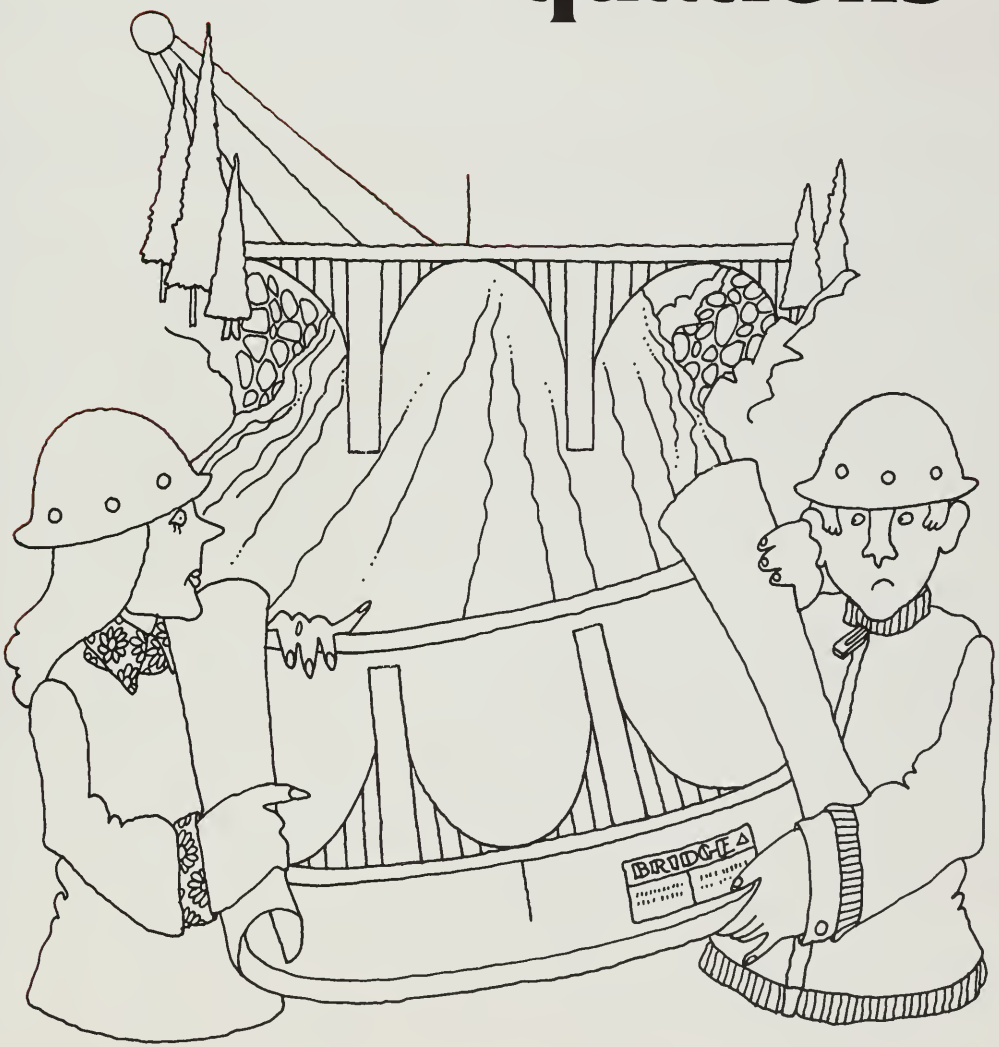
Here's what you'll learn in this chapter:

1. To solve quadratic equations by factoring.
2. To solve quadratic equations by taking square roots.
3. To use the quadratic formula to solve quadratic equations.
4. To use quadratic equations to solve word problems.
5. To draw the graphs of quadratic functions.



Chapter 12

Quadratic Equations



1 • Zero Products

Can you solve the puzzle below?



I'm thinking of two numbers.
Their product is zero.
Tell me one of the numbers.



If two numbers have a zero product, one of the numbers must be zero.

The rule above is used to solve some equations.

The binomials $(x - 2)$ and $(x + 3)$ have a zero product.

$$(x - 2)(x + 3) = 0$$

Therefore, one of the numbers must be zero.

$$x - 2 = 0 \quad or \quad x + 3 = 0$$
$$x = 2 \qquad\qquad x = -3$$

Answer: $x = 2$ *or* $x = -3$

Study these examples.

EXAMPLE 1

$$x(x + 1) = 0$$
$$x = 0 \quad or \quad x + 1 = 0$$
$$x = -1$$

Answer: $x = 0$ *or* $x = -1$

EXAMPLE 2

$$(n - 5)(2n - 6) = 0$$
$$n - 5 = 0 \quad or \quad 2n - 6 = 0$$
$$n = 5 \qquad\qquad 2n = 6$$
$$n = 3$$

Answer: $n = 5$ *or* $n = 3$

Classroom Practice

Complete.

1. If $x(x + 3) = 0$, then either $x =$ $\underset{?}{0}$ or $x + 3 =$ $\underset{?}{0}$.
2. If $x(x - 2) = 0$, then either $x =$ $\underset{?}{0}$ or $x - 2 =$ $\underset{?}{0}$.
3. If $(x + 3)(x - 4) = 0$, either $x + 3 =$ $\underset{?}{0}$ or $\underset{?}{x - 4} = 0$.

Solve.

4. $x(x - 3) = 0$ 0 or 3
5. $a(a + 5) = 0$ 0 or −5
6. $3n(n - 2) = 0$ 0 or 2
7. $(x - 1)(x - 3) = 0$ 1 or 3
8. $(x - 5)(x + 2) = 0$ 5 or −2
9. $(2x - 8)(x + 1) = 0$ 4 or −1

Written Exercises

Solve.

A
1. $n(n + 3) = 0$ 0 or −3
2. $n(n - 3) = 0$ 0 or 3
3. $x(x + 9) = 0$ 0 or −9
4. $y(y + 6) = 0$ 0 or −6
5. $5m(m - 1) = 0$ 0 or 1
6. $2m(m - 4) = 0$ 0 or 4
7. $2n(n - 9) = 0$ 0 or 9
8. $4x(x + 8) = 0$ 0 or −8
9. $(y - 3)(y - 1) = 0$ 3 or 1
10. $(a + 8)(a + 1) = 0$ −8 or −1
11. $(x + 4)(x + 3) = 0$ −4 or −3
12. $(b + 1)(b + 6) = 0$ −1 or −6
13. $(y + 3)(y + 8) = 0$ −3 or −8
14. $(x - 10)(x - 20) = 0$ 10 or 20
15. $(n - 14)(n - 2) = 0$ 14 or 2
16. $(y - 10)(y - 3) = 0$ 10 or 3
17. $(x + 5)(x + 6) = 0$ −5 or −6
18. $(n + 20)(n + 40) = 0$ −20 or −40
19. $(a - 8)(a + 8) = 0$ 8 or −8
20. $(x - 25)(x + 25) = 0$ 25 or −25
21. $(y + 12)(y - 12) = 0$ −12 or 12
22. $(n - 6)(n + 4) = 0$ 6 or −4
23. $(x + 100)(x - 100) = 0$ −100 or 100
24. $(x - 4)(x - 75) = 0$ 4 or 75

B
25. $(y - 12)(4y + 20) = 0$ 12 or −5
26. $(m - 8)(3m - 12) = 0$ 8 or 4
27. $(6r - 18)(7r + 49) = 0$ 3 or −7
28. $(4n - 12)(6n + 72) = 0$ 3 or −12
29. $(12x - 72)(5x + 20) = 0$ 6 or −4
30. $x(x + 1)(x + 5) = 0$ 0, −1, or −5

2 • Solving Quadratic Equations

A **quadratic equation** in x is an equation which has an x^2 term but no term with a higher power of x. Here are three examples of quadratic equations.

$$2x^2 + 6x = 0 \qquad x^2 - 5x + 6 = 0 \qquad x^2 = 5$$

Many quadratic equations can be solved by factoring.

EXAMPLE 1

$2x^2 + 6x = 0$

Factor.

$2x(x + 3) = 0$

$2x = 0$ *or* $x + 3 = 0$

$x = 0$ $\qquad$ $x = -3$

Answer: $x = 0$ or $x = -3$

To solve a quadratic equation like $x^2 - 5x + 6 = 0$, you'll have to factor the trinomial $x^2 - 5x + 6$. Recall how to factor.

Factor $x^2 - 5x + 6$.

Step 1: $(\quad)(\quad)$

Step 2: $(x \quad)(x \quad)$

Step 3: $(x \ ? \)(x \ ? \)$

What two numbers have a product of 6 and a sum of -5?

-2 and -3

Step 4: $(x - 2)(x - 3)$

Now see how the equation $x^2 - 5x + 6 = 0$ is solved.

EXAMPLE 2

$x^2 - 5x + 6 = 0$

Factor.

$(x - 2)(x - 3) = 0$

$x - 2 = 0$ *or* $x - 3 = 0$

$x = 2$ $\qquad$ $x = 3$

Answer: $x = 2$ or $x = 3$

EXAMPLE 3

$x^2 - x - 12 = 0$

Factor.

$(x - 4)(x + 3) = 0$

$x - 4 = 0$ *or* $x + 3 = 0$

$x = 4$ $\qquad$ $x = -3$

Answer: $x = 4$ or $x = -3$

Classroom Practice

1. Factor: $x^2 - 3x$ $x(x - 3)$
Solve: $x^2 - 3x = 0$ 0 or 3

2. Factor: $x^2 + 2x$ $x(x + 2)$
Solve: $x^2 + 2x = 0$ 0 or -2

3. Factor: $5x^2 - 10x$ $5x(x - 2)$
Solve: $5x^2 - 10x = 0$ 0 or 2

4. Factor: $x^2 - 6x + 8$ $(x - 2)(x - 4)$
Solve: $x^2 - 6x + 8 = 0$ 2 or 4

5. Factor: $x^2 - x - 20$ $(x - 5)(x + 4)$
Solve: $x^2 - x - 20 = 0$ 5 or -4

6. Factor: $x^2 - 8x + 12$ $(x - 6)(x - 2)$
Solve: $x^2 - 8x + 12 = 0$ 6 or 2

7. Factor: $x^2 - 3x - 10$ $(x - 5)(x + 2)$
Solve: $x^2 - 3x - 10 = 0$ 5 or -2

Written Exercises

Factor.

A **1.** $x^2 - 2x$ $x(x - 2)$

2. $y^2 - 3y$ $y(y - 3)$

3. $2x^2 - 4x$ $2x(x - 2)$

4. $x^2 + 3x + 2$ $(x + 1)(x + 2)$

5. $x^2 + 2x - 3$ $(x + 3)(x - 1)$

6. $x^2 + x - 2$ $(x + 2)(x - 1)$

7. $x^2 + 6x + 8$ $(x + 2)(x + 4)$

8. $x^2 - 7x + 12$ $(x - 4)(x - 3)$

9. $x^2 - 3x - 18$ $(x - 6)(x + 3)$

Solve.

10. $x^2 + x = 0$ 0 or -1

11. $x^2 - 3x = 0$ 0 or 3

12. $x^2 - 5x = 0$ 0 or 5

13. $6x + 2x^2 = 0$ 0 or -3

14. $9x - 3x^2 = 0$ 0 or 3

15. $16x - 4x^2 = 0$ 0 or 4

16. $x^2 - 5x + 4 = 0$ 4 or 1

17. $x^2 - 9x + 8 = 0$ 8 or 1

18. $x^2 + 4x + 3 = 0$ -3 or -1

19. $x^2 - 8x + 7 = 0$ 7 or 1

20. $x^2 + 8x + 15 = 0$ -5 or -3

21. $x^2 - 10x + 21 = 0$ 3 or 7

22. $y^2 + 4y + 4 = 0$ -2

23. $y^2 + 4y - 21 = 0$ -7 or 3

24. $y^2 - 7y + 6 = 0$ 6 or 1

25. $x^2 - 2x - 8 = 0$ 4 or -2

26. $x^2 + x - 6 = 0$ -3 or 2

27. $n^2 - 5n - 14 = 0$ 7 or -2

28. $a^2 + 4a - 32 = 0$ -8 or 4

29. $a^2 + 10a + 25 = 0$ -5

30. $a^2 - 12a + 35 = 0$ 7 or 5

31. $x^2 - 11x + 28 = 0$ 7 or 4

32. $x^2 - 16 = 0$ -4 or 4

33. $x^2 - 9 = 0$ -3 or 3

34. $x^2 + 12x + 36 = 0$ -6

35. $x^2 - 13x + 22 = 0$ 11 or 2

36. $x^2 - 6x - 27 = 0$ -3 or 9

37. $x^2 - 6x - 55 = 0$ 11 or -5

38. $x^2 - 22x + 40 = 0$ 20 or 2

39. $x^2 - 13x + 40 = 0$ 5 or 8

40. $x^2 - 5x - 36 = 0$ 9 or -4

41. $a^2 + 6a - 40 = 0$ 4 or -10

42. $y^2 + 5y - 24 = 0$ -8 or 3

3 • Solving Quadratic Equations

Sometimes before you can solve a quadratic equation you must first rewrite the equation so that one side is zero.

EXAMPLE 1

$x^2 = 5x$

Rewrite.

$x^2 - 5x = 0$

$x(x - 5) = 0$

$x = 0$ *or* $x - 5 = 0$

$x = 5$

Answer: $x = 0$ or $x = 5$

EXAMPLE 2

$x^2 + 8x = -15$

Rewrite.

$x^2 + 8x + 15 = 0$

$(x + 3)(x + 5) = 0$

$x + 3 = 0$ *or* $x + 5 = 0$

$x = -3$ $\qquad$ $x = -5$

Answer: $x = -3$ or $x = -5$

EXAMPLE 3

$x(x - 1) = 20$

Rewrite.

$x(x - 1) - 20 = 0$

$x^2 - x - 20 = 0$

$(x - 5)(x + 4) = 0$

$x - 5 = 0$ *or* $x + 4 = 0$

$x = 5$ $\qquad$ $x = -4$

Answer: $x = 5$ or $x = -4$

Classroom Practice

Solve.

1. $x^2 = 4x$ 0 or 4
2. $x^2 - 4x = 5$ 5 or −1
3. $x^2 + 8 = 6x$ 4 or 2
4. $6 + x = x^2$ 3 or −2
5. $x(x - 1) = 12$ 4 or −3
6. $x(x + 2) = 15$ −5 or 3
7. $8 = (x + 1)(x - 1)$ −3 or 3
8. $\frac{x}{3} = \frac{x + 6}{x}$ 6 or −3
9. $\frac{x}{5} = \frac{2x - 5}{x}$ 5

Written Exercises

Solve.

A

1. $x^2 = 3x$ 0 or 3
2. $x^2 = -7x$ 0 or −7
3. $x^2 = -4x$ 0 or −4
4. $x^2 + 5x = 6x$ 0 or 1
5. $y^2 + 4y = 32$ −8 or 4
6. $x^2 + 5x = 6$ −6 or 1
7. $y^2 - 2y = 8$ 4 or −2
8. $6 - x = x^2$ −3 or 2
9. $6y - 9 = y^2$ 3
10. $x^2 + 22 = 13x$ 11 or 2
11. $x^2 + 20 = -12x$ −10 or −2
12. $x^2 - 6 = 5x$ 6 or −1
13. $y^2 + y = 42$ −7 or 6
14. $y^2 = 9y - 20$ 5 or 4
15. $x^2 = 7x + 18$ 9 or −2
16. $a^2 - 3a = 18$ 6 or −3
17. $a^2 = 12a - 35$ 7 or 5
18. $x^2 = 5x + 24$ 8 or −3
19. $x(x + 4) = 12$ −6 or 2
20. $y(y + 3) = 40$ −8 or 5
21. $(y - 1)(y + 1) = 48$ 7 or −7
22. $(x + 2)(x - 3) = 6$ 4 or −3
23. $(n - 2)(n + 5) = 18$ −7 or 4
24. $(a - 1)(a + 1) = 15$ −4 or 4

B

25. $\frac{x}{2} = \frac{6}{x + 1}$ −4 or 3
26. $\frac{x}{4} = \frac{7}{x + 3}$ −7 or 4
27. $\frac{x - 1}{3} = \frac{4}{x}$ 4 or −3
28. $\frac{x - 2}{x} = \frac{x}{8}$ 4
29. $\frac{y - 3}{5} = \frac{8}{y + 3}$ −7 or 7
30. $\frac{4}{x + 3} = \frac{x - 3}{4}$ −5 or 5

C

31. $2x^2 + x = 3$ $-1\frac{1}{2}$ or 1
32. $2x^2 - x = 1$ $-\frac{1}{2}$ or 1
33. $x + 10 = 2x^2$ $2\frac{1}{2}$ or −2
34. $(2x + 1)(2x - 1) = 0$ $\frac{1}{2}$ or $-\frac{1}{2}$
35. $(2n + 1)(n + 1) = 28$ $-4\frac{1}{2}$ or 3
36. $\frac{1 + n}{3} = 2n^2$ $-\frac{1}{3}$ or $\frac{1}{2}$

For more practice, see page 434.

SELF-TEST

Solve.

1. $x(x - 3) = 0$ 0 or 3
2. $y(y + 2) = 0$ 0 or −2
3. $2n(n + 7) = 0$ 0 or −7
4. $(y - 6)(y - 3) = 0$ 6 or 3
5. $(x + 4)(x - 3) = 0$ −4 or 3
6. $x^2 - 3x = 0$ 0 or 3
7. $2y^2 - 10y = 0$ 0 or 5
8. $x^2 + 8x + 15 = 0$ −5 or −3
9. $x^2 + 4x - 12 = 0$ −6 or 2
10. $x^2 = -8x$ 0 or −8
11. $y^2 + y = 30$ 5 or −6
12. $x^2 = 11x - 28$ 4 or 7

4 • Using Square Roots

A quadratic equation like $x^2 = 25$ can be solved by factoring or by taking square roots. In Chapter 11 you studied how to solve equations by taking square roots. Let's review that method here.

SOLVING BY FACTORING

$$x^2 = 25$$
$$x^2 - 25 = 0$$
$$(x - 5)(x + 5) = 0$$

$x - 5 = 0$ *or* $x + 5 = 0$

$x = 5$ $\qquad$ $x = -5$

Answer: $x = 5$ or $x = -5$

SOLVING BY TAKING SQUARE ROOTS

$$x^2 = 25$$

Take the square root. $\qquad$ Take the square root.

$$x = \pm 5$$

Read: "x equals $+5$ or -5." This means $x = 5$ or $x = -5$.

Answer: $x = 5$ or $x = -5$

You'll probably agree that the second method is a little easier. Study these examples of that method.

EXAMPLE 1

$$x^2 = 18$$

Take the square root. $\qquad$ Take the square root.

$$x = \pm\sqrt{18}$$
$$x = \pm 3\sqrt{2}$$

Answer: $x = 3\sqrt{2}$ or $x = -3\sqrt{2}$

EXAMPLE 2

$$2x^2 = 18$$
$$x^2 = 9$$

Take the square root. $\qquad$ Take the square root.

$$x = \pm 3$$

Answer: $x = 3$ or $x = -3$

Some quadratic equations have no real-number solutions.

EXAMPLE 3 $x^2 + 9 = 0$
$x^2 = -9$ ←

It's impossible to find a real-number square root for -9. The square of a real number is never negative.

Written Exercises

Solve. If there is no solution, write *no solution*.

A
1. $x^2 = 4$ ±2
2. $x^2 = 9$ ±3
3. $x^2 = 16$ ±4
4. $y^2 = 49$ ±7
5. $x^2 = 64$ ±8
6. $a^2 = 81$ ±9
7. $y^2 = 121$ ±11
8. $x^2 = 100$ ±10
9. $x^2 = 400$ ±20
10. $x^2 = 3$ $\pm\sqrt{3}$
11. $y^2 = 5$ $\pm\sqrt{5}$
12. $n^2 = 7$ $\pm\sqrt{7}$
13. $a^2 = 8$ $\pm 2\sqrt{2}$
14. $y^2 = 32$ $\pm 4\sqrt{2}$
15. $n^2 = 1$ ±1
16. $n^2 = 0$ 0
17. $a^2 = 12$ $\pm 2\sqrt{3}$
18. $2x^2 = 18$ ±3
19. $3x^2 = 12$ ±2
20. $2x^2 = 50$ ±5
21. $3y^2 = -6$ no solution
22. $2n^2 = -200$ no solution
23. $x^2 - 1 = 3$ ±2
24. $x^2 - 5 = 4$ ±3
25. $x^2 + 1 = 50$ ±7
26. $y^2 + 2 = 30$ $\pm 2\sqrt{7}$
27. $y^2 - 4 = 20$ $\pm 2\sqrt{6}$
28. $n^2 - 2 = 5$ $\pm\sqrt{7}$
29. $a^2 + 7 = 4$ no solution
30. $x^2 + 3 = 2$ no solution
31. $n^2 - 5 = 10$ $\pm\sqrt{15}$
32. $y^2 + 3 = 21$ $\pm 3\sqrt{2}$

B
33. $2x^2 - 1 = 17$ ±3
34. $3y^2 + 5 = 50$ $\pm\sqrt{15}$
35. $2n^2 - 7 = 13$ $\pm\sqrt{10}$
36. $3n^2 + 9 = 42$ $\pm\sqrt{11}$
37. $x^2 = \frac{1}{9}$ $\pm\frac{1}{3}$
38. $x^2 = \frac{4}{9}$ $\pm\frac{2}{3}$
39. $\frac{x}{4} = \frac{9}{x}$ ±6
40. $\frac{y}{4} = \frac{16}{y}$ ±8

Galileo discovered the following law for falling bodies.

distance it falls $= 5 \times (\text{time})^2$
$d = 5t^2$



Complete.

41.

time in seconds	1	2	4	?5	?10
distance in meters	?	?	?	125	500
	5	20	80		

5 • Using Square Roots

Equations like $(x + 1)^2 = 25$ can be solved in the same way you solved equations in the last section. All you do is take the square root of both sides of the equation.

EXAMPLE 1

$(x + 1)^2 = 25$

Take the square root. Take the square root.

$x + 1 = \pm 5$

$x + 1 = 5$ *or* $x + 1 = -5$

$x = 4$ $\qquad$ $x = -6$

Answer: $x = 4$ or $x = -6$

EXAMPLE 2

$(2x - 1)^2 = 49$

Take the square root. Take the square root.

$2x - 1 = \pm 7$

$2x - 1 = 7$ *or* $2x - 1 = -7$

$2x = 8$ $\qquad$ $2x = -6$

$x = 4$ $\qquad$ $x = -3$

Answer: $x = 4$ or $x = -3$

Classroom Practice

Complete.

1. $(x + 1)^2 = 9$

$x + 1 = \pm$?3

$x + 1 = 3$ *or* $x + 1 = -3$

$x =$?2 $\qquad$ $x =$?−4

2. $(x - 3)^2 = 16$

$x - 3 = \pm 4$

$x - 3 = 4$ *or* $x - 3 = -4$

$x =$?7 $\qquad$ $x =$?−1

Solve.

3. $(x - 1)^2 = 4$ 3 or −1

4. $(2x + 1)^2 = 25$ 2 or −3

5. $(x - 4)^2 = 36$ 10 or −2

Written Exercises

Solve.

A

1. $(x - 3)^2 = 36$ 9 or −3
2. $(x + 5)^2 = 49$ 2 or −12
3. $(x - 4)^2 = 25$ 9 or −1
4. $(x + 1)^2 = 16$ 3 or −5
5. $(y + 8)^2 = 100$ 2 or −18
6. $(y - 2)^2 = 81$ 11 or −7
7. $(y - 3)^2 = 64$ 11 or −5
8. $(n + 2)^2 = 9$ 1 or −5
9. $(a - 7)^2 = 121$ 18 or −4
10. $(2x - 1)^2 = 25$ 3 or −2
11. $(4n - 2)^2 = 36$ 2 or −1
12. $(2y - 2)^2 = 64$ 5 or −3
13. $(2x - 3)^2 = 49$ 5 or −2
14. $(2x - 1)^2 = 81$ 5 or −4
15. $(2x + 3)^2 = 25$ 1 or −4
16. $(5p + 5)^2 = 100$ 1 or −3
17. $(2p + 3)^2 = 81$ 3 or −6
18. $(3x - 6)^2 = 9$ 3 or 1
19. $(4a - 8)^2 = 0$ 2
20. $(3x - 3)^2 = 1$ $1\frac{1}{3}$ or $\frac{2}{3}$
21. $(6n - 6)^2 = 36$ 2 or 0

B

22. $(x - 1)^2 = 5$ $1 \pm \sqrt{5}$
23. $(x - 2)^2 = 7$ $2 \pm \sqrt{7}$
24. $(x - 4)^2 = 3$ $4 \pm \sqrt{3}$
25. $(y + 1)^2 = 2$ $-1 \pm \sqrt{2}$
26. $(y + 8)^2 = 6$ $-8 \pm \sqrt{6}$
27. $(a - 5)^2 = 8$ $5 \pm 2\sqrt{2}$
28. $3(x - 1)^2 = 12$ 3 or −1
29. $2(x - 7)^2 = 18$ 10 or 4
30. $6(a + 1)^2 = 24$ 1 or −3
31. $5(n + 2)^2 = 45$ 1 or −5
32. $3(a - 1)^2 = 75$ 6 or −4
33. $7(x + 3)^2 = 28$ −1 or −5
34. $3(y - 6)^2 = 27$ 3 or 9
35. $3(2x + 2)^2 = 48$ 1 or −3
36. $2(2x - 1)^2 = 50$ 3 or −2

For more practice, see page 434.

SELF-TEST

Solve.

1. $x^2 = 49$ ± 7
2. $y^2 = 64$ ± 8
3. $n^2 = 17$ $\pm \sqrt{17}$
4. $2x^2 = 14$ $\pm \sqrt{7}$
5. $5x^2 = 30$ $\pm \sqrt{6}$
6. $\frac{x}{4} = \frac{7}{x}$ $\pm 2\sqrt{7}$
7. $\frac{x}{5} = \frac{8}{x}$ $\pm 2\sqrt{10}$
8. $(x + 1)^2 = 36$ −7 or 5
9. $(x - 3)^2 = 64$ 11 or −5
10. $(x + 7)^2 = 25$ −2 or −12
11. $(2x + 6)^2 = 16$ −1 or −5
12. $(2x - 3)^2 = 81$ 6 or −3

6 • The Quadratic Formula

Not all quadratic equations can be solved by factoring or by taking the square roots. The formula below will help you find the solutions to all quadratic equations.



THE QUADRATIC FORMULA

The solution of an equation $ax^2 + bx + c = 0$ is either

$x = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ or $x = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$. Sometimes these are written in combined form: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

EXAMPLE 1 Solve $2x^2 + 3x - 9 = 0$.

general equation → $ax^2 + bx + c = 0$

$2x^2 + 3x - 9 = 0$

So, $a = 2$, $b = 3$, and $c = -9$.

Substitute. → $x = \dfrac{-3 \pm \sqrt{3^2 - (4 \cdot 2 \cdot -9)}}{2 \cdot 2}$

$x = \dfrac{-3 + \sqrt{3^2 - (4 \cdot 2 \cdot -9)}}{2 \cdot 2}$ *or* $x = \dfrac{-3 - \sqrt{3^2 - (4 \cdot 2 \cdot -9)}}{2 \cdot 2}$

$x = \dfrac{-3 + \sqrt{9 + 72}}{4}$ $x = \dfrac{-3 - \sqrt{9 + 72}}{4}$

$x = \dfrac{-3 + \sqrt{81}}{4}$ $x = \dfrac{-3 - \sqrt{81}}{4}$

$x = \dfrac{-3 + 9}{4}$ $x = \dfrac{-3 - 9}{4}$

$x = \dfrac{6}{4}$ $x = \dfrac{-12}{4}$

$x = \dfrac{3}{2}$ $x = -3$

Answer: $x = \dfrac{3}{2}$ or $x = -3$

Once you feel comfortable working with the $\pm$ symbol, you might simplify your computation in the following way.

EXAMPLE 2 Solve $x^2 - 6x + 2 = 0$.

$$\begin{array}{ccccccc} ax^2 & + & bx & + & c & = & 0 \\ \updownarrow & & \updownarrow & & \updownarrow & & \\ 1x^2 & - & 6x & + & 2 & = & 0 \end{array}$$

So, $a = 1$, $b = -6$, and $c = 2$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - (4 \cdot 1 \cdot 2)}}{2 \cdot 1}$$

$$x = \frac{6 \pm \sqrt{36 - 8}}{2}$$

$$x = \frac{6 \pm \sqrt{28}}{2}$$

$$x = \frac{6 \pm 2\sqrt{7}}{2}$$

$$x = \frac{2(3 \pm \sqrt{7})}{2}$$

$$x = 3 \pm \sqrt{7}$$

$x = 3 + \sqrt{7}$ *or* $x = 3 - \sqrt{7}$

Answer: $x = 3 + \sqrt{7}$ or $x = 3 - \sqrt{7}$

Classroom Practice

Tell what the following expressions mean.

1. $2 \pm \sqrt{5}$
$2 + \sqrt{5}$ or $2 - \sqrt{5}$

2. $1 \pm 3\sqrt{2}$
$1 + 3\sqrt{2}$ or $1 - 3\sqrt{2}$

3. $4 \pm 5\sqrt{6}$
$4 + 5\sqrt{6}$ or $4 - 5\sqrt{6}$

4. $2 \pm 3\sqrt{2}$
$2 + 3\sqrt{2}$ or $2 - 3\sqrt{2}$

State the values of a, b, and c for each equation.

5. $2x^2 + 3x + 1 = 0$
$a = 2, b = 3, c = 1$

6. $3x^2 - 7x + 2 = 0$
$a = 3, b = -7, c = 2$

7. $x^2 - 3x - 4 = 0$
$a = 1, b = -3, c = -4$

8. $x^2 - x - 3 = 0$
$a = 1, b = -1, c = -3$

9. $4x^2 + 4x + 1 = 0$
$a = 4, b = 4, c = 1$

10. $2y^2 - 3y - 7 = 0$
$a = 2, b = -3, c = -7$

11. $2y^2 + 5y = 0$
$a = 2, b = 5, c = 0$

12. $3x^2 - 15 = 0$
$a = 3, b = 0, c = -15$

13. $2x^2 + 2x - 12 = 0$
$a = 2, b = 2, c = -12$

Complete.

14. If $3x^2 - 5x = 2$,
then $3x^2 - 5x - \underline{?} = 0$. 2
$a = 3$, $b = \underline{?}$, and $c = \underline{?}$. -5, -2

15. If $x^2 + 2x = 7$,
then $\underline{?} = 0$. $x^2 + 2x - 7$
$a = \underline{?}$, $b = \underline{?}$, and $c = \underline{?}$. 1, 2, -7

16. If $2x^2 = 1 - 3x$,
then $\underline{?} = 0$. $2x^2 + 3x - 1$
$a = \underline{?}$, $b = \underline{?}$, and $c = \underline{?}$. 2, 3, -1

17. If $2x^2 - x = 15$,
then $\underline{?} = 0$. $2x^2 - x - 15$
$a = \underline{?}$, $b = \underline{?}$, and $c = \underline{?}$. 2, -1, -15

Written Exercises

Solve by using the quadratic formula. Leave irrational numbers in simplest radical form.

A **1.** $2x^2 + 3x + 1 = 0$ $-\frac{1}{2}, -1$ **2.** $4x^2 + 5x + 1 = 0$ $-\frac{1}{4}, -1$ **3.** $3x^2 - 7x + 2 = 0$ $\frac{1}{3}, 2$

4. $2x^2 - 5x - 3 = 0$ $3, -\frac{1}{2}$ **5.** $4x^2 + 7x - 2 = 0$ $\frac{1}{4}, -2$ **6.** $x^2 - 5x - 14 = 0$ $-2, 7$

7. $2x^2 + 7x - 4 = 0$ $-4, \frac{1}{2}$ **8.** $x^2 + 4x - 21 = 0$ $-7, 3$ **9.** $3y^2 - 7y + 2 = 0$ $\frac{1}{3}, 2$

10. $5x^2 + 3x - 2 = 0$ $-1, \frac{2}{5}$ **11.** $3x^2 - x - 2 = 0$ $-\frac{2}{3}, 1$ **12.** $2x^2 - 2x - 3 = 0$

13. $4x^2 - 4 = 0$ $1, -1$ **14.** $x^2 - 9 = 0$ $3, -3$ **15.** $4x^2 - 1 = 0$ $\frac{1}{2}, -\frac{1}{2}$

16. $y^2 - y - 1 = 0$ $\frac{1 \pm \sqrt{5}}{2}$ **17.** $2x^2 + x - 2 = 0$ $\frac{-1 \pm \sqrt{17}}{4}$ **18.** $x^2 - 3x + 1 = 0$

19. $5y^2 + 7y + 2 = 0$ $-\frac{2}{5}, -1$ **20.** $2y^2 + 4y + 1 = 0$ $\frac{-1 \pm \sqrt{2}}{2}$ **21.** $x^2 - 2x - 5 = 0$ $1 \pm \sqrt{6}$

22. $2x^2 - 3x = 5$ $-1, 2\frac{1}{2}$ **23.** $2x^2 + x = 7$ $\frac{-1 \pm \sqrt{57}}{4}$ **24.** $3x^2 = x + 1$ $\frac{1 \pm \sqrt{13}}{6}$

25. $2x^2 = 5 + x$ $\frac{1 \pm \sqrt{41}}{4}$ **26.** $3x^2 = 8 - x$ $\frac{-1 \pm \sqrt{97}}{6}$ **27.** $x^2 + 3 = 6x$ $3 \pm \sqrt{6}$

28. $2x^2 = 5x - 3$ $1, 1\frac{1}{2}$ **29.** $2x^2 = 7x - 2$ $\frac{7 \pm \sqrt{33}}{4}$ **30.** $4y^2 = 6y + 2$ $\frac{3 \pm \sqrt{17}}{4}$

Solve by two methods.

a. Factoring
b. Using the quadratic formula

12. $\frac{1 \pm \sqrt{7}}{2}$ 18. $\frac{3 \pm \sqrt{5}}{2}$

B **31.** $x^2 - 5x = 0$ $0, 5$ **32.** $x^2 + 8x + 7 = 0$ $-1, -7$ **33.** $x^2 - 4x - 5 = 0$ $5, -1$

34. $y^2 + y = 0$ $0, -1$ **35.** $y^2 - 7y + 10 = 0$ $2, 5$ **36.** $s^2 - 6s + 9 = 0$ 3

37. $x^2 - 6x + 8 = 0$ $2, 4$ **38.** $x^2 - 6x - 7 = 0$ $-1, 7$ **39.** $x^2 - x - 12 = 0$ $-3, 4$

Mixed Practice Exercises

Solve by factoring.

1. $x^2 - 2x = 0$ 0, 2
2. $y^2 + 3y = 0$ 0, −3
3. $x^2 - 4x = 0$ 0, 4
4. $2x^2 - 4x = 0$ 0, 2
5. $5x^2 - 10x = 0$ 0, 2
6. $2x^2 + x = 0$ $0, -\frac{1}{2}$
7. $x^2 + 3x + 2 = 0$ −2, −1
8. $x^2 - 2x - 3 = 0$ −1, 3
9. $x^2 + 2x - 8 = 0$ −4, 2
10. $x^2 + 10x + 21 = 0$ −7, −3
11. $x^2 - x = 30$ 6, −5
12. $x^2 + 2x = 35$ −7, 5

Solve by taking square roots.

13. $x^2 = 9$ ± 3
14. $x^2 = 36$ ± 6
15. $y^2 = 35$ $\pm\sqrt{35}$
16. $x^2 = 42$ $\pm\sqrt{42}$
17. $y^2 = 19$ $\pm\sqrt{19}$
18. $x^2 - 40 = 0$ $\pm 2\sqrt{10}$
19. $x^2 - 18 = 0$ $\pm 3\sqrt{2}$
20. $(x + 2)^2 = 25$ 3, −7
21. $(x - 1)^2 = 16$ 5, −3
22. $(x + 4)^2 = 49$ 3, −11
23. $(y - 3)^2 = 17$ $3 \pm \sqrt{17}$
24. $(y + 2)^2 = 28$ $-2 \pm 2\sqrt{7}$

Solve by using the quadratic formula.

25. $x^2 - 2x - 3 = 0$ −1, 3
26. $x^2 + 4x - 5 = 0$ −5, 1
27. $x^2 + 2x - 8 = 0$ −4, 2
28. $2x^2 - 3x + 1 = 0$ $\frac{1}{2}$, 1
29. $3x^2 + 4x + 1 = 0$ $-1, -\frac{1}{3}$
30. $x^2 - 5x + 3 = 0$ $\frac{5 \pm \sqrt{13}}{2}$
31. $3x^2 - 2x - 5 = 0$ $-1, 1\frac{2}{3}$
32. $2x^2 + 5x - 3 = 0$ $\frac{1}{2}$, −3
33. $x^2 - 3x + 2 = 0$ 1, 2
34. $x^2 + 4x - 3 = 0$ $-2 \pm \sqrt{7}$
35. $2x^2 - x = 4$ $\frac{1 \pm \sqrt{33}}{4}$
36. $2x^2 - 2x = 3$ $\frac{1 \pm \sqrt{7}}{2}$

For more practice, see page 434.

Puzzles & Things

Imaginary Numbers

You know that there are no real-number solutions to the equation $x^2 = -4$. If you take another course in algebra, you'll learn that equations like this have solutions which are **imaginary numbers.**

$$x^2 = 4(-1)$$
$$x = \pm 2\sqrt{-1}$$

The number $\sqrt{-1}$ is called imaginary, but actually it is very, very useful in technical fields such as electricity.

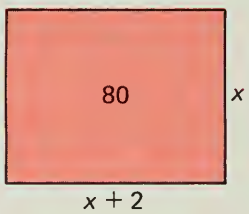
7 • Problem Solving

In this section you'll use quadratic equations to solve word problems. Your method should be the same as you used earlier in this course.

A Guide for Problem Solving

1. Read the problem. Read it more than once if you wish.
2. Let a variable stand for one of the unknowns. Show the other unknowns in terms of that variable.
3. Write an equation and solve it.
4. Answer the question.
5. Check your work.

EXAMPLE The length of a rectangle is 2 cm more than the width. The area is 80 cm^2. Find the length and width.



Let x = width
Then $x + 2$ = length

$$\underline{\text{width}} \times \underline{\text{length}} = \underline{\text{Area}}$$

$$\begin{aligned} x \times (x+2) &= 80 \\ x(x+2) &= 80 \\ x^2 + 2x &= 80 \\ x^2 + 2x - 80 &= 0 \\ (x+10)(x-8) &= 0 \end{aligned}$$

$x + 10 = 0$ *or* $x - 8 = 0$
$x = -10$ $\qquad$ $x = 8$

Answer: The width must be a positive number, so the value $x = -10$ is *not* a solution to the problem.

width: $x = 8 \longrightarrow 8$ cm
length: $x + 2 = 10 \longrightarrow 10$ cm

Check: length × width = Area
10 × 8 = 80 ✓

Written Exercises

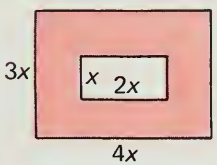
A **1.** The length of a rectangle is 3 cm more than the width. The area is 40 cm^2. Find the length and width.
length: 8 cm; width: 5 cm

2. The length of a rectangle is twice its width. The area is 72 cm^2. Find the length and width.
length: 12 cm; width: 6 cm

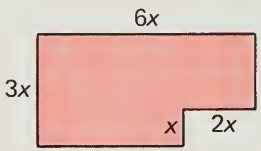
3. One number is 2 more than another. The product of the numbers is 35. Find the numbers.
-5 and -7, or 5 and 7

4. The square of a number is 30 more than the number. The number is positive. Find the number. 6

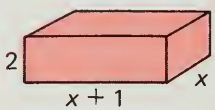
5. Shaded area = 40
Find x. 2



6. Shaded area = 400
Find x. 5

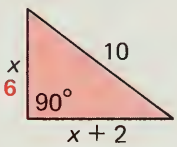


7. Volume = 60
Find x. 5

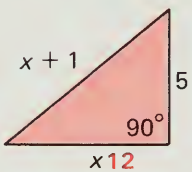


Use the Law of Pythagoras to find x.

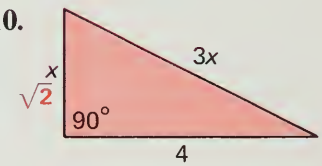
8.



9.



10.



B **11.** The perimeter of a rectangle is 32 cm. The area is 48 cm^2. Find the dimensions. 12 cm by 4 cm

12. The sum of two numbers is 13. The product is 42. Find the numbers. 6 and 7

C **13.** Vince bikes 3 km/h faster than Al. It takes Al 1 hour more than Vince to travel 36 km. Find how fast each bikes.
Al: 9 km/h
Vince: 12 km/h

	D	$\div$ r	$=$ t
Al	36	x	$\frac{36}{x}$
Vince	? 36	? $x + 3$	$\frac{36}{x+3}$

For more practice, see page 435.

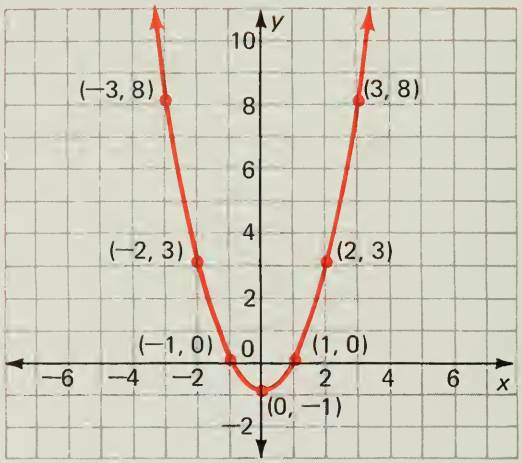
8 • Quadratic Graphs

The equation $y = x^2 - 1$ states that y is a **quadratic function** of x, or that y *depends* upon x. This equation has two variables, so its solutions are pairs of numbers.

You name a number as a value for x.

Then you substitute that value for x in the equation to find a value for y.

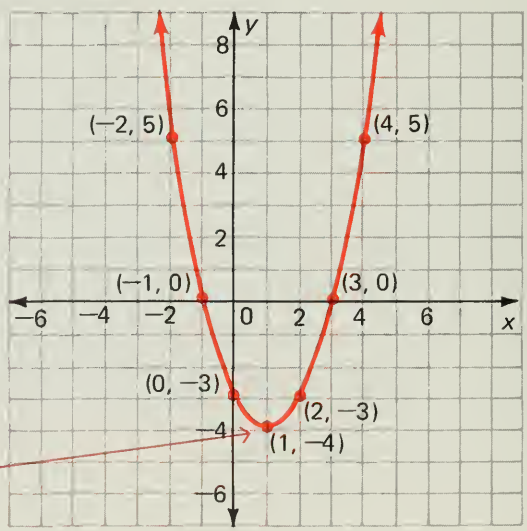
$y = x^2 - 1$	
x	y
−3	8
−2	3
−1	0
0	−1
1	0
2	3
3	8



The graphs of quadratic functions all curve in a similar way. The graph is called a **parabola.** The "tip" of a parabola is called its **vertex.** Study a few more examples of these special graphs.

EXAMPLE 1

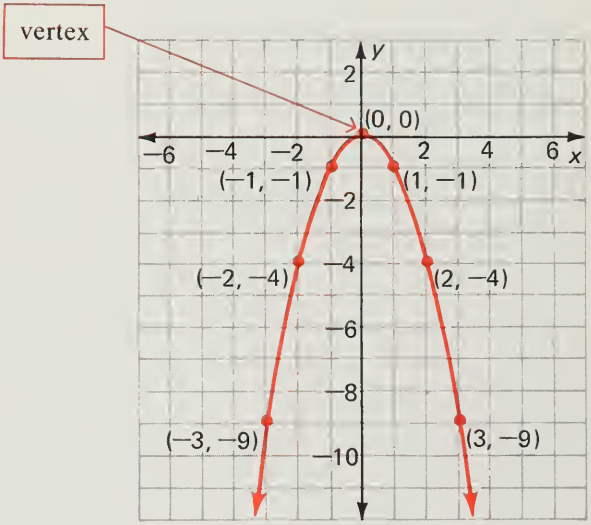
$y = x^2 - 2x - 3$	
x	y
−2	5
−1	0
0	−3
1	−4
2	−3
3	0
4	5



Some graphs of quadratic functions curve upside down.

EXAMPLE 2

$y = -x^2$	
x	y
-3	-9
-2	-4
-1	-1
0	0
1	-1
2	-4
3	-9



Classroom Practice

Study the graph of $y = x^2 - 1$ shown on the last page.

1. What are the coordinates of the vertex of this parabola? 0, −1
2. How large a value can be substituted for x? How small a value? any value; any value
3. How many values of x make $y = 3$? two
4. How many values of x make $y = -3$? none
5. Suppose you are told that the point (2.3, 4.29) is on the graph. Then you also know that the point $(-2.3, \underline{?})$ is on the graph. 4.29

Study the graph of $y = x^2 - 2x - 3$ shown on the last page.

6. What is the x-coordinate of the vertex of this parabola? 1
7. What is the y-coordinate of the vertex? −4
8. How many values of x make $y = -4$? one
9. How many values of x make $y = -3$? two

Written Exercises

For each equation do three things.

a. Make a table of values. Check student's charts.

b. Draw the graph. Check student's graphs.

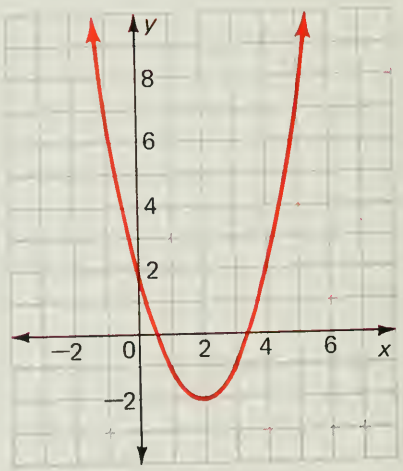
c. Give the coordinates of the vertex. Coordinates given below.

A

1. $y = x^2$ (0, 0)
2. $y = x^2 + 1$ (0, 1)
3. $y = x^2 - 4$ (0, −4)
4. $y = 2x^2$ (0, 0)
5. $y = -2x^2$ (0, 0)
6. $y = \frac{1}{2}x^2$ (0, 0)
7. $y = x^2 + 2x$ (−1, −1)
8. $y = x^2 - 2x$ (1, −1)
9. $y = 2x - x^2$ (1, 1)
10. $y = x^2 - 2x - 1$ (1, −2)
11. $y = x^2 - 4x + 3$ (2, −1)
12. $y = x^2 + 4x - 2$ (−2, −6)
13. $y = x^2 - 2x - 2$ (1, −3)
14. $y = x^2 - 6x + 8$ (3, −1)
15. $y = x^2 - 6x + 9$ (3, 0)
16. $y = x^2 + 6x - 6$ (−3, −15)
17. $y = (3 - x)(3 + x)$ (0, 9)
18. $y = (2 - x)(2 - x)$ (2, 0)

19. On the same set of axes, graph both equations:
$y = x^2 + 2$ and $y = -(x^2 + 2)$. Check student's graphs.

The graph of $y = x^2 - 4x + 2$ is shown at the right.



20. What is the vertex? (2, −2)

21. For what values of x does $y = -1$? 1 and 3

22. How many values of x make $y = 2$? two

23. How many values of x make $y = 7$? two

24. How many values of x make $y = -2$? one

B

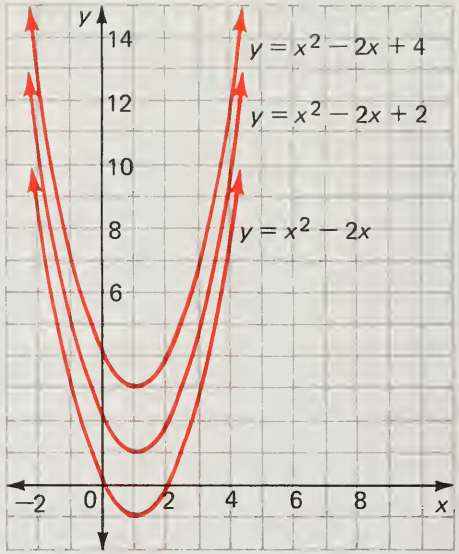
25. Use the graph of $y = x^2 - 4x + 2$ shown at the right to draw the graph of $y = -(x^2 - 4x + 2)$. Check student's graphs.

26. Suppose you are told that (4,1) is the vertex of the parabola $y = x^2 - 8x + 17$. Can you give the vertex of the parabola $y = -(x^2 - 8x + 17)$? (4, −1)

The graphs of three equations are shown at the right.

27. Give the vertex of each parabola.
(1, 3), (1, 1), and (1, −1)

28. What is the vertex of the parabola $y = x^2 - 2x + 6$? (1, 5)



C **The expression $b^2 - 4ac$ (from the quadratic formula) can tell you whether the graph of a quadratic function crosses the x-axis once, twice, or not at all.**

If $b^2 - 4ac > 0 \longleftrightarrow$ graph crosses x-axis 2 times
If $b^2 - 4ac = 0 \longleftrightarrow$ graph crosses x-axis 1 time
If $b^2 - 4ac < 0 \longleftrightarrow$ graph crosses x-axis 0 times

How many times does the graph of the equation cross the x-axis?

29. $y = x^2 - 5x + 2$
2 times

30. $y = x^2 - 4x + 6$
0 times

31. $y = x^2 - 10x + 25$
1 time

For more practice, see page 435.

SELF-TEST

Solve by using the quadratic formula. Leave irrational solutions in simplest radical form.

1. $x^2 + 7x - 10 = 0$ $\frac{-7 \pm \sqrt{89}}{2}$

2. $2x^2 + 4x - 12 = 0$ $-1 \pm \sqrt{7}$

3. $3x^2 - 2x - 6 = 0$ $\frac{1 \pm \sqrt{19}}{3}$

4. The length of a rectangle is 4 cm more than the width. The area is 45 cm^2. Find the length and width. length: 9 cm; width: 5 cm

5. Draw the graph of $y = x^2 - 2x - 2$. Check student's graphs.

CONSUMER CORNER

Averages

Have you ever heard a statement such as this?

The average family in this town has 4.2 members and owns 1.2 cars.

What does it mean to say, "The average family has 4.2 members" when we know that no family can have 4.2 members? Study this chart.

Family	Number of Members	Number of Cars
Nelson	5	2
Jones	6	2
Jackson	4	1
Bernstein	3	1
Marko	3	0
Totals	21	6

Average number of members $= \frac{21}{5} = 4.2$

Average number of cars $= \frac{6}{5} = 1.2$

The averages figured above are called arithmetic averages. There is another type of average, called the median. The median of several numbers is the number in the middle.

Be sure to arrange the numbers in order.

3,3,4,5,6 — median = 4

0,1,1,2,3,4,5 — median = 2

3,4,5,7 — median = 4.5

Sometimes in a wage dispute, labor and management will use the word "average" in two different ways.

Management says: *"The average yearly wage in this company is $17,900."*
Labor says: *"The average yearly wage in this company is $11,000."*

The figures below show that both statements can be correct.

President	$60,000
Vice President	$30,000
Sales Manager	$15,000
Two Supervisors	$12,000 each
Five Workers	$10,000 each

Arithmetic Average: $17,900

Median: $11,000
(5 wages above, 5 below)

1. Find the arithmetic average for the number of members and for the number of pets in the families listed below. members: 3; pets: 1.2

Family	Number of Members	Number of Pets
Gardner	6	2
Pollock	4	2
Shea	3	1
Rawlings	2	0
Lister	2	1
Samuels	1	1

2. Find the median number of members and pets.
members: 2.5; pets: 1

3. In the company where Fran works, there are 10 people. Last year people took the following number of sick days: Fran, 2; Art, 4; Sue, 4; Dick, 2; Amy, 1; Melba, 1; Ted, 22; Elena, 3; Hank, 3; Rose, 0.

a. What is the arithmetic average? 4.2
b. What is the median? 2.5
c. Most people took far fewer sick days than Ted. Which average best describes the number of sick days taken by most of the people in the company? median

Reviewing the Chapter

Solve. *(See pp. 380-381.)*

1. $x(x - 2) = 0$ 0, 2
2. $y(y + 4) = 0$ 0, −4
3. $x(x - 7) = 0$ 0, 7
4. $2a(a - 9) = 0$ 0, 9
5. $4x(x + 6) = 0$ 0, −6
6. $5n(n + 8) = 0$ 0, −8
7. $(x + 1)(x + 5) = 0$ −1, −5
8. $(x - 3)(x + 6) = 0$ 3, −6
9. $(y + 5)(y - 3) = 0$ −5, 3
10. $(x - 3)(x - 2) = 0$ 3, 2
11. $(n - 6)(n - 2) = 0$ 6, 2
12. $(x - 8)(x + 6) = 0$ 8, −6

Solve. *(See pp. 382-383.)*

13. $x^2 - 3x = 0$ 0, 3
14. $n^2 + 4n = 0$ 0, −4
15. $5x + x^2 = 0$ 0, −5
16. $2y^2 - 4y = 0$ 0, 2
17. $7n^2 + 14n = 0$ 0, −2
18. $x^2 + 4x + 3 = 0$ −3, −1
19. $x^2 + 5x + 6 = 0$ −3, −2
20. $x^2 - 8x + 15 = 0$ 5, 3
21. $x^2 - 6x + 8 = 0$ 4, 2
22. $x^2 - x - 20 = 0$ 5, −4
23. $x^2 + 3x - 18 = 0$ −6, 3
24. $x^2 + 2x - 35 = 0$ −7, 5

Solve. *(See pp. 384-385.)*

25. $x^2 = 2x$ 0, 2
26. $x^2 = -3x$ 0, −3
27. $y^2 = -y$ 0, −1
28. $n^2 = -4n$ 0, −4
29. $x^2 + 7x = -10$ −5, −2
30. $x^2 + 6x = -9$ −3
31. $x^2 - 10x = -16$ 2, 8
32. $x^2 - 11x = -28$ 4, 7
33. $x^2 + 4x = 21$ −7, 3
34. $x^2 = -11x - 30$ −5, −6
35. $x^2 = 9x - 14$ 2, 7
36. $x^2 = x + 42$ 7, −6

Solve. *(See pp. 386-387.)*

37. $x^2 = 9$ ± 3
38. $y^2 = 81$ ± 9
39. $x^2 = 36$ ± 6
40. $x^2 = 14$ $\pm \sqrt{14}$
41. $y^2 = 21$ $\pm \sqrt{21}$
42. $n^2 = 20$ $\pm 2\sqrt{5}$
43. $y^2 = 27$ $\pm 3\sqrt{3}$
44. $2x^2 = 50$ ± 5
45. $3y^2 = 18$ $\pm \sqrt{6}$
46. $n^2 - 3 = 13$ ± 4
47. $x^2 + 2 = 38$ ± 6
48. $y^2 - 6 = 43$ ± 7

Solve. *(See pp. 388-389.)*

49. $(x - 1)^2 = 16$ 5, −3

50. $(x + 3)^2 = 81$ 6, −12

51. $(x + 4)^2 = 64$ 4, −12

52. $(y - 2)^2 = 36$ 8, −4

53. $(y + 5)^2 = 49$ 2, −12

54. $(a - 7)^2 = 100$ 17, −3

55. $(3x + 3)^2 = 36$ 1, −3

56. $(2x - 4)^2 = 64$ 6, −2

57. $(2x + 6)^2 = 16$ −1, −5

58. $(4x + 4)^2 = 16$ 0, −2

59. $(3x - 9)^2 = 36$ 1, 5

60. $(5x - 15)^2 = 100$ 1, 5

Solve. *(See pp. 390-393.)*

61. $x^2 + 3x - 10 = 0$ 2, −5

62. $x^2 - 9x + 14 = 0$ 2, 7

63. $2x^2 + 5x - 3 = 0$ $\frac{1}{2}$, −3

64. $2x^2 - 2x - 4 = 0$ −1, 2

65. $x^2 + 8x + 15 = 0$ −3, −5

66. $3x^2 + 5x + 2 = 0$ $-\frac{2}{3}$, −1

67. $4x^2 - 2x - 2 = 0$ $-\frac{1}{2}$, 1

68. $2x^2 - x - 3 = 0$ $1\frac{1}{2}$, −1

69. $2x^2 + 5x + 3 = 0$ $-1, -1\frac{1}{2}$

70. $5x^2 + 2x - 2 = 0$ $\frac{-1 \pm \sqrt{11}}{5}$

71. $4x^2 + 7x + 2 = 0$ $\frac{-7 \pm \sqrt{17}}{8}$

72. $3x^2 - 2x - 4 = 0$ $\frac{1 \pm \sqrt{13}}{3}$

Solve. *(See pp. 394-395.)*

73. The length of a rectangle is 5 cm more than the width. The area is 36 cm^2. Find the length and width. length: 9 cm; width: 4 cm

74. One number is 6 more than another. The sum of their squares is 90. Find the numbers. 3 and 9, or −9 and −3

Do the following exercises. *(See pp. 396-399.)*

75. Make a table of values for the equation $y = x^2 + 2$. For what values of x does $y = 6$? −2 and 2

76. Make a table of values for the equation $y = x^2 + 3x - 5$. For what values of x does $y = -7$? −1 and −2

Draw the graph of the equation. *(See pp. 396-399.)* Check student's graphs.

77. $y = x^2 - 3$

78. $y = 2x^2 + 1$

79. $y = x^2 + 2x + 1$

A permission-to-reproduce chapter test can be found on page T22.

Cumulative Review

Simplify.

1. $-7a(4a)(-a)$ $28a^3$
2. $(10x + 3) - (4x - 7)$ $6x + 10$
3. $0.9m - 0.03m$ $0.87m$
4. $\dfrac{5 - x}{x^2 - 25}$ $\dfrac{-1}{x + 5}$
5. $-\sqrt{\dfrac{n^2}{16}}$ $-\dfrac{n}{4}$
6. $5\sqrt{x} \cdot 4\sqrt{x}$ $20x$

Factor.

7. $a^2 + 7a$ $a(a + 7)$
8. $8x^3 + x^2$ $x^2(8x + 1)$
9. $3m^2 + 12m$ $3m(m + 4)$
10. $x^2 + 7x + 12$ $(x + 3)(x + 4)$
11. $y^2 - 3y - 18$ $(y - 6)(y + 3)$
12. $n^2 - 11n + 18$ $(n - 9)(n - 2)$
13. $y^2 + 12y + 36$ $(y + 6)^2$
14. $a^2 - b^2$ $(a - b)(a + b)$
15. $x^2 - 5x - 24$ $(x - 8)(x + 3)$
16. $16x^2 - 25$ $(4x - 5)(4x + 5)$
17. $m^2 + 6m - 40$ $(m + 10)(m - 4)$
18. $x^2 - 6x + 9$ $(x - 3)^2$

Solve.

19. $6x + 4 = 28$ 4
20. $3y - 7 = -28$ -7
21. $7a + 3 = 4a - 9$ -4
22. $5(2m + 3) = 35$ 2
23. $-(a + 3) = 3a + 1$ -1
24. $-4(y - 1) = y - 6$ 2
25. $-(x - 10) = 4 + x$ 3
26. $-3x + 12 = -x - 4$ 8
27. $0.1y - 0.02y = 2.4$ 30
28. $\dfrac{x}{15} = \dfrac{1}{5}$ 3
29. $\dfrac{a}{3} = \dfrac{a + 7}{4}$ 21
30. $\dfrac{2x}{5} + \dfrac{x}{2} = 9$ 10
31. $\begin{cases} 3x - y = 10 \\ 4x + y = 11 \end{cases}$ $x = 3$; $y = -1$
32. $\begin{cases} 2a - b = 10 \\ a + 3b = -2 \end{cases}$ $a = 4$; $b = -2$
33. $\begin{cases} 4x - 3y = 15 \\ 3x + 2y = 7 \end{cases}$ $x = 3$; $y = -1$
34. 12 is 30% of what number? 40
35. What percent of 18 is 12? $66\frac{2}{3}\%$
36. $x(x - 4) = 0$ 0, 4
37. $6m(m + 3) = 0$ 0, -3
38. $(y - 4)(y + 3) = 0$ 4, -3
39. $m^2 - 5m = 0$ 0, 5
40. $x^2 + 4x + 3 = 0$ -1, -3
41. $y^2 - y - 6 = 0$ 3, -2
42. $n^2 + n = 6$ -3, 2
43. $2x^2 = 50$ ± 5
44. $4(x + 1)^2 = 36$ 2, -4
45. $2x^2 + 3x - 2 = 0$ $\frac{1}{2}$, -2
46. $x^2 + 5x + 5 = 0$ $\dfrac{-5 \pm \sqrt{5}}{2}$
47. $4x^2 - 6x + 2 = 0$ 1, $\frac{1}{2}$

Draw the graph of the equation. Check student's graphs.

48. $x + 2y = 0$
49. $y = x^2 - 2$
50. $y = x^2 - 8x + 16$

 A more extensive cumulative review for Chapters 1–12 can be found on page 444.

Extra Practice Exercises

CHAPTER 1

For use with pages 2–7

Find the value of the expression.

A **1.** $7 \cdot 6$ 42 **2.** $5 \cdot 8$ 40 **3.** $9 \cdot 7$ 63 **4.** $6 \cdot 4$ 24

5. $2 \cdot 3 + 4$ 10 **6.** $5 \cdot 2 - 6$ 4 **7.** $(6 - 1) \cdot 3$ 15 **8.** $(4 + 2) \div 3$ 2

9. $2(3 + 2)$ 10 **10.** $4(5 + 1)$ 24 **11.** $8 - (2 \cdot 3)$ 2 **12.** $10 + (6 \cdot 1)$ 16

If $n = 3$, find the value of the expression.

13. $9n$ 27 **14.** $9 + n$ 12 **15.** $9 - n$ 6 **16.** $9 \div n$ 3

If $x = 10$ and $y = 5$, find the value of the expression.

17. $x + y$ 15 **18.** $x - y$ 5 **19.** $x + x + y$ 25 **20.** $x + 5 - y$ 10

If $y = 2$, find the value of the expression.

21. $6(y + 2)$ 24 **22.** $6y + 2$ 14 **23.** $(y - 1) \cdot 4$ 4 **24.** $2y - 3$ 1

For use with pages 8–11

Simplify.

A **1.** $n + n$ $2n$ **2.** $n + n + n$ $3n$ **3.** $2n + n$ $3n$ **4.** $n + 2n$ $3n$

5. $3x - x$ $2x$ **6.** $5y - 2y$ $3y$ **7.** $7m + 3m$ $10m$ **8.** $10x - x$ $9x$

9. $4a + 5a$ $9a$ **10.** $6x + 3x$ $9x$ **11.** $4a + 2a - 3a$ $3a$ **12.** $5x + 5x - 3x$ $7x$

13. $3x + 8x - x$ $10x$ **14.** $n + 6n - 7$ $7n - 7$ **15.** $a + 2a - b$ $3a - b$ **16.** $3n - n + m$ $2n + m$

17. $2x - 5 + 3x$ $5x - 5$ **18.** $6n - 6 + 2n$ $8n - 6$ **19.** $5m + 2 + 6m - 1$ $11m + 1$

20. $6 + 7n - 6n + 7$ $n + 13$ **21.** $8 + 6y + 5y - 7$ $11y + 1$ **22.** $7 + 3x + 9x + 9$ $12x + 16$

23. $5x + 3y - y + 11x$ $16x + 2y$ **24.** $2x + 5y - y + 9x$ $11x + 4y$ **25.** $4n + r + r - 3n$ $n + 2r$

For use with pages 12–15

Find the value of the expression.

A **1.** 3^2 9 **2.** 4^2 16 **3.** 2^2 4 **4.** 6^2 36

5. 4^1 4 **6.** 2^3 8 **7.** 3^3 27 **8.** 5^2 25

Simplify.

9. $y \cdot y \cdot y$ y^3 **10.** $x \cdot x$ x^2 **11.** $a \cdot a \cdot a$ a^3 **12.** $c \cdot c \cdot c \cdot c$ c^4

13. $r \cdot s \cdot s$ rs^2 **14.** $a \cdot a \cdot b \cdot b$ a^2b^2 **15.** $x \cdot y \cdot y \cdot y$ xy^3 **16.** $a \cdot b \cdot b \cdot c \cdot c$ ab^2c^2

17. $3 \cdot a \cdot a$ $3a^2$ **18.** $2 \cdot c \cdot c$ $2c^2$ **19.** $5 \cdot 2 \cdot n \cdot n$ $10n^2$ **20.** $6 \cdot a \cdot a \cdot b$ $6a^2b$

21. $2 \cdot (4r)$ $8r$ **22.** $6 \cdot 3n$ $18n$ **23.** $(4n) \cdot 4$ $16n$ **24.** $6n \cdot 5$ $30n$

25. $(2r)(4r)$ $8r^2$ **26.** $(10x)(5x)$ $50x^2$ **27.** $7y \cdot 8y$ $56y^2$ **28.** $6s \cdot 5s$ $30s^2$

29. $(2a)(4b)$ $8ab$ **30.** $(3x)(4y)$ $12xy$ **31.** $2 \cdot a \cdot 5 \cdot a$ $10a^2$ **32.** $x \cdot x \cdot 4 \cdot x$ $4x^3$

For use with pages 16–17

State the expression without parentheses.

A **1.** $3(x + 3)$ $3x + 9$ **2.** $7(y + 1)$ $7y + 7$ **3.** $5(a - 1)$ $5a - 5$ **4.** $3(n + 2)$ $3n + 6$

5. $8(n + 4)$ $8n + 32$ **6.** $6(c - 2)$ $6c - 12$ **7.** $4(x + 3)$ $4x + 12$ **8.** $a(a + 1)$ $a^2 + a$

9. $x(x - 2)$ $x^2 - 2x$ **10.** $y(y + 7)$ $y^2 + 7y$ **11.** $n(n + 2)$ $n^2 + 2n$ **12.** $c(c + 9)$ $c^2 + 9c$

13. $6(2n - 8)$ $12n - 48$ **14.** $4(5x - 4)$ $20x - 16$ **15.** $3(2a - 1)$ $6a - 3$ **16.** $5(3x + 2)$ $15x + 10$

17. $3(2a - 3b)$ $6a - 9b$ **18.** $2(4x - 2y)$ $8x - 4y$ **19.** $a(6a + b)$ $6a^2 + ab$ **20.** $x(4x - 2y)$ $4x^2 - 2xy$

Use the distributive property. Then combine like terms.

21. $2(n + 3) + 1$ $2n + 7$ **22.** $4(y + 1) + 3$ $4y + 7$ **23.** $3(n + 1) + 8$ $3n + 11$

24. $4(x + 3) + 2x$ $6x + 12$ **25.** $2(x + 4) - 2x$ 8 **26.** $8(3n + 1) - 2$ $24n + 6$

B **27.** $4(2a + 3) - 7a + 1$ $a + 13$ **28.** $3(5x + 7) + 2x - 20$ $17x + 1$ **29.** $2(3y + 5) - 5y + 4$ $y + 14$

30. $5(2 + 3x) - 4x - 7$ $11x + 3$ **31.** $6(a + 1) + 4(a + 2)$ $10a + 14$ **32.** $7(n + 3) + 2(n - 6)$ $9n + 9$

For use with pages 18–21

Find the value if possible. If not, write *impossible*.

A **1.** $6 \cdot 0$ 0 **2.** $5 \cdot 1$ 5 **3.** $8 \cdot 0$ 0 **4.** $\frac{0}{4}$ 0

5. $\frac{0}{7}$ 0 **6.** $\frac{9}{0}$ impossible **7.** $6 \div 1$ 6 **8.** $\frac{8}{0}$ impossible

9. $\frac{5 \cdot 0}{3}$ 0 **10.** $\frac{7}{1}$ 7 **11.** $\frac{7 + 1}{0}$ impossible **12.** $12 \div 0$ impossible

Let $x = 1$. Find the value of the expression.

13. $7(x - 1)$ 0 **14.** $x \div x$ 1 **15.** $3x \div x$ 3 **16.** $\frac{4}{4}x$ 1

Let $a = 6$. Find the value of the expression.

17. $(a - 1) \div 5$ 1 **18.** $(2a - 2) \div 10$ 1 **19.** $\frac{a + 2}{8}$ 1 **20.** $\frac{a}{6} \cdot 1$ 1

Simplify.

21. $3 + 2x - 3$ $2x$ **22.** $7 + 4y - 7$ $4y$ **23.** $4a + 5 + 2a - 5$ $6a$ **24.** $2(x + 4) - 8$ $2x$

25. $\frac{6}{6} \cdot n$ n **26.** $\frac{9}{9} \cdot c$ c **27.** $\frac{x}{3} \cdot 3$ x **28.** $\frac{2n}{2}$ n

For use with pages 22–25

Tell which of the numbers shown in color is a solution.

A **1.** $n + 3 = 4$ 2, 1, or 0 **2.** $x - 2 = 6$ 7, 8, or 9

3. $5 + y = 8$ 3, 2, or 1 **4.** $a - 4 = 8$ 14, 13, or 12

5. $6 - a = 4$ 4, 3, or 2 **6.** $n - 5 = 10$ 14, 15, or 16

Tell which of the numbers shown in color are solutions.

7. $n < 4$ 1, 2, 4, 5 **8.** $x > 6$ 6, 7, 9, 10

9. $y < 17$ 15, 16, 19, 20 **10.** $a > 9$ 6, 8, 10, 12

11. $x + 1 > 3$ 1, 3, 4, 10 **12.** $2c > 3$ 4, 2, 1, 9

CHAPTER 2

For use with pages 34–37

Solve.

A

1. $x - 2 = 4$ 6
2. $y - 4 = 2$ 6
3. $n - 3 = 3$ 6
4. $t - 4 = 4$ 8
5. $y - 4 = 7$ 11
6. $x - 3 = 6$ 9
7. $n - 7 = 6$ 13
8. $m - 3 = 8$ 11
9. $n + 2 = 6$ 4
10. $x + 1 = 7$ 6
11. $x + 7 = 8$ 1
12. $y + 6 = 10$ 4
13. $x + 5 = 20$ 15
14. $y + 6 = 14$ 8
15. $n + 9 = 15$ 6
16. $m + 12 = 18$ 6

Solve.

17. $y - 5 = 20$ 25
18. $x - 6 = 13$ 19
19. $x + 6 = 19$ 13
20. $n + 7 = 14$ 7
21. $n + 9 = 17$ 8
22. $n - 5 = 16$ 21
23. $x + 14 = 21$ 7
24. $r - 11 = 22$ 33
25. $t + 16 = 22$ 6
26. $n + 18 = 25$ 7
27. $x - 6 = 18$ 24
28. $30 = n - 5$ 35
29. $15 = y + 8$ 7
30. $x - 4 = 0$ 4
31. $y + 3 = 27$ 24
32. $16 = x - 14$ 30

For use with pages 38–41

Solve.

A

1. $2x = 12$ 6
2. $3y = 12$ 4
3. $4n = 12$ 3
4. $5n = 25$ 5
5. $3x = 15$ 5
6. $6x = 18$ 3
7. $7y = 28$ 4
8. $9x = 63$ 7
9. $\frac{x}{3} = 4$ 12
10. $\frac{n}{2} = 9$ 18
11. $\frac{a}{4} = 3$ 12
12. $\frac{x}{2} = 6$ 12

Solve.

13. $9y = 54$ 6
14. $7x = 21$ 3
15. $\frac{n}{3} = 8$ 24
16. $\frac{a}{6} = 7$ 42
17. $4x = 36$ 9
18. $8x = 72$ 9
19. $7y = 42$ 6
20. $9a = 81$ 9
21. $\frac{x}{7} = 5$ 35
22. $9 = \frac{a}{4}$ 36
23. $\frac{n}{8} = 8$ 64
24. $9x = 54$ 6

For use with pages 42–45

Solve.

A
1. $2x + 4 = 6$ 1
2. $4y - 3 = 9$ 3
3. $3n - 7 = 11$ 6
4. $5n - 3 = 22$ 5
5. $5y + 5 = 15$ 2
6. $6x + 18 = 36$ 3
7. $4n + 5 = 45$ 10
8. $2a - 6 = 20$ 13
9. $3a + 6 = 36$ 10
10. $7n - 8 = 41$ 7
11. $3x - 8 = 13$ 7
12. $8x - 5 = 35$ 5
13. $7a + 14 = 21$ 1
14. $2m + 1 = 13$ 6
15. $4n + 3 = 51$ 12
16. $9n - 40 = 32$ 8
17. $6x - 12 = 30$ 7
18. $4a + 2 = 46$ 11
19. $7x + 12 = 75$ 9
20. $5b - 13 = 17$ 6

For use with pages 48–49

Solve.

A
1. $2x + 3x = 15$ 3
2. $4a - 2a = 10$ 5
3. $6y + 3y = 18$ 2
4. $2x + 3x - 5 = 30$ 7
5. $5c - 2c + 8 = 20$ 4
6. $9a + a - 2 = 28$ 3
7. $2n - 5 + 8n = 45$ 5
8. $7x + 3 - 2x = 18$ 3
9. $4n + 7 + 3n = 21$ 2
10. $15n - 6 - 7n = 34$ 5
11. $8y - 3 + y = 24$ 3
12. $10a + 6 - 7a = 30$ 8
13. $28 = y - 4 + 3y$ 8
14. $40 = 13x - 6x - 2$ 6
15. $56 = 7a - 16 + 2a$ 8
16. $46 = 2x + 5x - 3$ 7
17. $19 = 8a + 7 - 6a$ 6
18. $21 = 6x + 3x - 4 - 4x$ 5

For use with pages 54–57

A
1. Three times a number plus 5 times the number is 64.
 What is the number? 8

2. Cynthia's father is 26 years older than Cynthia.
 Together their ages total 42.
 How old is Cynthia? 8 yrs. old

3. Amy has twice as much money as Barb.
 Together they have $15.
 How much does Amy have? $10

4. Our class has 90 students.
 There are 4 more boys than girls.
 How many boys are there? 47

For use with pages 58–59

Solve.

A **1.** $2n = 10 + n$ 10 **2.** $4x = x + 15$ 5 **3.** $8x = 2x + 6$ 1

4. $16 - 2y = 6y$ 2 **5.** $5x - 20 = 3x$ 10 **6.** $10x - 14 = 3x$ 2

7. $7x - 12 = 4x$ 4 **8.** $3y + 9 = 4y$ 9 **9.** $14 - 2x = 5x$ 2

10. $6y - 2 = 4y + 8$ 5 **11.** $3x + 1 = x + 7$ 3 **12.** $5a - 4 = 8 - a$ 2

13. $2n + 11 = 7 + 6n$ 1 **14.** $5x - 8 = 10 + 2x$ 6 **15.** $7c - 4 = 2c + 6$ 2

For use with pages 60–61

Solve.

A **1.** $2(n - 4) = 16$ 12 **2.** $7(a - 3) = 4a$ 7 **3.** $5(z + 2) = 10z$ 2

4. $6(y - 3) = 0$ 3 **5.** $8(x - 2) = 0$ 2 **6.** $5(m + 2) = 25$ 3

7. $4(x + 3) = 6x$ 6 **8.** $6(y + 3) = 18$ 0 **9.** $3(2 - x) = 3x$ 1

10. $4(x - 2) = 3x + 2$ 10 **11.** $3(x + 1) = 2x + 10$ 7 **12.** $2(5 - x) = 3x$ 2

B **13.** $2(c + 3) = 3(c - 1)$ 9 **14.** $4(r - 2) = 2(r + 1)$ 5 **15.** $4(x - 4) = 2(x - 1)$ 7

For use with pages 62–63

Solve.

A **1.** Al has 3 times as much money as Bob.
Bob has \$10 more than Ed.
Together the three boys have \$65.
How much money does each have? Ed: \$5; Bob: \$15; Al: \$45

2. Jo, May, and Sara save dimes.
Jo has twice as many dimes as May.
Sara has 4 more dimes than May.
Together the three have 100 dimes.
How many dimes has Jo? 48 dimes

B **3.** Ned is 11 years older than Rob.
Next year Ned will be twice as old as Rob.
How old is Ned? 21 yrs. old

CHAPTER 3

For use with pages 72–75

Compare the numbers. Write > or <.

A **1.** -2 ? -5 > **2.** 3 ? -2 > **3.** 0 ? -4 > **4.** -6 ? -1 <

5. 6 ? -7 > **6.** 3 ? 0 > **7.** 2 ? -2 > **8.** -7 ? -3 <

Graph the solutions on the number line. Check student's graphs.

9. $n + 1 = 4$ **10.** $y - 3 = 7$ **11.** $4 + x = 6$ **12.** $a - 4 = 3$

13. $x < 3$ **14.** $x > -1$ **15.** $y < -2$ **16.** $x > 3$

For use with pages 76–80

Add.

A **1.** $2 + (-6)$ -4 **2.** $6 + (-2)$ 4 **3.** $-8 + 8$ 0

4. $3 + (-1) + (-8)$ -6 **5.** $-4 + 8 + (-4)$ 0 **6.** $5 + 0 + (-7)$ -2

Subtract.

7. $4 - (-8)$ 12 **8.** $7 - 11$ -4 **9.** $-9 - (-2)$ -7 **10.** $-6 - 4$ -10

11. $18 - 4$ 14 **12.** $3 - (-9)$ 12 **13.** $-4 - 7$ -11 **14.** $8 - (-5)$ 13

For use with pages 82–83

Simplify.

A **1.** $2n - 5n$ $-3n$ **2.** $-a + 6a$ $5a$ **3.** $-4b - 6b$ $-10b$

4. $x + 9x$ $10x$ **5.** $-2y - 4y$ $-6y$ **6.** $x^2 - 3x^2$ $-2x^2$

7. $-5a + 6a + 1$ $a + 1$ **8.** $7x - (-x) + y$ $8x + y$ **9.** $-4y + 3 + 4y$ 3

10. $15 - (-3x) + x$ $4x + 15$ **11.** $-7x - (-8x) + 2$ $x + 2$ **12.** $6y + 2x - (-6y)$ $12y + 2x$

13. $x^2 - y + 2x^2$ $3x^2 - y$ **14.** $8a - (-2a) + a^2$ $10a + a^2$ **15.** $2n - n^2 - (-3n)$ $5n - n^2$

For use with pages 84–87

Simplify.

A **1.** $12 \cdot 4$ 48 **2.** $-3(6)$ -18 **3.** $-8(-7)$ 56 **4.** $3(-9)$ -27

5. 5^2 25 **6.** $(-5)^2$ 25 **7.** $-(5)^2$ -25 **8.** $-(-4)^2$ -16

9. $3n(-7)$ $-21n$ **10.** $-2y(-8x)$ $16xy$ **11.** $2(2a - b)$ $4a - 2b$ **12.** $-5(x + y)$ $-5x - 5y$

For use with pages 88–89

Divide.

A **1.** $72 \div -8$ -9 **2.** $14 \div -14$ -1 **3.** $-63 \div -9$ 7 **4.** $-54 \div 6$ -9

5. $56 \div -8$ -7 **6.** $-24 \div -6$ 4 **7.** $15 \div -5$ -3 **8.** $-30 \div -3$ 10

9. $\frac{40}{-5}$ -8 **10.** $\frac{-18}{-2}$ 9 **11.** $\frac{-16}{4}$ -4 **12.** $\frac{81}{9}$ 9

For use with pages 90–91

Solve for the variable. Check.

A **1.** $n + 3 = 6$ 3 **2.** $n - 3 = 7$ 10 **3.** $x + 9 = 6$ -3

4. $a + 4 = -4$ -8 **5.** $b - 2 = -2$ 0 **6.** $15 = y - 12$ 27

7. $3x = 15$ 5 **8.** $-3x = -15$ 5 **9.** $6x = -24$ -4

10. $33 = -11y$ -3 **11.** $5n - n = -16$ -4 **12.** $9y - 11y = 20$ -10

13. $5n = 50 - 45$ 1 **14.** $3n + 60 = 2n$ -60 **15.** $-3n + 2 = -4n$ -2

16. $3x - 9 = 12x$ -1 **17.** $5n - 4 = 3n + 8$ 6 **18.** $14x - 4 = 13x$ 4

19. $-4x + 1 = -x + 19$ -6 **20.** $6x - 14 = 2x - 2$ 3 **21.** $-7y - 3 = y + 29$ -4

B **22.** $x - (4 - x) = 20$ 12 **23.** $5n - (n + 3) = 25$ 7 **24.** $11x - (x - 20) = 100$ 8

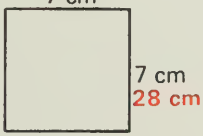
25. $4 - (10 - x) = 34$ 40 **26.** $-10(v + 4) - 20 = 10v$ -3 **27.** $(w - 4) - 3 = 11$ 18

CHAPTER 4

For use with pages 102–103

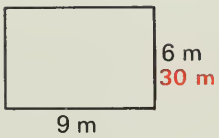
Find the perimeter.

A **1.**



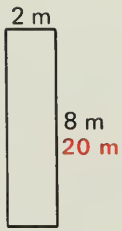
28 cm

2.



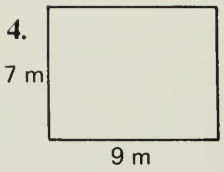
30 m

3.



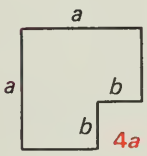
20 m

4.



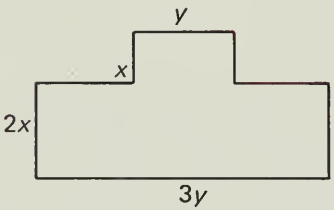
32 m

5.



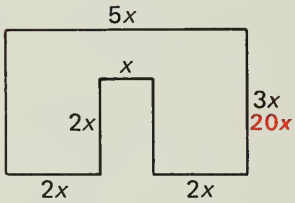
$4a$

6.



$6x + 6y$

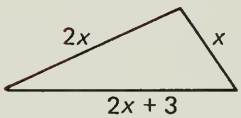
7.



$20x$

8. The length of a rectangle is 4 cm longer than the width. The perimeter of the rectangle is 20 cm. Find its length and its width. length: 7 cm; width: 3 cm

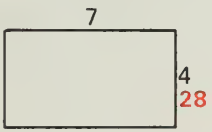
9. The perimeter of the triangle shown is 23. Find the value of x. 4



For use with pages 106–107

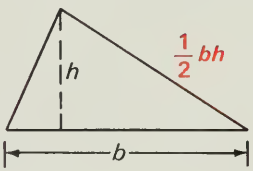
Find the area. Give your answer in terms of the variables.

A **1.**



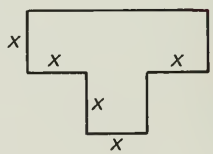
28

2.



$\frac{1}{2}bh$

3.



$4x^2$

4. A rectangular lot measures 30 m by 35 m. Find its value at \$5.50 per square meter. \$5775.00

5. Washable wallpaper costs \$3.95 per square meter. How much would it cost to paper a kitchen wall 3 m high and 4 m long? \$47.40

For use with pages 108–111

A **1.** State the formula for finding the volume of a prism. $V = Bh$

2. The area of the base of a rectangular box is 20 cm^2. Its height is 3 cm. What is its volume? 60 cm^3

3. Find the volume of a cube 6 cm on a side. 216 cm^3

4. Find the volume of a box that measures $6x$ units by $4x$ units by $2x$ units. $48x^3$

For use with pages 112–121

Complete.

A **1.** You walk K hours at h km/h. $D =$ ___?___ km. hK

2. You work x hours at \$2 per hour. You earn ___?___. $\$2 \cdot x$

3. John hikes from Midland to Newton at 20 km/h. He averages 40 km/h returning because of a ride part way. The trip going takes 1 hour more than returning. The distance from Midland to Newton is ___?___. 40 km

	rate ×	time =	Distance
Hike	20	$x + 1$	$20x + 20$
Ride	40	x	$40x$

4. Di earns \$3 an hour and so does her sister Jo. If Di works 3 hours more per week and together they earn \$249 per week, how many hours per week does Jo work? 40 hours per week

	pay per hour ×	number of hours =	pay per week
Di	3	$x + 3$	$3x + 9$
Jo	3	x	$3x$
			249

For use with pages 122–125

A **Solve for x.**

1. $y + x = n$ $n - y$

2. $2x - y = 4$ $\frac{4 + y}{2}$

3. $4y - x = 12$ $4y - 12$

4. $4x = y$ $\frac{y}{4}$

5. $nx = y$ $\frac{y}{n}$

6. $kx = y$ $\frac{y}{k}$

7. $10y + x = 15$ $15 - 10y$

8. $3y - x = 9$ $3y - 9$

9. $3x - y = 0$ $\frac{y}{3}$

10. $5x + 5 = 15y$ $\frac{15y - 5}{5}$

11. $x - y = k$ $k + y$

12. $-x + y = -2$ $y + 2$

For use with pages 136–139

Add.

A

1. $n + 3$ / $3n + 1$ / $4n + 4$
2. $7y^2 + 3$ / $8y^2 + 9$ / $15y^2 + 12$
3. $a^2 + b^2$ / $a^2 - b^2$ / $2a^2$
4. $3x + 7y$ / $-5x + 8y$ / $-2x + 15y$
5. $-4xy - z$ / $4xy + 6z$ / $5z$
6. $a^2 + ab$ / $3a^2 + ab$ / $4a^2 + 2ab$
7. $4x - 6y$ / $-7x + 4y$ / $-3x - 2y$
8. $9x^2 + 4y^2$ / $6x^2 - 8y^2$ / $15x^2 - 4y^2$
9. $(5a - b) + (8a + 4b)$ $13a + 3b$
10. $(x + 2y + 9) + (-3x - y + 7)$ $-2x + y + 16$
11. $(x^2 - 3x + 4) + (2x^2 + 3x - 4)$ $3x^2$
12. $(k^2 + 1) + (-3k^2 + 2k - 1)$ $-2k^2 + 2k$

Subtract.

13. $3a - b$ / $-(2a - b)$ / a
14. $-6n + 5$ / $-(-7n + 2)$ / $n + 3$
15. $y^3 + 4$ / $-(y^3 - 4)$ / 8
16. $3x + y$ / $-(7x - y)$ / $-4x + 2y$
17. $2n^2 + 2n$ / $-(4n^2 + 2n)$ / $-2n^2$
18. $5x - 3$ / $-(3x + 4)$ / $2x - 7$
19. $3a - 7$ / $-(7a + 1)$ / $-4a - 8$
20. $6x^2y^2 + 3$ / $-(x^2y^2 - 3)$ / $5x^2y^2 + 6$
21. $(2x + 1) - (x - 4)$ $x + 5$
22. $(3x - 6y) - (2x - 5y)$ $x - y$
23. $(6a + 2b) - (a - b)$ $5a + 3b$
24. $(7m - 2n) - (m + 3n)$ $6m - 5n$

B

25. Jon walked $2n$ km to the corner, $(n + 1)$ km to the store, and $(2n + 4)$ km back home. How far did he walk? $5n + 5$ km
26. May hiked $(5n + 4)$ km. Jo hiked $(4n - 1)$ km. May hiked how much farther than Jo? $n + 5$ km

For use with pages 140–143

Multiply.

A

1. $y \cdot y^3$ y^4
2. $(3x^2)(7x^2)$ $21x^4$
3. $(-2n)(-n^2)$ $2n^3$
4. $(-4x^2)(7x^3)$ $-28x^5$
5. $x(-xy)$ $-x^2y$
6. $(4x^2y)(3xy^2)$ $12x^3y^3$
7. $n^5 \cdot n^2$ n^7
8. $x^4 \cdot x^3$ x^7

Simplify.

9. $(n^3)^4$ n^{12}
10. $(x^2)^5$ x^{10}
11. $(y^5)^2$ y^{10}
12. $(x^4)^4$ x^{16}
13. $(ab)^2$ a^2b^2
14. $(xy)^2$ x^2y^2
15. $(mn)^4$ m^4n^4
16. $(2a)^2$ $4a^2$
17. $(-xy)^3$ $-x^3y^3$
18. $(3a^2)^2$ $9a^4$
19. $(-3x^2)^3$ $-27x^6$
20. $5(-ab^2)^5$ $-5a^5b^{10}$

B

21. $-(ab^2)(ab)^2$ $-a^3b^4$
22. $(-xy)^2(-xy)$ $-x^3y^3$
23. $(4rs)^3(2s)^3$ $512r^3s^6$
24. $(2xy^2)^2(-3x)^2$ $36x^4y^4$
25. $(3mn)^3(-3mn)$ $-81m^4n^4$
26. $(2x^2y^2)(2x^3y^3)^2$ $8x^8y^8$
27. $(5m^2n^2)^3(-2mn)^6$ $8000m^{12}n^{12}$
28. $-(2ab)^4(4a^4b^4)$ $-64a^8b^8$

For use with pages 144–147

Multiply.

A

1. $3(n + 5)$ $3n + 15$
2. $-4(x - y)$ $-4x + 4y$
3. $6(a^2 + b^2)$ $6a^2 + 6b^2$
4. $-2(a - 3b)$ $-2a + 6b$
5. $a(x - y)$ $ax - ay$
6. $-3x(2x + 3y)$ $-6x^2 - 9xy$
7. $-3(2a + 4b + 6c)$ $-6a - 12b - 18c$
8. $-1(5x + 2y + z)$ $-5x - 2y - z$
9. $4x(x^2y + xy^2 + 3y^3)$ $4x^3y + 4x^2y^2 + 12xy^3$
10. $5a(a^3 + a^2 - a + 1)$ $5a^4 + 5a^3 - 5a^2 + 5a$
11. $-x(3x^3 - 5x^2 + x - 1)$ $-3x^4 + 5x^3 - x^2 + x$
12. $m^2(m^2 + 2mn + n)$ $m^4 + 2m^3n + m^2n$
13. $(5 - a)(4 - a)$ $20 - 9a + a^2$
14. $(n + 5)(n - 3)$ $n^2 + 2n - 15$
15. $(x + 4)(x + 8)$ $x^2 + 12x + 32$
16. $(3a - 1)(a + 1)$ $3a^2 + 2a - 1$
17. $(5x - 2)(5x - 4)$ $25x^2 - 30x + 8$
18. $(n + 6)(n - 9)$ $n^2 - 3n - 54$

B

19. $(a - 1)(a^2 + 2a + 1)$ $a^3 + a^2 - a - 1$
20. $(x - y)(x^2 - 2xy - y^2)$ $x^3 - 3x^2y + xy^2 + y^3$
21. $(m - n)(m^2 - n^2)$ $m^3 - mn^2 - nm^2 + n^3$
22. $(x + y)(x^2 + 2xy + y^2)$ $x^3 + 3x^2y + 3xy^2 + y^3$
23. $(a^2 - 1)(a^2 + 1)$ $a^4 - 1$
24. $(n + 2)(n^2 + 4n + 4)$ $n^3 + 6n^2 + 12n + 8$

For use with pages 148–153

Multiply. You may use the FOIL method.

A

1. $(n + 3)(n + 4)$ $n^2 + 7n + 12$
2. $(x + 6)(x - 3)$ $x^2 + 3x - 18$
3. $(m - 5)(m - 2)$ $m^2 - 7m + 10$
4. $(3x - 2)(2x + 3)$ $6x^2 + 5x - 6$
5. $(5y - 1)(5y + 1)$ $25y^2 - 1$
6. $(7a + 3)(3a - 1)$ $21a^2 + 2a - 3$
7. $(2x + y)(4x - y)$ $8x^2 + 2xy - y^2$
8. $(2a + b)(a - 2b)$ $2a^2 - 3ab - 2b^2$
9. $(5x - y)(3x - y)$ $15x^2 - 8xy + y^2$
10. A rectangle measures $(3n + 1)$ cm by $(2n + 5)$ cm. What is its area? $6n^2 + 17n + 5$ cm^2

Express as a trinomial.

11. $(n + 2)^2$
$n^2 + 4n + 4$

12. $(y - 5)^2$
$y^2 - 10y + 25$

13. $(n + 7)^2$
$n^2 + 14n + 49$

14. $(x - 10)^2$
$x^2 - 20x + 100$

15. $(x + 3y)^2$
$x^2 + 6xy + 9y^2$

16. $(a - 2b)^2$
$a^2 - 4ab + 4b^2$

17. $(a - 10b)^2$
$a^2 - 20ab + 100b^2$

18. $(3m + 2n)^2$
$9m^2 + 12mn + 4n^2$

19. $-1(n + 2)^2$
$-n^2 - 4n - 4$

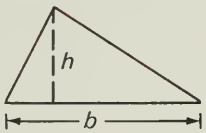
20. $-2(4x + y)^2$
$-32x^2 - 16xy - 2y^2$

21. $4(2r + s)^2$
$16r^2 + 16rs + 4s^2$

22. $-3(2x - 3y)^2$
$-12x^2 + 36xy - 27y^2$

Find the area. Use $A = \frac{1}{2}bh$ or $A = lw$.

B **23.**

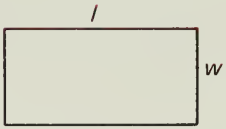


$b = 8n + 1$

$h = 4n$

$16n^2 + 2n$

24.



$l = 3n + 1$

$w = 6n$

$18n^2 + 6n$

For use with pages 154-155, 158-159

Divide.

A **1.** $\frac{3x}{x}$ 3

2. $\frac{a^8}{a^2}$ a^6

3. $\frac{5x}{5}$ x

4. $\frac{4ab}{4b}$ a

5. $\frac{6x^3}{2x}$ $3x^2$

6. $\frac{-12a^5}{2a^3}$ $-6a^2$

7. $\frac{63n^4}{-7n^2}$ $-9n^2$

8. $\frac{-49a^2b^2}{-7ab}$ $7ab$

9. $\frac{4x - 8y}{2}$ $2x - 4y$

10. $-\frac{6a - 6b}{6}$ $-a + b$

11. $\frac{x^2y^2 - 3xy}{xy}$ $xy - 3$

12. $\frac{15m + 30mn}{5m}$ $3 + 6n$

13. $\frac{20n^3 - 10n^2 - 5n}{5n}$ $4n^2 - 2n - 1$

14. $\frac{x^3 + 3x^4 - 5x^5}{-x^2}$ $-x - 3x^2 + 5x^3$

15. $\frac{18a^3b^3 - 12a^2b^2 + 6ab}{6ab}$ $3a^2b^2 - 2ab + 1$

16. $\frac{8x^2y + 12xy^2 - 16y^3}{4y}$ $2x^2 + 3xy - 4y^2$

17. $\frac{a^3 - 6a^5 + 8a^7}{-a^3}$ $-1 + 6a^2 - 8a^4$

18. $\frac{11x^4y^5 + 9x^3y^4 + 3x^3y^3}{x^2y^2}$ $11x^2y^3 + 9xy^2 + 3xy$

Simplify.

B **19.** $\frac{12xy}{4x} - \frac{10y^3}{5y^2}$ y

20. $\frac{-21ab}{7b} + \frac{4a^2}{a}$ a

21. $\frac{3x^3y^5}{xy^3} - \frac{x^4y^5}{x^2y^3}$ $2x^2y^2$

Divide.

22. $\frac{9x^4y^4 - 12x^3y^3 + 21x^2y^2 - 3xy}{-3xy}$ $-3x^3y^3 + 4x^2y^2 - 7xy + 1$

23. $\frac{8a^5b^5 + 6a^4b^6 - a^3b^7 + 3a^2b^8}{a^2b^3}$ $8a^3b^2 + 6a^2b^3 - ab^4 + 3b^5$

CHAPTER 6

For use with pages 170–173

Find the prime factors.

A **1.** 20 2, 2, 5 **2.** 40 2, 2, 2, 5 **3.** 250 2, 5, 5, 5 **4.** 110 2, 5, 11 **5.** 140 2, 2, 5, 7

Find the greatest common factor of each pair.

6. 10, 25 5 **7.** 32, 28 4 **8.** 26, 39 13 **9.** 63, 14 7

10. 15, 90 15 **11.** 40, 48 8 **12.** 70, 105 35 **13.** 55, 242 11

Factor.

14. $14 - 2x$ $2(7 - x)$ **15.** $20n - 4$ $4(5n - 1)$ **16.** $12 - 36x$ $12(1 - 3x)$

17. $2x^2 + 6x$ $2x(x + 3)$ **18.** $y^2 - 3y$ $y(y - 3)$ **19.** $14x^2 + 7x$ $7x(2x + 1)$

20. $10xy - y^2$ $y(10x - y)$ **21.** $11n^2 - 44n$ $11n(n - 4)$ **22.** $6ab^2 + 24a^2b$ $6ab(b + 4a)$

23. $3n^2 - 9n + 12$ $3(n^2 - 3n + 4)$ **24.** $4x^2 + 12xy + 24y^2$ $4(x^2 + 3xy + 6y^2)$ **25.** $7a + 14b + 70c$ $7(a + 2b + 10c)$

B **26.** $5 - 25n + 75n^2 + 125n^3$ $5(1 - 5n + 15n^2 + 25n^3)$ **27.** $6xy + 42x^2y^2 - 66x^3y^3 + 72x^4y^4$ $6xy(1 + 7xy - 11x^2y^2 + 12x^3y^3)$

28. $a^4b - a^3b^2 + a^2b^3 - ab^4$ $ab(a^3 - a^2b + ab^2 - b^3)$ **29.** $12a^2b - 36ab^2 + 48b^2$ $12b(a^2 - 3ab + 4b)$

30. $100x^4y^4 - 75x^3y^3 + 50x^2y^2 + 25xy$ $25xy(4x^3y^3 - 3x^2y^2 + 2xy + 1)$ **31.** $15m^2n^4 + 45m^3n^5 - 60m^4n^6$ $15m^2n^4(1 + 3mn - 4m^2n^2)$

For use with pages 176–181

Factor.

A **1.** $y^2 + 7y + 12$ $(y + 3)(y + 4)$ **2.** $n^2 + 10n + 24$ $(n + 6)(n + 4)$ **3.** $n^2 + 16n + 60$ $(n + 6)(n + 10)$

4. $x^2 + 14x + 33$ $(x + 11)(x + 3)$ **5.** $x^2 + 13x + 36$ $(x + 4)(x + 9)$ **6.** $n^2 + 16n + 55$ $(n + 11)(n + 5)$

7. $b^2 + 19b + 90$ $(b + 9)(b + 10)$ **8.** $m^2 + 15m + 36$ $(m + 12)(m + 3)$ **9.** $n^2 + 13n + 12$ $(n + 12)(n + 1)$

10. $a^2 + 14a + 24$ $(a + 12)(a + 2)$ **11.** $n^2 + 19n + 34$ $(n + 17)(n + 2)$ **12.** $m^2 + 17m + 16$ $(m + 16)(m + 1)$

13. $x^2 + 14x + 13$ $(x + 13)(x + 1)$ **14.** $y^2 + 10y + 16$ $(y + 8)(y + 2)$ **15.** $x^2 + 16x + 28$ $(x + 14)(x + 2)$

16. $b^2 - 10b + 16$
$(b - 8)(b - 2)$

17. $m^2 - 9m + 20$
$(m - 4)(m - 5)$

18. $n^2 - 8n + 15$
$(n - 3)(n - 5)$

19. $y^2 - 14y + 45$
$(y - 9)(y - 5)$

20. $n^2 - 10n + 9$
$(n - 9)(n - 1)$

21. $y^2 - 17y + 60$
$(y - 12)(y - 5)$

22. $x^2 - 19x + 90$
$(x - 10)(x - 9)$

23. $n^2 - 18n + 80$
$(n - 10)(n - 8)$

24. $y^2 - 15y + 44$
$(y - 11)(y - 4)$

B 25. $x^2 + 42x + 80$
$(x + 40)(x + 2)$

26. $x^2 + 29x + 100$
$(x + 25)(x + 4)$

27. $x^2 - 101x + 100$
$(x - 100)(x - 1)$

28. $y^2 - 34y + 93$
$(y - 31)(y - 3)$

29. $m^2 - 32m + 60$
$(m - 30)(m - 2)$

30. $y^2 - 22y + 72$
$(y - 18)(y - 4)$

31. $x^2 + 20x + 64$
$(x + 16)(x + 4)$

32. $n^2 - 40n + 144$
$(n - 36)(n - 4)$

33. $n^2 + 21n + 68$
$(n + 4)(n + 17)$

For use with pages 182–187

Factor. Many are trinomial squares.

A 1. $y^2 - 6y + 9$ $(y - 3)^2$

2. $x^2 + 16x + 64$ $(x + 8)^2$

3. $n^2 - 14n + 49$
$(n - 7)^2$

4. $36 - 12x + x^2$ $(6 - x)^2$

5. $n^2 + 20n + 100$ $(n + 10)^2$

6. $81 - 18x + x^2$
$(9 - x)^2$

7. $n^2 + 5n + 4$ $(n + 4)(n + 1)$

8. $x^2 - 13x + 42$
$(x - 6)(x - 7)$

9. $y^2 - 22y + 121$
$(y - 11)^2$

10. $25 - 10x + x^2$ $(5 - x)^2$

11. $m^2n^2 - 4mn + 4$
$(mn - 2)^2$

12. $b^2 - 10b + 25$
$(b - 5)^2$

Factor. Watch the signs.

13. $y^2 - 7y - 8$ $(y - 8)(y + 1)$

14. $n^2 - 2n - 15$ $(n - 5)(n + 3)$

15. $y^2 + 6y - 40$
$(y - 4)(y + 10)$

16. $n^2 - 8n - 20$
$(n - 10)(n + 2)$

17. $m^2 - 5m - 50$
$(m - 10)(m + 5)$

18. $x^2 - 7x - 60$
$(x - 12)(x + 5)$

19. $n^2 - 8n - 9$
$(n - 9)(n + 1)$

20. $n^2 + 13n - 14$
$(n - 1)(n + 14)$

21. $n^2 - 3n - 4$
$(n - 4)(n + 1)$

For use with pages 188–191

Multiply at sight.

A 1. $(x + 4)(x - 4)$ $x^2 - 16$

2. $(m - n)(m + n)$ $m^2 - n^2$

3. $(1 - 5x)(1 + 5x)$
$1 - 25x^2$

4. $(3y + 1)(3y - 1)$ $9y^2 - 1$

5. $(2x - y)(2x + y)$ $4x^2 - y^2$

6. $(a + 2b)(a - 2b)$
$a^2 - 4b^2$

7. $(5x + 2y)(5x - 2y)$
$25x^2 - 4y^2$

8. $(8m - n)(8m + n)$
$64m^2 - n^2$

9. $(4n + 3)(4n - 3)$
$16n^2 - 9$

10. $(1 - 6a)(1 + 6a)$
$1 - 36a^2$

11. $(9 - 2b)(9 + 2b)$
$81 - 4b^2$

12. $(8x - 3)(8x + 3)$
$64x^2 - 9$

Factor.

13. $x^2 - 25$
$(x - 5)(x + 5)$

14. $n^2 - 36$
$(n - 6)(n + 6)$

15. $x^2 - 100$
$(x - 10)(x + 10)$

16. $4a^2 - 9b^2$
$(2a - 3b)(2a + 3b)$

17. $9x^2 - 36y^2$
$9(x - 2y)(x + 2y)$

18. $m^2 - 49$
$(m - 7)(m + 7)$

19. $16a^2 - 1$
$(4a - 1)(4a + 1)$

20. $64 - 4a^2$
$(8 - 2a)(8 + 2a)$

21. $16s^2 - 4t^2$
$4(2s - t)(2s + t)$

22. $25a^2 - b^2$
$(5a - b)(5a + b)$

23. $121 - 9x^2$
$(11 - 3x)(11 + 3x)$

24. $9b^2 - 4c^2$
$(3b - 2c)(3b + 2c)$

Multiply.

B **25.** $(n^2 + 1)(n^2 - 1)$
$n^4 - 1$

26. $(x^2 - 5)(x^2 + 5)$
$x^4 - 25$

27. $(y^2 + 8)(y^2 - 8)$
$y^4 - 64$

Factor.

28. $-1 + x^2$
$(x - 1)(x + 1)$

29. $-9 + 16y^2$
$(4y - 3)(4y + 3)$

30. $-b^2 + a^2$
$(a - b)(a + b)$

31. $-64 + a^2$
$(a - 8)(a + 8)$

32. $-4m^2 + k^2$
$(k - 2m)(k + 2m)$

33. $-16b^2 + 81a^2$
$(9a - 4b)(9a + 4b)$

34. $m^2 - 144$
$(m - 12)(m + 12)$

35. $n^2 - 625$
$(n - 25)(n + 25)$

For use with pages 192–193

Factor.

A **1.** $2x^2 - 50$
$2(x - 5)(x + 5)$

2. $3y^2 - 12y + 12$
$3(y - 2)^2$

3. $4n^2 + 20n + 16$
$4(n + 4)(n + 1)$

4. $8 + 4n - 4n^2$
$-4(n - 2)(n + 1)$

5. $4n^2 + 12n - 16$
$4(n + 4)(n - 1)$

6. $4x^2 - 64$
$4(x - 4)(x + 4)$

7. $10x^2 - 40$
$10(x - 2)(x + 2)$

8. $a^3 - 16a$
$a(a - 4)(a + 4)$

9. $3n^2 - 9n - 12$
$3(n - 4)(n + 1)$

10. $5n^2 - 40n - 100$
$5(n - 10)(n + 2)$

11. $12x^2 + 36x + 24$
$12(x + 1)(x + 2)$

12. $3x^2 - 24x + 45$
$3(x - 3)(x - 5)$

13. $3n^2 + 12n + 12$
$3(n + 2)^2$

14. $4n^2 + 28n + 40$
$4(n + 5)(n + 2)$

15. $5a^2 - 5b^2$
$5(a - b)(a + b)$

16. $12x^2 + 36x + 24$
$12(x + 2)(x + 1)$

17. $4x^2 - 24x + 36$
$4(x - 3)^2$

18. $7x^2 + 42x + 63$
$7(x + 3)^2$

19. $4n^2 - 8n - 96$
$4(n - 6)(n + 4)$

20. $4x^2 - 36$
$4(x - 3)(x + 3)$

21. $x - 3x^2$
$x(1 - 3x)$

22. $7x^2 - 56x + 49$
$7(x - 7)(x - 1)$

23. $x^2y^2 - y^3$
$y^2(x^2 - y)$

24. $4x^2 + 48$
$4(x^2 + 12)$

B **25.** $10a^2 - 10ab - 60b^2$
$10(a + 2b)(a - 3b)$

26. $3x^2 - 6x - 189$
$3(x - 9)(x + 7)$

27. $-4b^2 + 9a^2$
$(3a - 2b)(3a + 2b)$

28. $-6 - 6x^4$
$-6(1 + x^4)$

29. $4r^2 - 24rs + 32s^2$
$4(r - 2s)(r - 4s)$

30. $9b^2 + 36b - 45$
$9(b - 1)(b + 5)$

31. $5x^2 - 55x - 300$
$5(x - 15)(x + 4)$

32. $24n^2 + 216n - 864$
$24(n + 12)(n - 3)$

33. $x^4 - y^4$
$(x - y)(x + y)(x^2 + y^2)$

CHAPTER 7

For use with pages 204–207

The table below shows the average lifespan of several animals.

Bat, 6 yr.	Fox, 8 yr.	Bear, 31 yr.
Monkey, 7 yr.	Dog, 16 yr.	Elephant, 47 yr.

A **1.** Make a bar graph using the information in the table. Check student's graphs.

2. Use your graph to answer:
a. A bear lives how many years longer than a fox? about 23 years
b. An elephant lives about how many times as long as a monkey? 7

For use with pages 212–215

Plot the following points on squared paper. Check student's graphs.

A **1.** $(3, -2)$ **2.** $(1, -2)$ **3.** $(1, 3)$ **4.** $(-3, -3)$

B **5.** Plot the following points on squared paper. Connect them in order.
Check student's graphs.

$(0, -3), (4, 6), (8, -3)$

For use with pages 216–219

Complete the tables to show four solutions for each equation.

A **1.** $y = x + 4$

x	y	
0	?	4
1	?	5
−1	?	3
2	?	6

2. $y = -x - 1$

x	y	
0	?	−1
−1	?	0
2	?	−3
3	?	−4

3. $y = 3x + 2$

x	y	
0	?	2
1	?	5
3	?	11
4	?	14

4. $y = -2x + 3$

x	y	
0	?	3
1	?	1
−1	?	5
2	?	−1

Solve the equation for y.

5. $x + y = 7$ — $7 - x$

6. $x + y = -7$ — $-7 - x$

7. $y - x = 3$ — $x + 3$

8. $4x + y = 8$ — $8 - 4x$

9. $x - y = 5$ — $x - 5$

10. $2x - y = 9$ — $2x - 9$

11. $10x + y = -2$ — $-10x - 2$

12. $5x - y = 0$ — $5x$

Find three solution pairs for the equation. Answers may vary.

13. $y = x - 3$
(−1, −4); (0, −3); (1, −2)

14. $y + x = 5$
(−1, 6); (0, 5); (1, 4)

15. $y + 2x = 1$
(−1, 3); (0, 1); (1, −1)

16. $y + 4x = 7$
(−1, 11); (0, 7); (1, 3)

17. $2x + y = 0$
(−1, 2); (0, 0); (1, −2)

18. $4x + y = 12$
(−1, 16); (0, 12); (1, 8)

19. $y - 2x = 0$
(−1, −2); (0, 0); (1, 2)

20. $y - 5x = 10$
(−1, 5); (0, 10); (1, 15)

Guess an equation to go with the table.

B **21.**

x	y
0	−5
1	−4
3	−2

$y = x - 5$

22.

x	y
0	0
1	2
−1	−2

$y = 2x$

23.

x	y
0	−1
1	2
2	5

$y = 3x - 1$

24.

x	y
0	3
1	7
2	11

$y = 4x + 3$

For use with pages 220–223

Draw the graph of the equation. Check student's graphs.

A **1.** $y = 5x$

2. $y = x + 5$

3. $y = 3x - 3$

4. $x + y = 7$

5. $y - 2x = 6$

6. $2x + y = 0$

7. $y = -2x$

8. $y = 3x - 1$

9. $y = 2x - 4$

What is the slope of the line when its graph is drawn?

10. $y = 2x$ 2

11. $y = -3x$ −3

12. $y = -x + 4$ −1

13. $y = \frac{1}{2}x$ $\frac{1}{2}$

For use with pages 224–227

State an equation relating x and y. y is a function of x.

A **1.**

x	1	2	3
y	3	6	9

$y = 3x$

2.

x	1	−1	0
y	3	1	2

$y = x + 2$

The distance a car goes in 3 hours is a function of its rate.

3. Complete the table.

Rate (km/h)	Distance (km)
10	30
30	? 90
40	? 120

4. Draw a graph based on the table. Show the rate along the x-axis and the distance along the y-axis as on page 226.

Check student's graphs.

CHAPTER 8

For use with pages 236-239

Solve by the graphing method.

A **1.** $y = -x$
$y = x + 2$ $(-1, 1)$

2. $y = 2x$
$y = 3x - 3$ $(3, 6)$

3. $x + y = 3$
$-2x + y = -3$ $(2, 1)$

4. $3x + 1 = y$
$2x - 1 = y$ $(-2, -5)$

B **5.** $y = x - 3$
$y = 3x + 2$
$\left(-\frac{5}{2}, -\frac{11}{2}\right)$

6. $2x + y = 3$
$x - y = 7$
$\left(\frac{10}{3}, -\frac{11}{3}\right)$

7. $y = 3x - 4$
$y = \frac{1}{2} - 3x$
$\left(\frac{3}{4}, -\frac{7}{4}\right)$

8. $x + y = 0$
$3x - y = 6$
$\left(\frac{3}{2}, -\frac{3}{2}\right)$

For use with pages 240-243

Find the slope of the line whose equation is given.

A **1.** $y = 3x$ 3

2. $y = 2x + 1$ 2

3. $y = -3x - 1$ -3

4. $y = \frac{1}{2}x + 5$ $\frac{1}{2}$

Do the equations share *one* solution pair, *no* solution pair, or *all* solution pairs?

5. $y = 3x$
$y = 3x - 2$ no

6. $x + y = 6$
$x - y = 2$ one

7. $x + y = 0$
$3x + 3y = 0$ all

8. $x + y = 3$
$x + y = 12$ no

For use with pages 244-249

Solve by the substitution method.

A **1.** $y = 3x$ $(3, 9)$
$x + y = 12$

2. $y = 4x$ $(2, 8)$
$y - x = 6$

3. $x - 2y = -4$
$3x + y = 2$ $(0, 2)$

4. $3x - y = 3$
$x + y = 5$ $(2, 3)$

Solve by the addition or subtraction method.

5. $x + y = 4$
$x - y = 6$ $(5, -1)$

6. $2x + 2y = 5$
$2x + 3y = 4$
$\left(\frac{7}{2}, -1\right)$

7. $3x - 2y = 0$
$x - 2y = 4$
$(-2, -3)$

8. $x - 2y = 18$
$5x + 2y = 0$
$\left(3, -\frac{15}{2}\right)$

Solve by the method easiest for you.

9. $3x - 7 = y$
$4x + 2 = y$
$(-9, -34)$

10. $y = 4x$
$3x - y = 0$
$(0, 0)$

11. $x = y$
$3x + 5y = 8$
$(1, 1)$

12. $x - 4y = 3$
$x - 6y = 1$
$(7, 1)$

B **13.** $x - 5y = 0$
$x - 4y = 9$
(45, 9)

14. $3x + 4y = 6$
$x + 4y = 7$
$\left(-\frac{1}{2}, \frac{15}{8}\right)$

15. $5x - y = 5$
$15x + y = 10$
$\left(\frac{3}{4}, -\frac{5}{4}\right)$

16. $4x - 12y = 3$
$4x + 10y = -2$
$\left(\frac{3}{44}, -\frac{5}{22}\right)$

For use with pages 250–257

Solve.

A **1.** $x + 2y = 5$
$2x + 3y = 4$
(−7, 6)

2. $2x - y = 4$
$3x + 2y = 6$
(2, 0)

3. $x - 4y = 2$
$3x - 10y = 4$
(−2, −1)

4. $4x - 5y = 9$
$x + y = 0$
(1, −1)

5. $3x - y = 6$
$x + 2y = 2$
(2, 0)

6. $2x + 3y = 1$
$x + y = 4$
(11, −7)

7. $x - 3y = 4$
$3x - y = 4$
(1, −1)

8. $3x - 5y = 2$
$x - 2y = 5$
(−21, −13)

Use two variables in solving these problems.

9. The sum of two numbers is 15. One number is twice the other. What are the numbers? 5, 10

10. Two hats and 3 ties cost \$15.50. One hat and 2 ties cost \$9.00. What does each item cost? hat: \$4.00; tie: \$2.50

11. Two bats and 1 ball cost \$10. One bat and 2 balls cost \$8. What does a bat cost? \$4.00

12. The sum of two numbers is 15. One number plus twice the other is 25. Find the numbers. 5, 10

13. Al's score is 6 more than Jo's. Their scores total 16. Find their scores. Jo: 5; Al: 11

14. One egg and 2 coffees cost \$1. Two eggs and 1 coffee cost \$1.25. What does one egg plus *one* coffee cost? \$.75

Solve.

B **15.** $2x + 3y = 0$
$3x + 4y = 6$ (18, −12)

16. $4x - 3y = 1$
$3x - 2y = 0$ (−2, −3)

17. $2x + 5y = 0$
$5x - 2y = 0$ (0, 0)

18. $3x - 4y = 5$
$2x - 5y = 1$ (3, 1)

19. $2x - 4y = 4$
$3x - 5y = 1$ (−8, −5)

20. $4x - 3y = -2$
$3x + 4y = 1$
$\left(-\frac{1}{5}, \frac{2}{5}\right)$

21. Tom's club buys some pencils at 25¢ each and some pens at 75¢ each. The total number of pens plus pencils was 25 and the total cost was \$6.75. How many of each did the club buy? 24 pencils; 1 pen

CHAPTER 9

For use after pages 268–273

Simplify.

A **1.** $\frac{4}{16}$ $\frac{1}{4}$ **2.** $\frac{9}{15}$ $\frac{3}{5}$ **3.** $\frac{3}{12}$ $\frac{1}{4}$ **4.** $\frac{4}{20}$ $\frac{1}{5}$ **5.** $\frac{6}{18}$ $\frac{1}{3}$

6. $\frac{5}{25a}$ $\frac{1}{5a}$ **7.** $-\frac{2x}{14x}$ $-\frac{1}{7}$ **8.** $\frac{21r}{7r^2}$ $\frac{3}{r}$ **9.** $\frac{5y}{-20xy^2}$ $\frac{-1}{4xy}$ **10.** $\frac{2mn^2}{10m^2n}$ $\frac{n}{5m}$

11. $\frac{-4xy^2}{12x^2}$ $\frac{-y^2}{3x}$ **12.** $\frac{5x^3y^3}{30xy}$ $\frac{x^2y^2}{6}$ **13.** $-\frac{40n^2s}{5n^3s^2}$ $-\frac{8}{ns}$ **14.** $\frac{6d^2e^2}{24d^4e^4}$ $\frac{1}{4d^2e^2}$ **15.** $\frac{24x^4y^3}{12x^3y^2}$ $2xy$

16. $\frac{n+1}{2n+2}$ $\frac{1}{2}$ **17.** $\frac{3x+3y}{10x+10y}$ $\frac{3}{10}$ **18.** $\frac{x-y}{x^2-y^2}$ $\frac{1}{x+y}$ **19.** $\frac{4a-4b}{12a-12b}$ $\frac{1}{3}$

Write the expression with a factor -1.

20. $1-n$ $-1(-1+n)$ **21.** x $-1(-x)$ **22.** $6-4y$ $-1(-6+4y)$ **23.** $-n^2+z^2$ $-1(n^2-z^2)$

Simplify.

24. $\frac{m-n}{n-m}$ -1 **25.** $\frac{d-z}{z-d}$ -1 **26.** $\frac{3x-3y}{y-x}$ -3 **27.** $\frac{x^2-y^2}{y-x}$ $-x-y$

28. $\frac{4n^2-4}{1-n}$ $-4n-4$ **29.** $\frac{x-3}{9-x^2}$ $-\frac{1}{3+x}$ **30.** $\frac{4n-8}{48-12n^2}$ $-\frac{1}{6+3n}$ **31.** $\frac{1-n}{3n^2-3}$ $-\frac{1}{3n+3}$

B **32.** $\frac{x^2+2x+1}{x^2+6x+5}$ $\frac{x+1}{x+5}$ **33.** $\frac{x^2-7x+12}{x^2-2x-8}$ $\frac{x-3}{x+2}$ **34.** $\frac{x^2-3x-10}{x^2-6x-16}$ $\frac{x-5}{x-8}$

For use with pages 274–283

Write the ratios in simplest form.

A **1.** 18 points to 12 points $\frac{3}{2}$

2. 4 days to 40 days $\frac{1}{10}$

3. \$6 to \$24 $\frac{1}{4}$

4. 9 students to 36 students $\frac{1}{4}$

5. There are 21 students in a math class.
The ratio of girls to boys is 3 to 4.
How many boys in the class? How many girls? 12; 9

Solve the proportion.

6. $\frac{2}{5} = \frac{x}{20}$ 8

7. $\frac{3}{9} = \frac{4}{x}$ 12

8. $\frac{5}{a} = \frac{10}{100}$ 50

9. $\frac{3}{1} = \frac{n}{8}$ 24

10. $\frac{x}{3+x} = \frac{2}{3}$ 6

11. $\frac{2n-3}{n+1} = \frac{3}{4}$ 3

12. $\frac{x+2}{x-2} = \frac{1}{5}$ −3

13. $\frac{2y+6}{y} = \frac{4}{5}$ −5

14. Five apples cost 30¢. How much will 15 apples cost? 90¢

15. You can walk 4 km in 2 hours. How many kilometers can you walk in 3 hours? 6 km

Give the ratio in simplest form. Use the same units in writing the ratio.

B 16. 2 weeks to 21 days $\frac{2}{3}$

17. 50¢ to \$2.50 $\frac{1}{5}$

18. 25 cm to 6 m $\frac{1}{24}$

19. 15 sec to 3 min $\frac{1}{12}$

20. A poll showed that TV audiences favored three programs in the ratio 6 to 5 to 2. Out of a viewing audience of 260,000, how many might be watching each program? 120,000; 100,000; and 40,000

For use with pages 284–293

Simplify.

A 1. $\frac{1}{2} \cdot \frac{1}{8}$ $\frac{1}{16}$

2. $\frac{4}{7} \cdot \frac{21}{2}$ 6

3. $\frac{a}{b} \cdot \frac{2a}{b}$ $\frac{2a^2}{b^2}$

4. $\frac{x^2-y^2}{2x+2y} \cdot \frac{6}{5}$ $\frac{3x-3y}{5}$

5. $\frac{9}{10} \div \frac{3}{20}$ 6

6. $\frac{3x-3}{2x} \div \frac{3}{4x}$ $2x-2$

7. $\frac{x^2-9}{3} \div (x+3)$ $\frac{x-3}{3}$

8. $\frac{14}{x-1} \div \frac{7}{x^2-1}$ $2x+2$

9. $\frac{5n}{3} - \frac{9n}{3}$ $\frac{-4n}{3}$

10. $\frac{12}{10x} - \frac{4}{10x}$ $\frac{4}{5x}$

11. $\frac{3n}{n-1} - \frac{1}{n-1}$ $\frac{3n-1}{n-1}$

12. $\frac{a-2}{a+b} - \frac{a-4}{a+b}$ $\frac{2}{a+b}$

13. $\frac{3}{4} + \frac{1}{2}$ $\frac{5}{4}$ or $1\frac{1}{4}$

14. $\frac{6}{5} + \frac{1}{4}$ $\frac{29}{20}$ or $1\frac{9}{20}$

15. $\frac{x}{2} - \frac{x}{5}$ $\frac{3x}{10}$

16. $\frac{2n}{3} - \frac{n}{2}$ $\frac{n}{6}$

17. $\frac{1}{x} + \frac{3}{y}$ $\frac{y+3x}{xy}$

18. $\frac{2}{a} - \frac{3}{a^2}$ $\frac{2a-3}{a^2}$

19. $\frac{9}{ab^2} - \frac{1}{a^2b}$ $\frac{9a-b}{a^2b^2}$

20. $\frac{3}{2xy} - \frac{2}{xy}$ $-\frac{1}{2xy}$

B 21. $\frac{x^2-y^2}{x^2-64} \cdot \frac{x-8}{x+y}$ $\frac{x-y}{x+8}$

22. $\frac{x-3}{1-x} \cdot \frac{x-1}{x^2-9}$ $-\frac{1}{x+3}$

23. $\frac{3a-6}{5a} \cdot \frac{5a^3-15a^2}{a-2}$ $3a^2-9a$

Solve.

A

1. $\frac{x}{5} - \frac{x}{2} = 30$ −100

2. $\frac{x}{6} - \frac{x}{5} = 3$ −90

3. $\frac{x}{3} = \frac{1}{6} + \frac{1}{2}$ 2

4. $\frac{1}{4} - \frac{1}{6} = \frac{3}{n}$ 36

5. $\frac{2x}{3} - \frac{x}{5} = \frac{14}{15}$ 2

6. $\frac{2}{n} + \frac{3}{4n} = \frac{1}{8}$ 22

7. $\frac{n}{3} + 10 = \frac{4n}{2}$ 6

8. $\frac{x}{2} + \frac{3x}{8} = 7$ 8

9. $\frac{3n}{4} + 5 = \frac{n}{3}$ −12

10. Frank can shovel snow from the driveway in 2 hours. His younger brother Mark would need 3 hours. If they work together, how long will it take? $\frac{6}{5}$ hour or $1\frac{1}{5}$ hour

	Frank	Mark	Together
Hours needed	2	3	h
Part done in one hour	? $\frac{1}{2}$	? $\frac{1}{3}$	? $\frac{1}{h}$

11. Mary would need 4 hours to paper a room. Her assistant, Brenda, would need 6 hours. How long would it take them together? $\frac{12}{5}$ hours or $2\frac{2}{5}$ hours

	Mary	Brenda	Together
Hours needed	? 4	? 6	? h
Part done in one hour	? $\frac{1}{4}$	? $\frac{1}{6}$	? $\frac{1}{h}$

Solve.

B

12. $\frac{x-1}{2} + \frac{x+2}{4} = 9$ 12

13. $\frac{12-x}{3} + \frac{3+x}{2} = \frac{1}{2}$ −30

14. $\frac{n+5}{2} - \frac{n-5}{6} = \frac{5}{8}$ $-\frac{65}{8}$ or $-8\frac{1}{8}$

15. A gardener can do his weekly job in 5 hours. One week he had a helper, and they finished in 3 hours. How long would the helper take working alone? $7\frac{1}{2}$ hours

CHAPTER 10

For use with pages 312–317

Add or subtract.

A **1.** $0.25 + 0.36$ 0.61 **2.** $0.05 + 9.12$ 9.17 **3.** $6.25 - 0.95$ 5.30 **4.** $9.14 - 8.39$ 0.75

5. $0.3 + 0.205$ 0.505 **6.** $6.27 - 4.39$ 1.88 **7.** $5.1 - 3.46$ 1.64 **8.** $2.41 + 1.995$ 4.405

Multiply.

9. 5×9.5 47.5 **10.** 9.3×0.1 0.93 **11.** 6.25×10 62.5 **12.** 100×4.5 450

13. 0.59×6.03 3.5577 **14.** 0.78×1.35 1.0530 **15.** 0.45×9.5 4.275 **16.** 0.7×6.703 4.6921

Divide. Round to one decimal place.

17. $3\overline{)3.7}$ 1.2 **18.** $0.03\overline{)3.74}$ 124.7 **19.** $0.9\overline{)6.17}$ 6.9 **20.** $1.5\overline{)10.06}$ 6.7

21. $6.2\overline{)3.06}$ 0.5 **22.** $7.8\overline{)9.31}$ 1.2 **23.** $0.16\overline{)8.45}$ 52.8 **24.** $0.55\overline{)6.95}$ 12.6

Express the fraction as a decimal. Use three dots with a repeating decimal.

25. $\frac{2}{5}$ 0.4 **26.** $\frac{1}{6}$ 0.16 . . . **27.** $\frac{1}{7}$ 0.142857 . . . **28.** $\frac{5}{6}$ 0.83 . . . **29.** $\frac{5}{7}$ 0.714285 . . .

30. $\frac{7}{8}$ 0.875 **31.** $\frac{1}{12}$ 0.083 . . . **32.** $\frac{1}{3}$ 0.3 . . . **33.** $\frac{2}{3}$ 0.6 . . . **34.** $\frac{1}{4}$ 0.25

For use with pages 318–321

Solve. Round answers to the nearest tenth.

A **1.** $0.20x = 0.1x + 1$ 10 **2.** $0.5 - 0.5n = 0.2n - 3$ 5 **3.** $0.06x = 360$ 6000

4. $0.05y - 5 = 3y$ −1.7 **5.** $0.07x + 25 = 0.124$ −355.4 **6.** $0.01(5 - n) = 0.04(6)$ −19

7. The sum of two numbers is 50. The product of 0.75 and the smaller is the same as the product of 0.5 and the greater. What are the numbers? 20 and 30

B **8.** $0.83n + 0.63(45 - n) = 0.71 - 45$ −363.2 **9.** $0.07(8000 + n) = 600 + 0.05n$ 2000

10. A rectangular garden is 1.4 times as long as it is wide. Its perimeter is 19.6 meters. What is the length of the garden? 5.7 m

For use with pages 322–327

Complete the table.

	Percent	Decimal	Fraction
A 1.	5%	? 0.05	? $\frac{1}{20}$
2.	? 10%	? 0.10	$\frac{1}{10}$
3.	? 80%	? 0.80	$\frac{4}{5}$
4.	?12.5%	0.125	? $\frac{1}{8}$
5.	9%	? 0.09	? $\frac{9}{100}$
6.	50%	? 0.50	? $\frac{1}{2}$
7.	? 6%	0.06	? $\frac{3}{50}$

Compute.

8. 20% of \$5.80 \$1.16 **9.** 10% of \$6.30 \$.63 **10.** 25% of \$5 \$1.25 **11.** 33% of \$9.30 \$3.07

Complete the table.

	On Sale, 50% off!		
	Item	Original price	Discount
12.	Skates	\$35	\$17.50 ?
13.	Cap	\$6.20	\$3.10 ?
14.	Gloves	\$12	\$6.00 ?

	On Sale, $33\frac{1}{3}$% off!		
	Item	Original price	Sale price
15.	Chair	\$15	? \$10
16.	Table	\$45	? \$30
17.	TV	\$210	?\$140

For use with pages 328–331

Solve.

A **1.** What percent of 20 is 4? 20%

2. 10% of \$50 is what amount? \$5

3. 35% of 1000 is what number? 350

4. 5% of what number is 45? 900

5. 60% of what number is 36? 60

6. What percent of 300 is 60? 20%

7. The meals tax in one state is 8%. What is the tax on a bill of \$12.25? \$.98

8. Air conditioning is included on 26% of new cars sold. This means that ___?___ new cars out of every 1,000 have air conditioning. 260

B 9. An automatic cruising control on one model car costs $75. What percent is this of the $3,000 selling price? $2\frac{1}{2}\%$

For use with pages 332–335

A 1. Find the interest on $6,500 at 6%. $390

2. How long will it take $2,000 to double at 5% interest? 20 years

3. How much must Cora invest at 5% to earn $300 in a year? $6000

4. A college invested $10,000, part at 5% and part at 7%. If it receives $620 in interest for a year, how much was invested at each rate? $4000 at 5% and $6000 at 7%

B 5. The Ramseys owe $200. On part of it they must pay 9% interest and on the rest 18%. At the end of the year they must pay $22.50 in interest. How much do they owe at each rate?
$150 at 9% and $50 at 18%

For use with pages 336–339

A 1. The Sports Club took in $4,125 at its annual show. Student tickets were $1.50; all others $2.25. If 2,000 people bought tickets, how many were student tickets? 500

	Number	×	unit price	=	receipts
Student	n		? 1.50		? $1.50n$
Others	$2000 - n$		? 2.25		? $2.25(2000 - n)$
					$4125

2. A solution of salt and water contains 4% salt. How much water should be added to 15 liters of the solution to reduce the salt content to 2%? 15 liters

	Amount of solution	×	% salt	=	Amount salt
Original solution	15		4%		0.04×15
Water added	x		0		? 0
New solution	$15 + x$		2%		? $0.02(15 + x)$

CHAPTER 11

For use with pages 348–353

Find the value. Use the table on page 350 where necessary.

A **1.** $\sqrt{49}$ 7 **2.** $\sqrt{100}$ 10 **3.** $\sqrt{25}$ 5 **4.** $-\sqrt{16}$ -4 **5.** $\sqrt{1}$ 1

6. $-\sqrt{64}$ -8 **7.** $\sqrt{0}$ 0 **8.** $\sqrt{4^2}$ 4 **9.** $\sqrt{6^2}$ 6 **10.** $\sqrt{8^2}$ 8

11. $-\sqrt{12}$ -3.464 **12.** $-\sqrt{7}$ -2.646 **13.** $-\sqrt{3}$ -1.732 **14.** $\pm\sqrt{66}$ ± 8.124 **15.** $-\sqrt{76}$ -8.718

Round the value to one decimal place.

16. $\sqrt{13}$ 3.6 **17.** $\sqrt{98}$ 9.9 **18.** $-\sqrt{52}$ -7.2 **19.** $\sqrt{63}$ 7.9 **20.** $\sqrt{70}$ 8.4

Is the number rational or irrational?

21. $\sqrt{16}$ rational **22.** $\frac{1}{2}$ rational **23.** $\sqrt{2}$ irrational **24.** 10 rational **25.** π irrational

B **26.** The area of a square is 60 cm^2.
Find the length of one side correct to two decimal places. 7.75 cm

For use with pages 354–355

Find the value. Leave your answers in simplest radical form.

A **1.** $\sqrt{500}$ $10\sqrt{5}$ **2.** $\sqrt{320}$ $8\sqrt{5}$ **3.** $\sqrt{6{,}400}$ 80 **4.** $\sqrt{405}$ $9\sqrt{5}$

5. $\sqrt{147}$ $7\sqrt{3}$ **6.** $\sqrt{112}$ $4\sqrt{7}$ **7.** $\sqrt{729}$ 27 **8.** $\sqrt{120}$ $2\sqrt{30}$

B **9.** $\sqrt{81x^2}$ $9x$ **10.** $\sqrt{49x^4}$ $7x^2$ **11.** $\sqrt{5x^4y^2}$ $x^2y\sqrt{5}$ **12.** $\sqrt{10a^2b^3}$ $ab\sqrt{10b}$

For use with pages 356–357

Find the two solutions to the equation.

A **1.** $x^2 = 49$ 7, -7 **2.** $n^2 = 25$ 5, -5 **3.** $y^2 = 36$ 6, -6

4. $\frac{n}{4} = \frac{4}{n}$ 4, -4 **5.** $x^2 = 28$ $2\sqrt{7}$, $-2\sqrt{7}$ **6.** $x^2 + 1 = 65$ 8, -8

Find the two solutions correct to one decimal place.

B **7.** $x^2 = 18$ ± 4.2 **8.** $y^2 = 24$ ± 4.9 **9.** $x^2 - 5 = 15$ ± 4.5

For use with pages 358–361

Solve. Give your results correct to one decimal place.

A **1.** The lengths of the sides of a right triangle are 4 cm and 6 cm. Find the length of the hypotenuse. 7.2 cm

2. The hypotenuse of a right triangle is 15 m long. Find the length of one side if the length of the other is 10 m. 11.2 m

3. The hypotenuse of a right triangle is 9 cm long. Find the length of one side if the length of the other is 5 cm. 7.5 cm

B **4.** A guy wire is needed to support a tree 10 meters tall. It is to be fastened at a point on the ground 5 meters from the foot of the tree. How long must the wire be? 11.2 m

For use with pages 362–367

Simplify. Leave results in simplest radical form.

A **1.** $\sqrt{\frac{1}{9}}$ $\frac{1}{3}$ **2.** $\sqrt{\frac{4}{49}}$ $\frac{2}{7}$ **3.** $\sqrt{\frac{9}{100}}$ $\frac{3}{10}$ **4.** $\sqrt{\frac{n^2}{4}}$ $\frac{n}{2}$ **5.** $\sqrt{\frac{x^2}{25}}$ $\frac{x}{5}$ **6.** $\sqrt{\frac{4n^2}{36}}$ $\frac{n}{3}$

7. $\sqrt{6} \cdot \sqrt{6}$ 6 **8.** $\sqrt{8} \cdot \sqrt{2}$ 4 **9.** $\sqrt{4} \cdot 2\sqrt{9}$ 12

10. $\sqrt{n} \cdot \sqrt{4n}$ $2n$ **11.** $5\sqrt{3x} \cdot 3\sqrt{12x}$ $90x$ **12.** $y\sqrt{6} \cdot 8\sqrt{2y^2}$ $16y^2\sqrt{3}$

B **13.** $\sqrt{3} \cdot \sqrt{5} \cdot \sqrt{15}$ 15 **14.** $-x\sqrt{5} \cdot x\sqrt{75}$ $-5x^2\sqrt{15}$ **15.** $\sqrt{\frac{1}{2}} \cdot \sqrt{8x^4}$ $x^2\sqrt{2}$

Find the decimal value correct to two decimal places.

16. $5\sqrt{\frac{2}{3}}$ 4.08 **17.** $\sqrt{\frac{5}{8}}$ 0.79 **18.** $\sqrt{\frac{1}{2}} \cdot \sqrt{\frac{1}{3}}$ 0.41 **19.** $\sqrt{\frac{1}{6}} \cdot \sqrt{\frac{5}{12}}$ 0.26

For use with pages 368–369

Express in simplest radical form.

A **1.** $3\sqrt{3} - 9\sqrt{3}$ $-6\sqrt{3}$ **2.** $10\sqrt{2} - \sqrt{2}$ $9\sqrt{2}$ **3.** $\sqrt{8} + \sqrt{2}$ $3\sqrt{2}$

4. $5\sqrt{5} + \sqrt{20}$ $7\sqrt{5}$ **5.** $-\sqrt{6} + 3\sqrt{6}$ $2\sqrt{6}$ **6.** $4\sqrt{18} + \sqrt{50}$ $17\sqrt{2}$

7. $-\sqrt{3} + \sqrt{2} + \sqrt{3}$ $\sqrt{2}$ **8.** $6\sqrt{5} - \sqrt{3} + \sqrt{5}$ $7\sqrt{5} - \sqrt{3}$

B **9.** $\sqrt{3n} - \sqrt{27n}$ $-2\sqrt{3n}$ **10.** $\sqrt{n} + 4\sqrt{n^3}$ $(4n + 1)\sqrt{n}$ **11.** $\sqrt{18} - \sqrt{\frac{1}{2}}$ $\frac{5\sqrt{2}}{2}$ **12.** $\sqrt{3} - (\sqrt{12} + \sqrt{45})$ $-\sqrt{3} - 3\sqrt{5}$

CHAPTER 12

For use with pages 380–385

Solve.

A **1.** $n(n + 5) = 0$ 0, −5 **2.** $x(x - 10) = 0$ 0, 10 **3.** $m(m - 2) = 0$ 0, 2

4. $(y + 2)(y + 3) = 0$ −2, −3 **5.** $(n - 5)(n - 2) = 0$ 5, 2 **6.** $(x + 3)(x - 4) = 0$ −3, 4

7. $n^2 + n = 0$ 0, −1 **8.** $x^2 - 4x = 0$ 0, 4 **9.** $y^2 - 9y = 0$ 0, 9

10. $x^2 + 2n - 15 = 0$ −5, 3 **11.** $n^2 + 11n + 24 = 0$ −3, −8 **12.** $y^2 + 3y - 18 = 0$ −6, 3

13. $n^2 = 3n - 2$ 2, 1 **14.** $x^2 + x = 12$ −4, 3 **15.** $y^2 - 10 = -3y$ −5, 2

B **16.** $(4x - 5)(x + 1) = 0$ $\frac{5}{4}$, −1 **17.** $(3n - 18)(4n - 4) = 0$ 6, 1 **18.** $\frac{x}{3} = \frac{5}{x - 2}$ 5, −3

For use with pages 386–389

Solve by using square roots. If there is no solution, write "no solution."

A **1.** $x^2 = 0$ 0 **2.** $2x^2 = 72$ ±6 **3.** $4y^2 = -8$ no solution

4. $x^2 + 64 = 100$ ±6 **5.** $(x - 2)^2 = 36$ −4, 8 **6.** $(2x - 1)^2 = 49$ 4, −3

7. $(x - 4)^2 = 2$ $4 \pm \sqrt{2}$ **8.** $n^2 = 1$ ±1 **9.** $(3n + 1)^2 = 8$ $\frac{\pm 2\sqrt{2} - 1}{3}$

B **10.** $2y^2 + 3 = 19$ $\pm 2\sqrt{2}$ **11.** $3n^2 - 9 = 15$ $\pm 2\sqrt{2}$ **12.** $3n^2 + 11 = 38$ ±3

For use with pages 390–393

Solve by using the quadratic formula. Leave irrational numbers in simplest radical form.

A **1.** $a^2 - 5a + 6 = 0$ 3, 2 **2.** $n^2 + 5n - 3 = 0$ $\frac{-5 \pm \sqrt{37}}{2}$ **3.** $x^2 - 5x + 1 = 0$

4. $x^2 - 2x - 3 = 0$ 3, −1 **5.** $x^2 + x - 20 = 0$ 4, −5 **6.** $3y^2 + 13y + 4 = 0$

7. $x^2 - x - 5 = 0$ $\frac{1 \pm \sqrt{21}}{2}$ **8.** $2x^2 - 11x + 12 = 0$ 4, $\frac{3}{2}$ **9.** $2x^2 - x - 4 = 0$

10. $x^2 - 16 = 0$ 4, −4 **11.** $4n^2 - 3n - 1 = 0$ 1, $\frac{-1}{4}$ **12.** $x^2 - 6x - 2 = 0$ $3 \pm \sqrt{11}$

3. $\frac{5 \pm \sqrt{21}}{2}$ 6. $-\frac{1}{3}$, −4 9. $\frac{1 \pm \sqrt{33}}{4}$

Solve first by the quadratic formula. Then solve by factoring.

B **13.** $x^2 - 5x - 14 = 0$ −2, 7 **14.** $n^2 - n = 0$ 0, 1 **15.** $y^2 - 8y + 16 = 0$ 4

16. $x^2 - 6x + 8 = 0$ 2, 4 **17.** $3x^2 - 9x + 6 = 0$ 1, 2 **18.** $5x^2 - 10x - 75 = 0$ −3, 5

For use with pages 394-395

A **1.** One number is 5 more than another number. Their product is 14. Find the numbers. 7 and 2, or −7 and −2

2. One number is 3 less than another number. Their product is 28. What are the numbers? 4 and 7, or −4 and −7

3. Find two consecutive numbers whose product is 210. 14 and 15, or −14 and −15

4. The width of a rectangle is 2 cm less than its length. Its area is 24 cm^2. Find its dimensions. 6 cm by 4 cm

B **5.** The sum of the squares of three consecutive numbers is 29. Find the numbers. 2, 3, 4 or −2, −3, −4

6. The sum of the length and width of a rectangle is 12 cm. Its area is 32 cm^2. Find its dimensions. 4 cm by 8 cm

For use with pages 396-399

For each equation do these three things:

a. Make a table of values.
b. Draw the graph.
c. Give the coordinates of the vertex.

A **1.** $y = x^2 - 1$ vertex: (0, −1) **2.** $y = x^2 - 4x$ vertex: (2, −4) **3.** $y = -x^2$ vertex: (0, 0)

4. $y = \frac{1}{4}x^2$ vertex: (0, 0) **5.** $y = x^2 - 2x + 1$ vertex: (1, 0) **6.** $y = x^2 - 2x - 3$ vertex: (1, −4)

7. $y = x^2 + 4x + 2$ vertex: (−2, −2) **8.** $y = (2 - x)(2 + x)$ vertex: (0, 4) **9.** $y = x^2 + 4x - 3$ vertex: (−2, −7)

Table of Squares

Number	Square	Number	Square	Number	Square	Number	Square
n	n^2	n	n^2	n	n^2	n	n^2
1	1	**26**	676	**51**	2601	**76**	5776
2	4	**27**	729	**52**	2704	**77**	5929
3	9	**28**	784	**53**	2809	**78**	6084
4	16	**29**	841	**54**	2916	**79**	6241
5	25	**30**	900	**55**	3025	**80**	6400
6	36	**31**	961	**56**	3136	**81**	6561
7	49	**32**	1024	**57**	3249	**82**	6724
8	64	**33**	1089	**58**	3364	**83**	6889
9	81	**34**	1156	**59**	3481	**84**	7056
10	100	**35**	1225	**60**	3600	**85**	7225
11	121	**36**	1296	**61**	3721	**86**	7396
12	144	**37**	1369	**62**	3844	**87**	7569
13	169	**38**	1444	**63**	3969	**88**	7744
14	196	**39**	1521	**64**	4096	**89**	7921
15	225	**40**	1600	**65**	4225	**90**	8100
16	256	**41**	1681	**66**	4356	**91**	8281
17	289	**42**	1764	**67**	4489	**92**	8464
18	324	**43**	1849	**68**	4624	**93**	8649
19	361	**44**	1936	**69**	4761	**94**	8836
20	400	**45**	2025	**70**	4900	**95**	9025
21	441	**46**	2116	**71**	5041	**96**	9216
22	484	**47**	2209	**72**	5184	**97**	9409
23	529	**48**	2304	**73**	5329	**98**	9604
24	576	**49**	2401	**74**	5476	**99**	9801
25	625	**50**	2500	**75**	5625	**100**	10,000

Table of Square Roots

The square roots are given correct to three decimal places.

Number	Positive Square Root	Number	Positive Square Root	Number	Positive Square Root	Number	Positive Square Root
n	$\sqrt{n}$	n	$\sqrt{n}$	n	$\sqrt{n}$	n	$\sqrt{n}$
1	1	**26**	5.099	**51**	7.141	**76**	8.718
2	1.414	**27**	5.196	**52**	7.211	**77**	8.775
3	1.732	**28**	5.292	**53**	7.280	**78**	8.832
4	2	**29**	5.385	**54**	7.348	**79**	8.888
5	2.236	**30**	5.477	**55**	7.416	**80**	8.944
6	2.449	**31**	5.568	**56**	7.483	**81**	9
7	2.646	**32**	5.657	**57**	7.550	**82**	9.055
8	2.828	**33**	5.745	**58**	7.616	**83**	9.110
9	3	**34**	5.831	**59**	7.681	**84**	9.165
10	3.162	**35**	5.916	**60**	7.746	**85**	9.220
11	3.317	**36**	6	**61**	7.810	**86**	9.274
12	3.464	**37**	6.083	**62**	7.874	**87**	9.327
13	3.606	**38**	6.164	**63**	7.937	**88**	9.381
14	3.742	**39**	6.245	**64**	8	**89**	9.434
15	3.873	**40**	6.325	**65**	8.062	**90**	9.487
16	4	**41**	6.403	**66**	8.124	**91**	9.539
17	4.123	**42**	6.481	**67**	8.185	**92**	9.592
18	4.243	**43**	6.557	**68**	8.246	**93**	9.644
19	4.359	**44**	6.633	**69**	8.307	**94**	9.695
20	4.472	**45**	6.708	**70**	8.367	**95**	9.747
21	4.583	**46**	6.782	**71**	8.426	**96**	9.798
22	4.690	**47**	6.856	**72**	8.485	**97**	9.849
23	4.796	**48**	6.928	**73**	8.544	**98**	9.899
24	4.899	**49**	7	**74**	8.602	**99**	9.950
25	5	**50**	7.071	**75**	8.660	**100**	10

Cumulative Review for Chapters 1-3

Basic Skills

Find the value of the expression.

1. $(5 + 3) \cdot 8$ 64
2. $6 - (3 \cdot 1)$ 3
3. $9 + (6 - 3)$ 12
4. 6^2 36
5. 5^1 5
6. 3^3 27
7. $5 \cdot 0$ 0
8. $5 + 0$ 5
9. $\frac{5}{5}$ 1
10. $\frac{5}{1}$ 5
11. $5 \cdot 1$ 5
12. $(5 - 5)(2 \cdot 5)$ 0
13. $-1 + (-4)$ −5
14. $-1(-4)$ 4
15. $-1 - (-4)$ 3
16. $(-4)^2$ 16
17. $-(4)^2$ −16
18. $-(-4)^2$ −16

Find the value of the expression. $a = 1$, $b = 2$, $c = 3$.

19. $a + b$ 3
20. $-a + b$ 1
21. $-ab$ −2
22. $c \div (-a)$ −3
23. $2(a + c)$ 8
24. $2a + c$ 5
25. $4(a - b)$ −4
26. $4a - b$ 2
27. $-abc$ −6

Simplify.

28. $3x + 4x$ $7x$
29. $10b - b$ $9b$
30. $2y - 4y$ $-2y$
31. $4x + 5x - 2$ $9x - 2$
32. $3a - 4a - 7a$ $-8a$
33. $2a + 4 - 2a$ 4
34. $3a - 5a + c - c$ $-2a$
35. $3 \cdot a \cdot a \cdot b \cdot b$ $3a^2b^2$
36. $3(x - 1) + 2$ $3x - 1$
37. $4 \cdot \frac{x}{4}$ x
38. $3x - (-2x)$ $5x$
39. $-(a - 2)$ $2 - a$
40. $(x + y)(-2)$ $-2x - 2y$
41. $4(x - 3) - x + 15$ $3x + 3$
42. $-(x^2 + x - 2)$ $-x^2 - x + 2$

Equations and Inequalities

1. Is $x + 2 = 7$ true if $x = -5$? no
2. Is -1 a solution of $x - 1 = -1$? no

3. **a.** Which of numbers 1, 2, 3, 4 are solutions of $x > 3$? 4
 b. Are there other solutions of $x > 3$? yes
 c. Graph the solution set of $x > 3$ on the number line. Check student's graphs.

Solve.

4. $x - 2 = 4$ 6
5. $y + 6 = 5$ −1
6. $x + 3 = 3$ 0
7. $9 = x + 3$ 6
8. $3x = 18$ 6
9. $-3x = 18$ −6
10. $\frac{x}{2} = 9$ 18
11. $3x - 1 = 20$ 7
12. $5x + 25 = 0$ −5
13. $3 + 3y - y = 21$ 9
14. $7x = 25 + 2x$ 5
15. $3n - n = -16$ −8
16. $2(x - 4) = -4$ 2
17. $5 - (3 - y) = 10$ 8
18. $4 - (2x - 1) = x - 1$ 2
19. $-(y - 4) + 5 = 9y - 1$ 1
20. $\frac{x}{9} = -9$ −81
21. $\frac{x}{-3} = -3$ 9

Write >, <, or = in place of the ?.

22. -2 ? -4 (>)
23. $3 - (-4)$? $-3(-4)$ (<)
24. $6 - (-2)$? $(-2)(-4)$ (=)
25. Can you give a value for $\frac{0}{6}$? for $\frac{6}{0}$? State the value if possible. 0; impossible

Problems

1. A number is doubled. Then 4 is added. The sum is 80. What is the number? 38
2. Find two consecutive whole numbers whose sum is 65. 32, 33
3. Mary has twice as many dimes as Don. Together they have 105 dimes. How many dimes does each have? Mary: 70; Don: 35
4. South High School played 32 games. They won three times as many as they lost. How many did they win? 24
5. Tom says "If I triple the number I'm thinking of and then subtract 5, I get 25." What is the number? 10

Cumulative Review for Chapters 1-6

Basic Skills

Simplify.

1. $15 \cdot \frac{0}{15x}$ 0
2. $-(-3)^2$ -9
3. $-4a + a - 7a$ $-10a$
4. $2(x - 8) + 2x$ $4x - 16$
5. $10x^2 - x^2$ $9x^2$
6. $(-a)^2 + 4a + 1$ $a^2 + 4a + 1$
7. $(x - 2y) + (3x + 2y)$ $4x$
8. $(x^2 - 2x + 1) - (x^2 + 2x - 1)$ $-4x + 2$

Multiply.

9. $4a \cdot a^3$ $4a^4$
10. $(-3xy)(4x^2y^2)$ $-12x^3y^3$
11. $(-3ab)^3$ $-27a^3b^3$
12. $2x(5 - y)$ $10x - 2xy$
13. $(x + 6)(x - 1)$ $x^2 + 5x - 6$
14. $(n - 4)(n + 4)$ $n^2 - 16$
15. $(y + 2)(y - 3)$ $y^2 - y - 6$

Express as a trinomial.

16. $(x - 3)^2$ $x^2 - 6x + 9$
17. $(x + 4)^2$ $x^2 + 8x + 16$
18. $(2x - 3)^2$ $4x^2 - 12x + 9$
19. $(3x - 2y)^2$ $9x^2 - 12xy + 4y^2$

Divide.

20. $\frac{6n^4}{-n^2}$ $-6n^2$
21. $\frac{8n^3}{2n}$ $4n^2$
22. $\frac{-24a^2b^2}{-12ab^2}$ $2a$
23. $\frac{-72xy^2}{8y^2}$ $-9x$
24. $\frac{x^2 - 3x^3 + 5x^4}{-x^2}$ $-1 + 3x - 5x^2$
25. $\frac{5a^2 - a^3 - a^4}{a}$ $5a - a^2 - a^3$
26. $\frac{5xy - 15x^2y^2}{-5x}$ $-y + 3xy^2$

Factor.

27. $3xy - 6x^2y^2$ $3xy(1 - 2xy)$
28. $x^2 + 5x + 6$ $(x + 2)(x + 3)$
29. $y^2 + 7y + 6$ $(y + 1)(y + 6)$
30. $x^2 + 4x + 4$ $(x + 2)(x + 2)$
31. $n^2 + 9n - 22$ $(n + 11)(n - 2)$
32. $x^2 - 9y^2$ $(x + 3y)(x - 3y)$

Equations and Formulas

Solve.

1. $x - 5 = 20$ 25
2. $40 = 20 - n$ -20
3. $4a + 1 = 13$ 3

4. $\frac{n}{4} = -4$ −16

5. $2(x - 1) = 14$ 8

6. $5x + 30 = 3x$ −15

7. $-10 = -3x + 2$ 4

8. $3x - (2x + 2) = 5x + 6$ −2

9. $(2a + 1) - (5a - 1) = -1$ 1

Solve for x.

10. $6x = 24$ 4

11. $6x = y$ $\frac{y}{6}$

12. $kx = y$ $\frac{y}{k}$

Solve for y.

13. $y + 7 = 10$ 3

14. $y + x = 2$ $2 - x$

15. $4x - y = 0$ $4x$

16. Solve $P = 8x$ for x. $\frac{P}{8}$

17. Solve $D = rt$ for r. $\frac{D}{t}$

Problems

1. Find the perimeter of a 4-cm by 4-cm square. 16 cm

2. Find the area of a rectangle with length $2x$ and width $4x$. $8x^2$

3. A triangle has an 8-cm base and a 4-cm height. Find its area. 16 cm^2

4. Write an expression for the volume of a rectangular box whose dimensions are $4x$, $3x$, and $2x$. $24x^3$

5. Find two consecutive numbers whose sum is 25. 12, 13

6. Wendy bought some cupcakes at 12¢ each. She ate two and sold the rest at 20¢ each. Her profit was $1.20. How many cupcakes did she buy? 20

	price × number = Cost		
Sell	? 20	$?n - 2$	? $20(n - 2)$
Buy	? 12	? n	? $12n$
			120

Cumulative Review for Chapters 1-9

Basic Skills

Let $x = 3$. Find the value if possible. If not, write *impossible*.

1. $\frac{x}{0}$ impossible
2. $\frac{x-5}{x-5}$ 1
3. $\frac{x-3}{x-3}$ impossible
4. $\frac{x}{3}$ 1
5. $\frac{0}{x}$ 0

Simplify.

6. $3n - 15n$ $-12n$
7. $x(3xy)(-3y)$ $-9x^2y^2$
8. $5(2a - 3b) + 15b$ $10a$
9. $(2ab^2)^2$ $4a^2b^4$
10. $(-4x^2y^2)^2$ $16x^4y^4$
11. $-10(-3y^2)^3$ $270y^6$
12. $(3x - 4y) - (9x - 4y)$ $-6x$
13. $\frac{4x^2y}{-4y}$ $-x^2$
14. $\frac{x}{3} - \frac{2x}{6}$ 0

Multiply.

15. $(x + 4)(x + 2)$ $x^2 + 6x + 8$
16. $(n - 5)(n + 5)$ $n^2 - 25$
17. $(2x - 1)(4x + 3)$ $8x^2 + 2x - 3$
18. $(2x + 5)^2$ $4x^2 + 20x + 25$
19. $x(x - 2)(x + 3)$ $x^3 + x^2 - 6x$
20. $(2a + b)(3a - 2b)$ $6a^2 - ab - 2b^2$

Factor.

21. $2x^2 - 4y^2$ $2(x^2 - 2y^2)$
22. $3a^2 - 3b^2$ $3(a + b)(a - b)$
23. $n^2 - 3n + 2$ $(n - 1)(n - 2)$
24. $m^2 - 5m - 14$ $(m - 7)(m + 2)$
25. $x^2 + 8x + 16$ $(x + 4)(x + 4)$
26. $x^2 - 6x - 27$ $(x - 9)(x + 3)$
27. $4y^2 - 9$ $(2y + 3)(2y - 3)$
28. $n^2 - 2n - 8$ $(n - 4)(n + 2)$
29. $x^2 + x - 6$ $(x + 3)(x - 2)$

Equations and Graphs

Solve for x.

1. $4(x + 1) = 16$ 3
2. $5x - 15 = 2x$ 5
3. $x + 8 = -3x$ -2
4. $x + 3x + 2 = 18$ 4
5. $6x - 1 = 3x + 11$ 4
6. $-x + 9 = 4x + 4$ 1

Solve for y.

7. $3 + y = x$ $x - 3$
8. $2x + y = 4$ $4 - 2x$
9. $5x - y = 10$ $5x - 10$
10. $x - y = 4$ $x - 4$
11. $y - 2x = 8$ $8 + 2x$
12. $5x + y = 0$ $-5x$

Draw the graph of the equation. Check student's graphs.

13. $y = x - 1$
14. $x + y = 2$
15. $y + 2x = 4$

Give the slope of the line without drawing the graph.

16. $y = -x - 9$ -1
17. $y = \frac{1}{2}x + 2$ $\frac{1}{2}$
18. $y = 2x + 7$ 2

Solve by graphing.

19. $\begin{cases} y - x = 0 \\ y + x = 6 \end{cases}$ (3, 3)
20. $\begin{cases} x + y = 2 \\ x + 2 = y \end{cases}$ (0, 2)
21. $\begin{cases} x + 1 = -y \\ y - x = 3 \end{cases}$ (−2, 1)

Solve by substitution.

22. $\begin{cases} y = x - 1 \\ x + y = 3 \end{cases}$ (2, 1)
23. $\begin{cases} x = 2y \\ x + y = 9 \end{cases}$ (6, 3)
24. $\begin{cases} x = y + 2 \\ y = 3x - 4 \end{cases}$ (1, −1)

Solve by addition or subtraction.

25. $\begin{cases} 5x - y = 1 \\ 3x + y = 15 \end{cases}$ (2, 9)
26. $\begin{cases} 2x - 3y = 1 \\ 3x - 2y = 4 \end{cases}$ (2, 1)
27. $\begin{cases} 3x + y = 4 \\ x + 2y = 3 \end{cases}$ (1, 1)

Problems

1. You can do a typing job in 2 hours. If Jean works alone, it takes 4 hours. How long would it take if you work together? $\frac{4}{3}$ hour or $1\frac{1}{3}$ hour

	You	Jean	Together
Hours needed	2	4	n
Part done in 1 hour	$?\ \frac{1}{2}$	$?\ \frac{1}{4}$	$?\ \frac{1}{n}$

2. *Use two variables for this problem.* The sum of two numbers is 75. Their difference is 25. What are the numbers? 50 and 25

Cumulative Review for Chapters 1-12

Basic Skills

Simplify.

1. $(3x - 4) - (5x + 6)$ $-2x - 10$
2. $(n + 9) + (4n - 18)$ $5n - 9$
3. $(a^2 + 3a + 1) - (a^2 - 2a)$ $5a + 1$
4. $(n^2 - 3) - (n + 8)$ $n^2 - n - 11$
5. $(5x + y) - (6x + y)$ $-x$
6. $3(n + 2) + 4(n - 2)$ $7n - 2$
7. $\dfrac{b^2 - 4}{b + 2}$ $b - 2$
8. $\dfrac{x^2 - 4}{8} \cdot \dfrac{4}{x - 2}$ $\dfrac{x + 2}{2}$
9. $\dfrac{3x^2 - 3}{2} \div \dfrac{x + 1}{16}$ $24x - 24$

Multiply.

10. $(3n - 1)(n + 6)$ $3n^2 + 17n - 6$
11. $(n - 4)(n + 4)$ $n^2 - 16$
12. $(n + 3)(n + 3)$ $n^2 + 6n + 9$
13. $(5n - 2)(2n + 5)$ $10n^2 + 21n - 10$
14. $(4x - 1)(x - 4)$ $4x^2 - 17x + 4$
15. $(2x + 7)(7 - x)$ $-2x^2 + 7x + 49$

Factor completely.

16. $5n^2 + 25n + 100$ $5(n^2 + 5n + 20)$
17. $3n^2 + 9n + 6$ $3(n + 1)(n + 2)$
18. $2n^2 - 8$ $2(n - 2)(n + 2)$
19. $x^2 - 7x + 6$ $(x - 1(x - 6)$
20. $x^2 - 5x - 6$ $(x - 6)(x + 1)$
21. $x^2 - 5x - 14$ $(x - 7)(x + 2)$

Find the square root. Leave your answer in simplest radical form.

22. $\sqrt{\dfrac{2}{16}}$ $\dfrac{\sqrt{2}}{4}$
23. $\sqrt{\dfrac{4y^2}{9}}$ $\dfrac{2y}{3}$
24. $\sqrt{\dfrac{1}{200}}$ $\dfrac{\sqrt{2}}{20}$
25. $\dfrac{1}{4}\sqrt{\dfrac{16}{x^2}}$ $\dfrac{1}{x}$

Simplify.

26. $\sqrt{3} \cdot \sqrt{12}$ 6
27. $3\sqrt{2} \cdot \sqrt{2x^2}$ $6x$
28. $\sqrt{\dfrac{1}{4}} \cdot \sqrt{\dfrac{1}{3}}$ $\dfrac{\sqrt{3}}{6}$
29. $\dfrac{\sqrt{2}}{\sqrt{5}}$ $\dfrac{\sqrt{10}}{5}$
30. $\sqrt{8} - \sqrt{2}$ $\sqrt{2}$
31. $3\sqrt{3} - 2\sqrt{12}$ $-\sqrt{3}$
32. $2\sqrt{27} - \sqrt{3}$ $5\sqrt{3}$
33. $4\sqrt{2} - \sqrt{18}$ $\sqrt{2}$

Equations and Inequalities

1. On the number line, graph the solution of $y < 2$. Check student's graphs.

Solve. (There will be two solutions to some equations.)

2. $3x - 1 = 4x - 9$ 8 **3.** $2(2x - 1) = 6$ 2 **4.** $2n + 3 - 3n + 5 = 0$ 8

5. $\frac{2}{x} = \frac{x}{8}$ 4, −4 **6.** $\frac{n}{3} - \frac{n}{5} = 16$ 120 **7.** $\frac{x}{x+2} = \frac{4}{3}$ −8

8. $a^2 - 3 = 13$ 4, −4 **9.** $y^2 = 50$ $\pm 5\sqrt{2}$ **10.** $0.3x + 0.9x = 3.6$ 3

11. $0.4x = 4.8$ 12 **12.** $0.04(100 - x) = 12 - 0.2x$ 50 **13.** $(n + 4)(n - 6) = 0$ −4, 6

14. $(x + 2)(x - 2) = 0$ 2, −2 **15.** $(3x - 6)(2x + 14) = 0$ 2, −7 **16.** $x(x - 4) = 0$ 0, 4

17. Solve for y: **a.** $3y = 24$; 8 **b.** $ay = 24$; $\frac{24}{a}$ **c.** $ay = c$ $\frac{c}{a}$

Solve for x and y.

18. $\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$ (3, 1) **19.** $\begin{cases} x + 3y = 4 \\ 2x - 5y = -3 \end{cases}$ (1, 1) **20.** $\begin{cases} 4x - 3y = 1 \\ 3x + 4y = 7 \end{cases}$ (1, 1)

Solve by factoring.

21. $n^2 - 7n + 6 = 0$ 1, 6 **22.** $y^2 - 2y - 3 = 0$ 3, −1 **23.** $x^2 - 16 = 0$ ± 4

Solve by the quadratic formula.

24. $2n^2 + 3n + 1 = 0$ $-1, -\frac{1}{2}$ **25.** $3x^2 - 2x - 3 = 0$ $\frac{1 \pm \sqrt{10}}{3}$ **26.** $2n^2 - 3n = 5$ $\frac{5}{2}, -1$

Problems

1. A wholesaler sells coffee at 50¢ a kilogram. She mixes one kind worth 65¢ a kilogram with another worth 40¢. How much of each kind does she use to make 100 kg of the mixture?

	$p \times n = C$		
65¢ Kind	65	n ?	?$65n$
40¢ Kind	40 ?	100?− n	?$40(100 - n)$
Mixture	50 ?	100	? 5000

40 kg of the 65¢ kind; 60 kg of the 40¢ kind

2. A lot is shaped like a right triangle. What is the length of the longest side of the lot if the other sides measure 30 m and 40 m? 50 m

Cumulative Review of Word Problems

Note: Students who have difficulty solving any of these problems should turn to the page number in parentheses for help.

1. I'm thinking of a number. If I add 7 to it and multiply the sum by 4, I get 68. What is the number? (p. 49) 10

2. The Eagles played 27 games and won 5 more games than they lost. How many games did they win? (p. 54) 16

3. Cindy is 6 years older than Phil. In two years the sum of their ages will be 30. How old is Cindy now? (p. 63) 16

4. After Sam and Pat bike in opposite directions for $2\frac{1}{2}$ hours, they are 80 km apart. If Pat's rate is 2 km/h more than Sam's rate, find how far each travels. (p. 114) Sam travels 37.5 km. Pat travels 42.5 km.

5. Tracy bought some donuts at 12 cents each and sold all but 2 for 20 cents each. If her profit was \$1.52, how many donuts did she buy? (p. 119) 24

6. Bob has \$3 more than Carol, and Carol has twice as much as Leon. Together they have \$108. How much does each person have? (p. 55) Bob has \$45. Carol has \$42. Leon has \$21.

7. Five times a number exceeds twice the number by 36. What is the number? (p. 59) 12

8. A woman invests \$6000, part at 6% interest and the rest at 5% interest. Her annual interest is \$335. How much is invested at 6% interest? (p. 334) \$3,500

9. If I use 21 L of gasoline to go 70 km, how many liters will I need to go 100 km? (p. 280) 30 L

10. The ratio of Carl's money to Lisa's money is 4:5. Together they have \$279. How much does Lisa have? (p. 276) \$155

11. Ralph jogs from his home to the stadium at 10 km/h. His trip home takes exactly one hour less because he takes a car traveling at 60 km/h. How far is it from Ralph's home to the stadium? (p. 114) 12 km

12. Bill earns \$1.00 per hour more than Bob. After working for 8 hours last Saturday, their combined pay was \$48. How much does each earn per hour? (p. 120) Bob earns \$2.50/hr. Bill earns \$3.50/hr.

13. It takes Stan 3 hours to mow the lawn and it takes Roger 2 hours. How long would it take them if they worked together? (p. 298)
$1\frac{1}{5}$ hours

14. In order to make a certain kind of concrete, you must mix sand, gravel, and cement in the ratio of 3 to 4 to 2. How much sand will be needed to make 45 cubic meters of concrete? (p. 276) 15 m^3

15. 120% of 55 is what number? (p. 328) 66

16. What percent of 75 is 30? (p. 328) 40%

17. 30% of what number is 72? (p. 328) 240

18. A motorboat went 32 km upstream in 4 hours. The return trip took 2 hours. Find the rate of the current. (p. 258) 4 km/h

19. The sum of 2 numbers is 73, and their difference is 11. Find the numbers. (p. 251) 31, 42

20. I am thinking of two numbers whose difference is 32. Twice the smaller number is 12 more than the larger number. What are the numbers? (p. 251) 44, 76

21. The length of a rectangle is 5 more than twice its width. The perimeter is 46 cm. Find the length and width. (p. 253)
17 cm

22. Find 3 consecutive integers whose sum is 363. (p. 56) 120, 121, 122

23. Two hamburgers and 3 root beers cost \$2.85. Four hamburgers and 5 root beers cost \$5.25. How much does each item cost? (p. 256)
hamburger: 75¢; root beer: 45¢

24. Marcia is 4 years younger than Don. She is also twice as old as Ken. The sum of their ages is 64. How old is Marcia? (pp. 63 and 252) Marcia is 24.

25. The length of a rectangle is 5 cm more than its width. The area is 150 cm^2. Find the length and width. (p. 394) width: 10 cm; length: 15 cm

26. The square of a positive number is 20 more than 8 times the number. Find the number. (p. 395) 10

27. A specialty food store mixes Brazil nuts and cashews to get mixed nuts, which they sell for \$5.00 per kilogram. If the Brazil nuts cost \$4.50 per kilogram and the cashews cost \$6.00 per kilogram, how many kilograms of each should be combined to give 30 kilograms of mixed nuts? (p. 336) 20 kg of Brazil nuts; 10 kg of cashews

28. Tom buys several notebooks for \$2.00 each. He sells some of them for \$3.00 each, but then has to lower his price to \$2.40 to sell the remaining 20. His profit is \$18.00. How many notebooks did he buy? (p. 119) 30

29. Frank needs 5 hours to rake his lawn. But when Sylvia helps him, they can do the job in 3 hours. How long would it take Sylvia to rake the lawn by herself? (p. 301) $7\frac{1}{2}$ hours

30. A baseball team buys 12 dozen game balls and 15 dozen practice balls for \$720. For the same amount of money they could have bought 8 dozen game balls and 20 dozen practice balls. Find the cost of a dozen game balls. (p. 256) \$30

31. A sport coat usually priced at \$48 is on sale at a 20% discount. If the sales tax is 5%, how much do you pay for the coat? (p. 326) \$40.32

32. A pump can fill a 2700 L tank in 36 minutes. How long will it take to fill an 1800 L tank? (p. 282) 24 min

33. The perimeter of a rectangle is 40 cm, and the area is 64 cm^2. Find the dimensions of the rectangle. (p. 395) 16 cm

34. A saltwater solution contains 25% salt. How much water should be added to 12 L of solution to get a mixture which is only 20% salt? (p. 337) 3 L

35. I am thinking of two consecutive even integers. The sum of their squares is 340. What are the integers? (p. 395) 12, 14

Answers to Self-Tests

CHAPTER 1

Page 11

1. 20 **2.** 8 **3.** 42 **4.** 2 **5.** 11 **6.** 8
7. 0 **8.** 7 **9.** $2xy$ **10.** Not Possible
11. $8y$ **12.** $9w - 5$ **13.** $2a - 5$
14. $5m - 3$ **15.** $3x + 2y$ **16.** $6k + 5j$
17. $9x$ **18.** $13a - b - 3$

Page 21

1. r^3s^2 **2.** $6m^2np$ **3.** $30x$ **4.** $28a^2$ **5.** $2a$
6. m **7.** $3x - 6$ **8.** $3y^2 + y$
9. $8a - 12b$ **10.** $18d + 30$

Page 25

1. $<$ **2.** $<$ **3.** $>$ **4.** $<$ **5.** 13 **6.** 7
7. 7 **8.** 7 **9.** 3, 5, 21 **10.** 4, 5

CHAPTER 2

Page 41

1. 22 **2.** 20 **3.** 21 **4.** 13 **5.** 11 **6.** 12
7. 10 **8.** 10 **9.** 23 **10.** 9 **11.** 7 **12.** 8
13. 3 **14.** 12 **15.** 11 **16.** 15 **17.** 42
18. 77

Page 45

1. 2 **2.** 3 **3.** 4 **4.** 6 **5.** 7 **6.** 6 **7.** 9
8. 11 **9.** 4 **10.** 7 **11.** 20 **12.** 5

Page 49

1. 6 **2.** 14 **3.** 3 **4.** 8 **5.** 5 **6.** 11
7. $3x + 5x = 40$; 5 **8.** $2x + 9 = 23$; 7

Page 57

1. $m + 3$ **2.** $x + 3$ **3.** \$69 **4.** 108

Page 63

1. 12 **2.** 5 **3.** 9 **4.** Jo has \$25. Lou has \$50. Sherry has \$14. **5.** Vic has \$30. Al has \$60. Connie has \$35.

CHAPTER 3

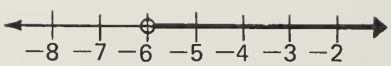
Page 75

1. $>$ **2.** $<$ **3.** $<$ **4.** $<$ **5.** $<$ **6.** $>$
7. $<$ **8.** $>$

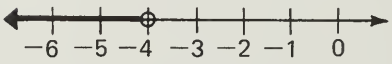
9.



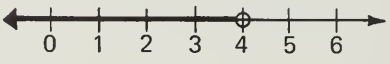
10.



11.



12.



Page 83

1. 6 **2.** -6 **3.** -1 **4.** -2 **5.** 2
6. -11 **7.** 12 **8.** 2 **9.** x **10.** n
11. $-11c$ **12.** $-2m^2$

Page 91

1. $-27x$ **2.** $6ab$ **3.** $-4y + 20$
4. $-5c - 5d$ **5.** $-x - 6$ **6.** $-a - 5$
7. $2y + 7$ **8.** $-x + 3$ **9.** 4 **10.** 3
11. -3 **12.** 3 **13.** -7 **14.** -2 **15.** -3

CHAPTER 4

Page 111

1. 20 **2.** 14 **3.** $b + 2c$ **4.** $\frac{1}{2}ab$
5. 648 cm^3

Page 121

1. km **2.** xy **3.** st **4.** $c - d$
5. 80 km/h **6.** Lou worked 35 hours and Bill worked 41 hours.

Page 125

1. **a.** 5 **b.** $\frac{y}{9}$ **c.** $\frac{y}{a}$ **2.** **a.** 18 **b.** $y - 2$
c. $y - k$ **3.** $x + 10$ **4.** $8 - 2x$
5. $2x - 8$ **6.** $2x + 8$ **7.** $9 - 3x$ **8.** $\frac{12}{x}$

CHAPTER 5

Page 139

1. $7x + 5$ **2.** $2m + 4$ **3.** $-2x + 6$
4. $a^2 - b + 1$ **5.** $5m - 1$
6. $-3b^2 + b - 9$

Page 145

1. $28x^4$ **2.** $-2m^3n^4$ **3.** a^3b^3 **4.** $49x^4y^2$
5. $xy + 7x$ **6.** $m^2 - mn$ **7.** $-a^3 + ab$

8. $-6x + 8y$ **9.** $20x^2 + 5x^3$
10. $3m^5 - m^4 + 2m^3$

Page 153

1. $x^2 + 6x + 8$ **2.** $a^2 - 6a + 5$
3. $y^2 + y - 12$ **4.** $m^2 - 4m - 5$
5. $4a^2 - 8ab - 21b^2$ **6.** $6m^2 + 17m + 5$
7. $a^2 + 8a + 16$ **8.** $n^2 - 6n + 9$
9. $36x^2 + 12xy + y^2$
10. $16a^2 - 56ab + 49b^2$

Page 159

1. 2 **2.** $2a^3$ **3.** $2n$ **4.** $-3y$ **5.** $3m - 4n$
6. $2a + 8$ **7.** $xy - y$ **8.** $a - 2b^2$

CHAPTER 6

Page 175

1. 8 **2.** 8 **3.** $3x$ **4.** $2mn$
5. $5xy^2(3x + 5y)$ **6.** $9mn^2(2 - m)$
7. $8a^2(b^3 + 7c^3)$

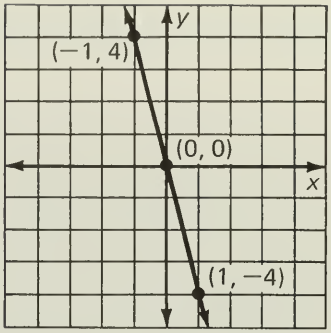
Page 187

1. $(x + 9)(x + 2)$ **2.** $(y + 12)(y + 2)$
3. $(a - 4)(a - 2)$ **4.** $(m + 13)(m - 2)$
5. $(x - 6)(x - 6)$ **6.** $(x + 12)(x - 4)$
7. $(a - 3)(a - 9)$ **8.** $(y - 6)(y - 7)$
9. $(b + 27)(b - 3)$ **10.** $(m - 14)(m - 4)$
11. $(x + 10)(x + 10)$ **12.** $(m + 8)(m + 8)$

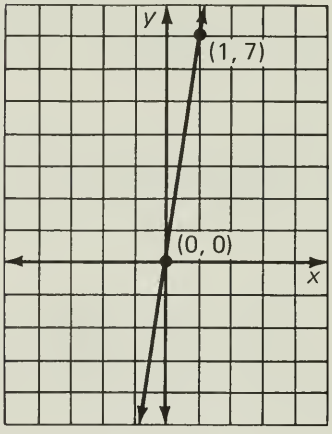
Page 193

1. $m^2 - 169$ **2.** $9 - 16x^2$ **3.** $25a^2 - 49$
4. $(a - 8)(a + 8)$ **5.** $(m - 6n)(m + 6n)$
6. $(4x - 3y)(4x + 3y)$
7. $3(x + 3)(x + 2)$ **8.** $2(y - 5)(y + 3)$
9. $2(m - 2n)(m + 2n)$

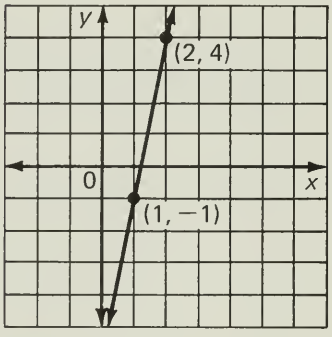
7.



8.



9.



CHAPTER 7

Page 215

1. $(0,-2)$ **2.** $(-2,1)$ **3.** $(-2,0)$
4. $(-1,-2)$ **5.** $(2,-2)$ **6.** $(-2,-2)$
7. H **8.** J **9.** I **10.** L **11.** K **12.** G

Page 227

Note: Answers to Ex. 1-3 may vary.

1. (0,0), (1,5), (2,10) **2.** $(0,-6)$, $(1,-3)$, (2,0) **3.** (0,1), (1,5), (2,9) **4.** 9 **5.** -3
6. -1

CHAPTER 8

Page 243

1. (3,9) **2.** $\left(1\frac{2}{3}, 4\frac{2}{3}\right)$ **3.** $(1,-1)$ **4.** 3
5. 3 **6.** -1 **7.** no **8.** one **9.** all

Page 249

1. (2,3) **2.** (3,1) **3.** $(-6,-8)$ **4.** (2,4)
5. $(1,-1)$ **6.** $(2,-1)$ **7.** (0,2) **8.** $(-2,5)$
9. $(3,-1)$

Page 257

1. (2,0) **2.** $\left(-1\frac{1}{5},1\frac{1}{5}\right)$ **3.** $(-2,1)$

4. Yvette: 15; Lauren: 18 **5.** hamburger: 95¢; milkshake: 60¢

CHAPTER 9

Page 273

1. $\frac{8}{7x}$ **2.** $\frac{-1}{3}$ **3.** $\frac{4}{3s}$ **4.** 5 **5.** $3 - m$

6. $\frac{2}{3}$ **7.** -1 **8.** -2

Page 283

1. 39 cm and 26 cm **2.** $x = 28$ **3.** $x = 8$

4. $a = 3$ **5.** $x = -4$ **6.** 100 km

7. 40 minutes

Page 289

1. $\frac{3}{2}$ **2.** $\frac{3}{2}$ **3.** $\frac{a+1}{3}$ **4.** $\frac{x}{3}$ **5.** $\frac{5m}{3}$

6. $\frac{b}{a^2}$ **7.** $\frac{x-1}{x^2+1}$ **8.** $\frac{9x}{7}$ **9.** $\frac{2}{9a}$

10. $\frac{5a-6}{b}$ **11.** $\frac{2x}{y}$ **12.** $\frac{m-5}{2n}$

Page 293

1. 28 **2.** $4x$ **3.** $3n$ **4.** $5x$ **5.** $\frac{8}{15}$ **6.** $\frac{8x}{21}$

7. $\frac{31}{36a}$ **8.** $\frac{9m-4}{3m^2}$

Page 301

1. $x = 132$ **2.** $a = 5$ **3.** $x = 36$

4. $n = 5$ **5.** $x = 22$ **6.** $x = 14$

CHAPTER 10

Page 321

1. 3.65 **2.** 4.485 **3.** 0.432 **4.** 6.74

5. 0.625 **6.** $x = 5$ **7.** 3 km and 4.5 km

Page 331

1. 52% **2.** 4% **3.** 60% **4.** 75%

5. $37\frac{1}{2}\%$ **6.** 0.35 **7.** 0.14 **8.** 0.07

9. 0.65 **10.** 0.0125 **11.** 19.5 **12.** 9.75

13. 20% **14.** 25 **15.** 44¢ **16.** $66\frac{2}{3}\%$

Page 339

1. \$500 **2.** 50 kilograms of each kind

CHAPTER 11

Page 355

1. 7 **2.** 1 **3.** -3 **4.** -10 **5.** 100

6. $5\sqrt{3}$ **7.** $3\sqrt{10}$ **8.** 60 **9.** rational

10. rational **11.** rational **12.** irrational

Page 361

1. ± 7 **2.** ± 10 **3.** ± 6 **4.** ± 4 **5.** ± 5

6. ± 3 **7.** 15 **8.** $2\sqrt{13}$ **9.** $4\sqrt{2}$

Page 369

1. $\frac{1}{2}$ **2.** $\frac{\sqrt{5}}{4}$ **3.** $\frac{2\sqrt{2}}{5}$ **4.** $\frac{2\sqrt{5}}{7}$

5. $4\sqrt{10}$ **6.** $\sqrt{7}$ **7.** $36\sqrt{2}$ **8.** $10\sqrt{3}$

9. $4\sqrt{2}$ **10.** $-2\sqrt{3}$

CHAPTER 12

Page 385

1. 0 or 3 **2.** 0 or -2 **3.** 0 or -7

4. 6 or 3 **5.** -4 or 3 **6.** 0 or 3

7. 0 or 5 **8.** -3 or -5 **9.** -6 or 2

10. 0 or -8 **11.** -6 or 5 **12.** 7 or 4

Page 389

1. ± 7 **2.** ± 8 **3.** $\pm\sqrt{17}$ **4.** $\pm\sqrt{7}$

5. $\pm\sqrt{6}$ **6.** $\pm 2\sqrt{7}$ **7.** $\pm 2\sqrt{10}$

8. 5, -7 **9.** 11, -5 **10.** -2, -12

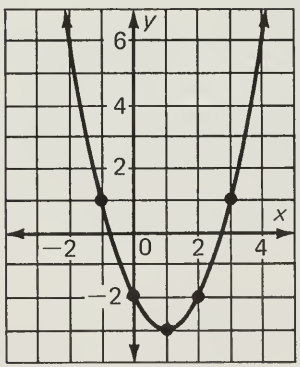
11. -1, -5 **12.** 6, -3

Page 399

1. $\frac{-7 \pm \sqrt{89}}{2}$ **2.** $-1 \pm \sqrt{7}$

3. $\frac{1}{3} \pm \frac{\sqrt{19}}{3}$ **4.** length: 9 cm; width: 5 cm

5.



Glossary

A

absolute value (p. 92) $|-5| = 5$; $|5| = 5$; $|0| = 0$ The symbol $|\,|$ means "absolute value of".

approximate values (p. 350) Values which are not exact.

In $\frac{1}{3} = 0.33\ldots$, the 0.33 is an approximate value.

area (p. 106) The amount of surface of a figure. Area is measured in square units.

associative property of addition (p. 10) $(a + b) + c = a + (b + c)$. It tells us that the way terms are grouped makes no difference.

associative property of multiplication (p. 14) $(ab)c = a(bc)$. The way the factors are grouped makes no difference.

average (p. 94) The sum of n numbers divided by n. If $n = 3$ and the numbers are 11, 20, and 35, the average is $\frac{11 + 20 + 35}{3} = \frac{66}{3} = 22$.

B

bar graph (p. 204) A statistical graph in which bars of different lengths stand for quantities.

binomial (p. 136) A polynomial with two terms. $2x + 3y$ is a binomial.

broken-line graph (p. 208) A statistical graph in which points are joined by straight lines.

C

centimeter (p. 28) $\frac{1}{100}$ of a meter.

circle graph (p. 324, 325) A circular region which shows the relation of parts to a whole.

coefficient (p. 8) The number 5 is the (numerical) coefficient of x in the expression $5x$.

common factor (p. 170) A number which is a factor of two or more numbers. 2 is a common factor of 4 and 14.

common monomial factor (p. 172) A monomial which is a common factor of each of the terms of a polynomial.

commutative property of addition (p. 10) The property $a + b = b + a$. It tells us that the order of terms makes no difference.

commutative property of multiplication (p. 14) $ab = ba$. The order of the factors makes no difference.

compound interest (p. 340) When interest is added to the principal to form a new principal, we say that interest is compound.

consecutive numbers (p. 56) Numbers which follow each other in order.

coordinates (p. 212) The numbers in an ordered pair representing a point; in $(3, -2)$, 3 and -2 are coordinates of point $(3, -2)$.

cubed (p. 12) The third power of x, namely x^3, may be read "x cubed."

D

denominator (p. 268) In the fraction $\frac{1}{x}$, x is called the denominator.

distributive property (p. 16) $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$.

E

equation (p. 22) Any number sentence containing the $=$ sign. $6 + 4 = 10$ and $x + 2 = 7$ are equations.

evaluate (p. 14) To find the value of.

exponent (p. 12) In a power, the number of times the base appears as a factor. In 5^3, the number 5 is the base and 3 is the exponent.

F

factor (p. 12) In a multiplication, each number multiplied is a factor of the product. In $2 \cdot 4$, 2 and 4 are factors of the product 8. In $2y$, 2 and y are factors of $2y$.

formula (p. 102) A rule for finding a desired value. $P = 4s$ is a formula for the perimeter P of a square when the length of one side s is known.

function (p. 224) A quantity which depends on a second quantity is a function of it.

G

gram (p. 64) A metric unit of quantity.

graph of an equation (p. 220) The line representing all the solutions of an equation.

graph of a solution (p. 74) The point or points on the number line paired with the solution of an equation or inequality.

greatest common factor (p. 171) The greatest number which is a common factor of two or more numbers. 9 is the greatest common factor of 18 and 81.

H

horizontal axis (p. 212) Horizontal number line on which one can locate the first number in an ordered pair.

hypotenuse (p. 358) Side opposite the right angle in any right triangle.

I

imaginary numbers (p. 393) A number such as $\sqrt{-1}$ is an imaginary number.

inequality (p. 24) A number sentence stating that two numbers are not equal. $a > 6$, $b < 2$, $6 \neq 3$ are all inequalities.

integers (p. 72) The positive and negative whole numbers and zero; . . . -3, -2, -1, 0, 1, 2, 3 . . .

interest (p. 332) Money paid for the use of money.

intersection (p. 236) The point where two lines cross.

irrational numbers (p. 352) Numbers which cannot be expressed as the ratio of two integers. $\sqrt{2}$ and $\sqrt{7}$ are irrational numbers; 2 and 7 are not perfect squares. The decimal form of an irrational number never repeats and never ends.

K

kilogram (p. 64) 1000 grams = 1 kilogram.

kilometer (p. 29) 1000 meters = 1 kilometer.

L

law of Pythagoras (p. 358) In any right triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the other two sides.

like terms (p. 8) $2x$ and $5x$ are like terms. The variable part of each term is the same.

liter (pp. 89, 110) A metric measure of capacity.

M

meter (p. 28) The basic unit of length in the metric system.

milligram (p. 64) $\frac{1}{1000}$ of a gram.

millimeter (p. 28) $\frac{1}{1000}$ of a meter.

monomial (p. 136) A polynomial of one term. $5x$ is a monomial.

N

negative numbers. (p. 72) All numbers less than zero.

number line (p. 72) A line on which numbers may be pictured.

numerator (p. 268) In the fraction $\frac{2}{3}$, 2 is called the numerator.

O

opposite of a number (p. 78) The opposite of 8 is -8; the opposite of -4 is 4.

ordered pair (pp. 212, 216) A pair of numbers in which their order is important. The point (2,5) is not the same as the point (5,2).

origin (p. 212) Point (0,0) intersection of horizontal axis and vertical axis.

P

parabola (p. 396) The graph of a quadratic function.

parallel lines (p. 240) Lines (in a plane) which never intersect.

parentheses (p. 6) The marks () used to group terms.

percent (p. 322) A number like 25% is called a percent. It expresses the ratio of 25 to 100. $25\% = \frac{25}{100}$.

perfect square (p. 348) Any number which is the square of an integer.

perimeter (p. 104) The distance around a figure.

pi (π) (pp. 174, 352) The Greek letter standing for the ratio of the circumference of a circle to its diameter, approximately 3.14 or $\frac{22}{7}$.

pictograph (p. 204) A kind of bar graph in which rows of symbols are used instead of bars.

plotting a point (p. 213) Locating it by its coordinates.

polynomial (p. 136) An algebraic expression of one or more terms connected by $+$ or $-$ signs. $3x + y$ and $x^2 + 2xy + y^2$ are polynomials.

positive numbers (p. 72) All numbers greater than zero.

power (p. 12) A product in which all the factors, except 1, are the same. x^1, x^2, and x^3 are called powers of x.

prime factors (p. 170) Factors which are prime numbers.

prime number (p. 170) A number with no factors except itself and 1. 2 and 5 are prime numbers.

prism (p. 108) A 3-dimensional figure with two parallel bases. An ordinary box is an example of a prism.

proportion (p. 278) A statement that two ratios are equal. $\frac{4}{6} = \frac{2}{3}$ is a proportion.

Q

quadratic equation (p. 382) If an equation in x has an x^2 term but no term with a higher power of x, it is called a quadratic equation. $x^2 + 6x + 5 = 0$ is a quadratic equation but $x^3 + x^2 + 1 = 0$ is not.

quadratic formula (p. 390) The formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ may be used to solve an equation of the form $ax^2 + bx + c = 0$.

quadratic function (p. 396) The equation $y = x^2 - 1$ is an example of such a function. y depends on the value of x.

R

radical (p. 348) A number in which the radical sign $\sqrt{\ }$ appears. $\sqrt{5}$ is a radical.

radius of a circle (p. 174) The distance from the center to the circle.

ratio (p. 274) The ratio of two numbers is their quotient. The ratio of 6 to 4 is $\frac{6}{4}$, or $\frac{3}{2}$.

rational numbers (pp. 316, 352) Fractions whose numerators and denominators are integers.

$\frac{1}{2}$, $\frac{2}{1}$, and $\frac{-5}{7}$ are rational numbers.

rationalizing the denominator (p. 366) When a fraction has a radical in the denominator, it can be simplified by multiplying both numerator and denominator by a number that will make the denominator a whole number. This is called "rationalizing the denominator."

$$\frac{1}{\sqrt{3}} = \frac{1 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{\sqrt{3}}{3}.$$

real numbers (p. 352) The positive numbers, negative numbers, and zero.

reciprocals (p. 286) $\frac{3}{4}$ and $\frac{4}{3}$ are reciprocals. Their product is 1.

repeating decimal (p. 316) When a fraction is changed to decimal form, sometimes a digit or group of digits repeats without end. Such a decimal is called a repeating decimal.

In $\frac{2}{3} = 0.666\ldots$, the 6 repeats indefinitely. In $\frac{3}{11} = 0.2727\ldots$, the 27 repeats indefinitely.

right angle (p. 358) An angle of 90°.

right triangle (p. 358) Triangle with a right angle.

root of an equation (p. 22) A value of the variable which makes the equation true. 4 is a root of $2 + x = 6$.

S

simplify (p. 9) To replace a number or expression by the simplest equivalent one.

slope (pp. 222, 240) The ratio of rise to run; the coefficient of x when an equation is solved for y. In the equation $y = \frac{1}{2}x + 2$, $\frac{1}{2}$ is the slope.

solution of an equation (p. 22) A value of the variable which makes the equation true. 2 is a solution of $x + 2 = 4$.

solution of two equations (p. 236) The ordered pair which is the solution of both equations. (1,3) is the solution of $y = 3x$ and $y = -x + 4$.

solution pair (p. 216) A pair of numbers which is a solution of an equation in two variables.

square root (p. 348) Each of the numbers 5 and -5 is a square root of 25. 5 is the positive, or principal, square root of 25. -5 is the negative square root of 25. $\sqrt{25}$ means the positive square root. $-\sqrt{25}$ means the negative square root. $\pm\sqrt{25}$ means both roots.

squared (p. 12) The second power of x, namely x^2, may be read "x squared."

T

term (p. 8) In the expression $2a + 3b - 4c$ the terms are $2a$, $3b$, and $4c$.

terminating decimal (p. 316) When a fraction is changed to its decimal form, the decimal terminates if the division "comes out even." Such a decimal is called a terminating decimal.

In $\frac{3}{4} = 0.75$, the 0.75 is a terminating decimal.

trinomial (p. 136) A polynomial with three terms. $a^2 + 2ab - 4b^2$ is a trinomial.

trinomial square (pp. 152, 182) The square of a binomial. $(x + 2)^2 = x^2 + 4x + 4$ (a trinomial square).

U

unlike terms (p. 8) $6x + 3y$ contains unlike terms. Their variable parts are not the same.

V

value of an expression (p. 3) The value when numbers are substituted for letters in the expression.

variable (p. 3) A letter used to represent a number.

variable expression (p. 3) An expression containing a variable.

vertex of a parabola (p. 396) Highest or lowest point.

vertical axis (p. 212) Vertical number line on which one can locate the second number in an ordered pair.

volume (p. 108) The amount of space in a 3-dimensional figure. Volume is measured in cubic units.

X

x-axis (p. 220) Horizontal axis.

Y

y-axis (p. 220) Vertical axis.

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Answers to Odd-Numbered Exercises

CHAPTER 1

Pages 4–5

1. 1 **3.** 10 **5.** 3 **7.** 10 **9.** 16 **11.** 0
13. 9 **15.** 24 **17.** 24 **19.** 4 **21.** 17
23. 10 **25.** 10 **27.** 10 **29.** 26 **31.** 96
33. $x + 7$ **35.** $2n$ **37.** ba
39. Subtract 4. **41.** Divide by 8. **43.** t

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1. 24 **3.** 36 **5.** 2 **7.** 17 **9.** 7 **11.** 31
13. 14 **15.** 18 **17.** 9 **19.** 20 and 14
21. 40 and 68 **23.** 12 and 18 **25.** 6, 13
27. $2x, 2x + 7$ **29.** 11, 22

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1. $6x$ **3.** $10a$ **5.** $3y$ **7.** $7a + 6$ **9.** $12a$
11. $4 + 11y$ **13.** $6s - 2$ **15.** $3 + 9y$
17. $4a - 4$ **19.** $4 + 4x$ **21.** $5xy + 7$
23. $8d$ **25.** $5y + 9x$ **27.** $5ab + 5b$
29. Not possible **31.** Not possible
33. $5ar$ **35.** Not possible

Page 11

1. 46 **3.** 143 **5.** 1571 **7.** $8n + 6$
9. $7c + 4$ **11.** $31 + 6a$ **13.** $10m + 11$
15. $ab + 10$ **17.** $4s + 5$ **19.** $4k + 4n$
21. $5n - 9b$ **23.** $7a + 6b$
25. $8a + 5b + 4$ **27.** $7d - 7a$
29. $3a + 4b$

Page 13

1. 9 **3.** 4 **5.** 100 **7.** y^4 **9.** x^2y^3 **11.** rs^3
13. t^2m^2 **15.** f^3g^2h **17.** de^2f^2 **19.** $7m^3$
21. $2a^2m^2$ **23.** $5n^3p$ **25.** $6a^2b$

Page 15

1. 2900 **3.** 3300 **5.** 730 **7.** $6x$ **9.** $15a$
11. $2b^2$ **13.** $18y$ **15.** $20n$ **17.** $72f$
19. $4a^2$ **21.** $8ab$ **23.** $16x^2$ **25.** $35a^3$
27. $10b^3$ **29.** $6x$ **31.** $2x^2$
33. Multiply by 4.

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1. $3(4 + 5) = 3 \cdot 9 = 27$ and $12 + 15 = 27$ **3.** $5(7 - 3) = 5 \cdot 4 = 20$ and $35 - 15 = 20$ **5.** $2x + 6$ **7.** $4x + 8$
9. $7a + 21$ **11.** $9x + 36$ **13.** $3x - 6$
15. $c^2 - 3c$ **17.** $14x + 7$ **19.** $15x - 10$
21. $6x + 8y$ **23.** $3x^2 - xy$ **25.** $2x + 13$
27. $7x + 15$ **29.** $36 + 5y$ **31.** $8x + 23$
33. $8 + 32x$ **35.** $20b + 4$

Page 19

1. 0 **3.** 35 **5.** 0 **7.** Impossible **9.** 0
11. 0 **13.** $8x$ **15.** $6c$ **17.** $15a$ **19.** x
21. x **23.** x **25.** Add 7.
27. Subtract 9. **29.** Add 16. **31.** Add 11.

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1. 1 **3.** 1 **5.** 1 **7.** 3 **9.** c **11.** 4 **13.** x
15. a **17.** x **19.** x **21.** Divide by 6.
23. x **25.** Multiply by 7.

Page 23

1. 3 **3.** 12 **5.** 2 **7.** 4 **9.** 3 **11.** 5
13. 5 **15.** Answers may vary.
17. Answers may vary. **19.** 8 **21.** 6
23. 8 **25.** Answers may vary. **27.** 40
29. Answers may vary. **31.** 2 **33.** 0

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1. $<$ **3.** $>$ **5.** $>$ **7.** $>$ **9.** 4
11. 1, 3, 7 **13.** 20 **15.** 6, 2 **17.** 4, 6
19. 10, 11 **21.** 2, 5 **23.** 4, 5, 6 **25.** $<$
27. $=$ **29.** $<$

Page 26

1. 56 **3.** 993 **5.** 229 **7.** 712 **9.** 822
11. 34 **13.** 77 **15.** 535 **17.** 354
19. 217 **21.** 288 **23.** 1825 **25.** 2040
27. 43,680 **29.** 75,705 **31.** 17
33. 68R2 **35.** 74R1 **37.** 237R5
39. 104R13 **41.** 221R14

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1. length: about 23 cm, width: about 19 cm. **3.** Answers may vary; for example, 150 cm.

Pages 30–31

1. 11 **3.** 2 **5.** 26 **7.** 15 **9.** 3 **11.** 35
13. 14 **15.** 21 **17.** 7 **19.** 6 **21.** 24
23. 16 **25.** $7x$ **27.** $7b + 2$ **29.** $6c + 3$
31. $8x + 3$ **33.** $10a + b$
35. $8x + 3y - 3$ **37.** 16 **39.** 125
41. 25 **43.** 49 **45.** $10x$ **47.** $16b$
49. $6b^3$ **51.** $6x^2$ **53.** $6n^3$ **55.** $30s^2$
57. $2x + 10$ **59.** $20x - 10y$
61. $8c - 12d$ **63.** $3a - 18b$

65. $3a + 6b$ **67.** $18x - 36y$
69. $15m + 5n$ **71.** $20m + 28k$ **73.** 0
75. Impossible **77.** Impossible **79.** 0
81. $8x$ **83.** a **85.** $6x$ **87.** $10s$ **89.** $5n$
91. 0 **93.** 4 **95.** 3 **97.** 8 **99.** 10 **101.** 2

CHAPTER 2

Page 35

1. 16 **3.** 10 **5.** 12 **7.** 13 **9.** 24 **11.** 19
13. 15 **15.** 9 **17.** 9 **19.** 30 **21.** 25
23. 8 **25.** 13 **27.** 5 **29.** 35 **31.** 51
33. 10 **35.** 6 **37.** 11 **39.** 21 **41.** 15
43. 9 **45.** 15 **47.** 25

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1. 10 **3.** 8 **5.** 4 **7.** 12 **9.** 14 **11.** 15
13. 17 **15.** 12 **17.** 5 **19.** 17 **21.** 12
23. 13 **25.** 10 **27.** 13 **29.** 15 **31.** 8

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1. 9 **3.** 18 **5.** 11 **7.** 18 **9.** 6 **11.** 5
13. 6 **15.** 17 **17.** 19 **19.** 29 **21.** 33
23. 30 **25.** 4 **27.** 17 **29.** 27 **31.** 46

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1. 4 **3.** 7 **5.** 12 **7.** 5 **9.** 10 **11.** 3
13. 3 **15.** 7 **17.** 9 **19.** 12 **21.** 13
23. 14 **25.** 180 **27.** 25 **29.** 18 **31.** 44
33. 54 **35.** 12 **37.** 15 **39.** 56 **41.** 48
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1. 11 **3.** 5 **5.** 4 **7.** 7 **9.** 20 **11.** 6
13. 23 **15.** 14 **17.** 6 **19.** 11 **21.** 21
23. 21 **25.** 2 **27.** 21 **29.** 10 **31.** 5
33. 14 **35.** 27 **37.** 51 **39.** 18 **41.** 30
43. 5 **45.** 3 **47.** 21 **49.** 20 **51.** 13
53. 19 **55.** 21 **57.** 19 **59.** 28 **61.** 6

Puzzles and Things, page 41

The first equation is true only in the sense that both glasses have the same amount in them. Multiplying by 2, or doubling the amount, results in a full glass in either case.

Pages 43–44

1. $5x = 10$; $x = 2$ **3.** $3b = 3$; $b = 1$
5. $2a = 18$; $a = 9$ **7.** $4c = 12$; $c = 3$
9. $6n = 18$; $n = 3$ **11.** 3 **13.** 3 **15.** 4
17. 2 **19.** 5 **21.** 5 **23.** 6 **25.** 5 **27.** 10
29. 5 **31.** 2 **33.** 5 **35.** 3 **37.** 3 **39.** 7
41. 6 **43.** 3 **45.** 5 **47.** 7 **49.** 6 **51.** 19
53. 4 **55.** $2n + 8 = 32$; $n = 12$
57. $2n + 4 = 16$; $n = 6$
59. $4n + 5 = 17$; $n = 3$

Page 45

Across **1.** 15 **3.** 302 **5.** 4329 **11.** 172
13. 865 **25.** 2042 **27.** 500 *Down*
3. 3921 **7.** 27 **9.** 41 **15.** 3204
17. 1120 **19.** 46 **21.** 925 **23.** 10

Page 47

Arithmetic: 40, 44, 880, 800; Algebra: $5n$, $5n + 4$, $100n + 80$, $100n$ **3.** 9 **5.** 12
7. If you use algebra to solve this problem, you will notice that the next to the last step is $n + 20$. It doesn't matter what number you begin with. Subtracting the original number from $n + 20$ will always obtain 20.

Page 48

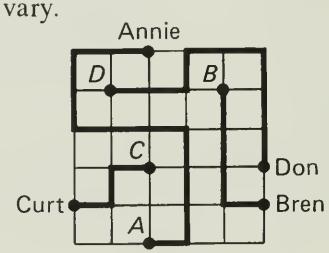
1. 6 **3.** 4 **5.** 8 **7.** 8 **9.** 5 **11.** 1 **13.** 2
15. 2 **17.** 2 **19.** 5 **21.** 5 **23.** 7
25. $4x + 3x = 63$; 9
27. $2x + 5 = 73$; 34 **29.** $2x + 9 = 25$; 8
31. $2x + 8 = 24$; 8

Pages 51–53

1. $n + 5$ **3.** $j + 2$ **5.** $b + 5$ **7.** $3x$
9. $4x$ **11.** $3x$ **13.** $x + 1$ **15.** $r - 1$
17. $x + 6$; $x + 11$ **19.** $c + 2$; $c + 1$; $c + 3$ **21.** $x + 2$; $x + 4$

Puzzles and Things, page 53

Answers may vary.



Pages 55–57

1. 384 boys **3.** 790 **5.** 14 **7.** The soup has 165 calories and the hamburger has 330 calories. **9.** 102 **11.** 37, 38 **13.** 130
15. Sue gets \$25, Liz gets \$50, and Wendy gets \$75.

Page 59

1. 14 **3.** 4 **5.** 3 **7.** 5 **9.** 2 **11.** 4

13. 1 **15.** 2 **17.** 1 **19.** 1 **21.** 2 **23.** 2
25. 7 **27.** 3 **29.** 2 **31.** 9 **33.** 1 **35.** 10
37. $7n = 3n + 12$; 3 **39.** $2n + 24 = 8n$; 4

Page 61

1. $6x - 15$ **3.** $16x - 40$ **5.** $48y + 24$
7. $24 + 48b$ **9.** 2 **11.** 3 **13.** 4 **15.** 10
17. 2 **19.** 4 **21.** 3 **23.** 0 **25.** 3 **27.** 5
29. 0 **31.** 15 **33.** 3 **35.** 14 **37.** 4
39. c **41.** a **43.** c

Pages 62–63

1. The peach has 45 calories and the apple has 75 calories. **3.** 18 and 25
5. Paul, \$28; Maria, \$56; Rocky, \$20
7. 20 **9.** The man is 41 years old.

Page 66

1. 18 **3.** 8 **5.** 8 **7.** 14 **9.** 28 **11.** 21
13. 2 **15.** 34 **17.** 5 **19.** 70 **21.** 72
23. 9 **25.** $2n + 7 = 25$; $n = 9$
27. $x + 5$ **29.** $h + 6$, $h + 9$ **31.** 32
33. 167 cm **35.** 5 **37.** 5 **39.** 2 **41.** 5
43. 7 **45.** 3

Page 68

1. $4a + 7$ **3.** $10b^3c$ **5.** 19 **7.** $3n + 24$
9. $x + 4y$ **11.** m **13.** $5x - 65$
15. $42m - 6n$ **17.** 12 **19.** 0 **21.** $s - 2$
23. 22 **25.** 12 **27.** 9 **29.** 30 **31.** 23
33. 9

CHAPTER 3

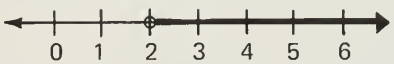
Page 73

1. +25 **3.** −10 **5.** −200 **7.** $<$ **9.** $>$
11. $>$ **13.** $<$ **15.** $<$ **17.** $>$
19. −274 **21.** Answer will vary with location.

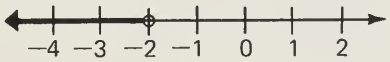
Page 75

1. $-2 < 3$ or $3 > -2$ **3.** $-1 > -6$ or $-6 < -1$ **5.** $5 > -4$ or $-4 < 5$
7. $-1 > -7$ or $-7 < -1$ **9.** $8 > -8$ or $-8 < 8$ **11.-17.** Check to be sure you have graphed each solution correctly.
11. 6 **13.** 3 **15.** 8 **17.** 10

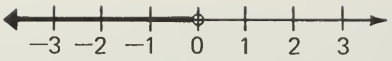
19.



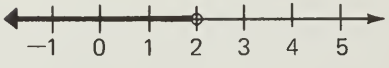
21.



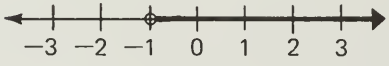
23.



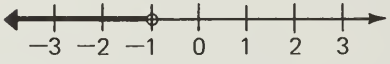
25.



27.



29.



31. $-4 < 0 < 4$ **33.** $-5 < -3 < 0$
35. $-4 < -2 < 6$

Page 77

1. −5 **3.** −1 **5.** −8 **7.** −8 **9.** 3
11. 2 **13.** −6 **15.** −3 **17.** −6 **19.** 0
21. 0 **23.** −6 **25.** −9 **27.** −5
29. −3 **31.** −1 **33.** 10

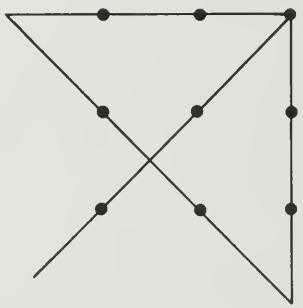
Pages 80–81

1. −10, 5 **3.** −7, −23 **5.** 2, 6 **7.** −1
9. −3 **11.** −7 **13.** −13 **15.** −12
17. 8 **19.** 8 **21.** 13 **23.** −1 **25.** −6
27. −1 **29.** −2 **31.** 8 **33.** −3
35. −7 **37.** −5 **39.** 37 degrees
41. −6 **43.** 53 **45.** −30

Mixed Practice Exercises, page 81

1. −1 **3.** −5 **5.** 9 **7.** −7 **9.** 4 **11.** 8
13. 0 **15.** −6 **17.** −1 **19.** 0 **21.** 15
23. −9 **25.** 11 **27.** −4 **29.** 11 **31.** 10

Puzzles and Things, page 81



Page 83

1. $4x$ **3.** $10x$ **5.** $-y$ **7.** $2x^2$ **9.** $6c^2$
11. $-a$ **13.** $6x$ **15.** $-10a$ **17.** $-3n^2$
19. $-9y + 10$ **21.** 1 **23.** x **25.** x
27. $-2x^2 + y$ **29.** $10 - 7r$

Page 85

1. 32 **3.** 120 **5.** -14 **7.** 20 **9.** -24
11. -6 **13.** 30 **15.** -7 **17.** -12
19. -4 **21.** 4 **23.** -28 **25.** -16
27. -25 **29.** 25 **31.** 36 **33.** 16
35. -100 **37.** -25 **39.** 100 **41.** -36
43. -100 **45.** 0 **47.** -48 **49.** 25
51. 16

Page 87

1. $2x$ **3.** $-6x$ **5.** 0 **7.** $-12x$ **9.** $-6ab$
11. $6xy$ **13.** $-8xy$ **15.** $-14ab$
17. $36 - 4x$ **19.** $2a - 2b$
21. $-6x + x^2$ **23.** $-3a - 3b$
25. $-a - 4$ **27.** $-y + 1$ **29.** $x - 5$
31. $-7 + a$ **33.** -3 **35.** $x - 4$
37. $7a + 4$ **39.** $7x - 3$ **41.** 13 **43.** -7
45. $28x - 4 - 24x + 6 = 4 + 3x - 2$;
$4x + 2 = 3x + 2$; $x = 0$

Page 89

1. -1 **3.** -8 **5.** -2 **7.** 2 **9.** -7
11. 5 **13.** -7 **15.** 3 **17.** -6 **19.** -2
21. 7 **23.** 3 **25.** 5 **27.** -4 **29.** 8
31. -8

Puzzles and Things, page 89

Fill the large pitcher. Pour water from it to fill the small pitcher. Empty the small pitcher. Pour the 2 L from the large pitcher into it. Fill the large pitcher again. Pour water from it into the small pitcher until the small one is full. The large pitcher now has 4 L!

Page 91

1. -6 **3.** -24 **5.** 1 **7.** -3 **9.** -5
11. -1 **13.** 9 **15.** 4 **17.** 3 **19.** -6
21. -5 **23.** 1 **25.** 4 **27.** -2 **29.** -14
31. 5 **33.** 2 **35.** 1

Pages 92–93

1. false **3.** true **5.** true **7.** false
9. false **11.** false **13.** $<$ **15.** $>$
17. $<$ **19.** $>$ **21.** $>$ **23.** -2 **25.** 11
27. -1 **29.** 4 or -4 **31.** 6 or -6 **33.** 14 or -14

Puzzles and Things, page 93

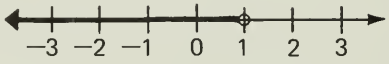
The lion tamer must be bald; since Geraldine and Amos have hair, Christopher is the lion tamer. Then, by clue 3, the ringmaster is Geraldine. Thus, Amos is the elephant trainer.

Page 94

1. about 80 **3.** about 110 **5.** about 90
7. about 120 **9.** about 200 **11.** about 190 **13.** about 120 **15.** about 80
17. about \$1.60 **19.** about \$1.40
21. about \$3.60 **23.** about \$2.70 **25.** Not reasonable **27.** Not reasonable **29.** Not reasonable **31.** Not reasonable **33.** Not reasonable **35.** reasonable **37.** 13
39. 50 **41.** 9 **43.** 14

Pages 98–99

1. $<$ **3.** $>$ **5.** $>$ **7.** $>$ **9.** $<$ **11.** $>$
13. $x = 2$ **15.** $x = -6$
17.



19. -13 **21.** -3 **23.** 1 **25.** -11
27. -6 **29.** -11 **31.** 18 **33.** 4
35. -13 **37.** -12 **39.** -5 **41.** 1
43. $-3x$ **45.** -15 **47.** $2x - y$ **49.** $8b^2$
51. $-6x + 3y$ **53.** -4 **55.** -9
57. -7 **59.** -17 **61.** -24 **63.** 24
65. -72 **67.** $y^2 - 5y$ **69.** $-2x - 6$
71. $-3a + 3b$ **73.** $-x^2$ **75.** $-y + 7$
77. $x + 9$ **79.** y^2 **81.** $-y - 6x + 9$
83. -9 **85.** -4 **87.** -10 **89.** -4
91. -5 **93.** -2 **95.** -8 **97.** 9 **99.** 2
101. 5

CHAPTER 4

Page 103

1. 22 **3.** $9x$ **5.** $3x + 3$ **7.** $12a$ **9.** $14x$
11. The width is 10 cm and the length is 15 cm.

Page 106

1. 49 cm^2 **3.** 16 m^2 **5.** 12 cm^2
7. 16 cm^2 **9a.** $6x^2$ **b.** $5x^2$ **11a.** $12xy$
b. $10xy$ **13.** $7xy$ **15.** $25y^2$ **17.** $4x^2$
19. centimeters **21.** \$480.00
23. $s^2 + \frac{1}{2}sh$ **25.** $41bh$

Page 110–111

1. 180 cm^3 **3.** 3000 cm^3 or 3 ℓ. **5.** $24x^3$
7. 48 **9.** 20 **11.** 40,000 cm^3
13. 40,000 grams **15.** more **17.** $16x^3$

Page 113

1. xt **3.** $6x$ **5.** $2b$ **7.** $2b + f$ **9.** $3x$
11. $s - b$ **13.** $3h + 2c$ **15.** $3b + c$

Page 116–117

1. 2 hours **3.** Barb travels at 18 km/h and Kate travels at 22 km/h. **5.** 60 km
7. 12 km/h

Page 119–121

1. 60 donuts **3.** 6 pens **5.** Lana earns $4 an hour and Tom earns $3 an hour.
7. There were 480 adult tickets and 240 student tickets.

Page 124–125

1a. 12 **b.** $x + 7$ **c.** $x + t$ **3.** 3 **5.** 8
7. $2x + 5$ **9.** 2 **11a.** 3 **b.** $\frac{y}{8}$ **c.** $\frac{y}{a}$
13. 8 **15.** $4 - 2y$ **17.** $6 - 4y$
19. $y - 5$ **21.** $w = \frac{A}{8}$ **23.** $t = \frac{D}{r}$
25. $b = N - a$ **27.** $d = \frac{W}{f}$
29. $H = AT$ **31.** 2 **33.** 128 km/h
35. 12,000 km/h

Page 127

1. clockwise **3.** 4 turns per second
5. 12 m **7.** 750 kilograms

Page 130–131

1. $4x + 6$ **3.** $16x$ **5.** $12x^2$ **7.** $10xy$
9. 16 cans **11.** $30x^3$ **13.** $30x^3$ **15.** $4x$
17. 70 km/h **19.** 98 calculators
21a. $7 - y$ **b.** $y + 7$ **c.** $y - 7$
23. $a = \frac{F}{m}$

Page 132

1. $3b + 3$ **3.** $5a + 19$ **5.** $4s - 24t$
7. $2a^2$ **9.** $-12a - 8$ **11.** 0 **13.** 21
15. 5 **17.** 7 **19.** 13 **21.** 10 **23.** 7
25. -6 **27.** -2 **29.** -2 **31.** 2
33. $t + s$ **35.** 56 kilograms **37.** 75 km/h

CHAPTER 5

Page 137

1. $3y + 11$ **3.** $2m^2$ **5.** $5ab + 2b^2$
7. $2x^2 + xy + 2y^2$ **9.** $-5y^2 + 1$
11. $12n + 2$ **13.** $10m + 12n + 15$
15. $-4m + 8$ **17.** $10x^2 - 20x + 2$
19. $5x^2 + 2x - 1$
21. $k^3 + k^2 - 2k + 6$ **23.** $x^3 + 6x^2 - 3$
25. $a^2 - ab - b^2$

Page 139

1. $x + 4y$ **3.** $-2x^2 + 2y^2$ **5.** $2x^2 + 4$
7. $-4xy$ **9.** $x^2y + 4xy^2 + 2$ **11.** $b + 2$
13. $4m - 18$ **15.** $4y + 14$
17. $-3x^2 + 5x - 6$
19. $-a^2 + 5ab - 10c$
21. $2y^2 + 5y - 18$ **23.** $n = 10$
25. $n = 80$

Page 141

1. a^3 **3.** $3x^3$ **5.** n^7 **7.** $4x^3$ **9.** $-5c^5$
11. $-6x^7$ **13.** $12x^6$ **15.** $-5a^2b^2$
17. $-3cd^4$ **19.** $20x^3y^3$ **21.** $10r^4s^3$
23. $3x^5yz^2$ **25.** $-5a^4b^3$ **27.** $-2x^4y^5$

Puzzles and Things, page 141

2 dimes, 7 pennies; 1 dime, 3 nickels, 2 pennies; 1 dime, 2 nickels, 7 pennies; 1 dime, 1 nickel, 12 pennies; 1 dime, 17 pennies; 5 nickels, 2 pennies; 4 nickels, 7 pennies; 3 nickels, 12 pennies; 2 nickels, 17 pennies; 1 nickel, 22 pennies; 27 pennies; 13 different combinations

Page 143

1. x^6 **3.** b^{12} **5.** c^{15} **7.** $4x^2$ **9.** a^4b^4
11. $36a^2x^2$ **13.** $25a^2$ **15.** $10x^2$ **17.** $9a^4$
19. $25x^{10}$ **21.** $4a^4b^2$ **23.** x^6y^6 **25.** $3x^4y^2$
27. $-16n^4$ **29.** $-x^3y^5$ **31.** $72\,m^3n^5$
33. $-r^{11}s^{12}$

Page 145

1. $2x + 8$ **3.** $4x + 4y$ **5.** $-6n - 12m$
7. $a^2 - ab$ **9.** $-ca - cb$
11. $-15x^2 - 10xy$
13. $4a^2 + 8ab + 12a$ **15.** $-2x - y - z$
17. $-4y^4 + 8y^2 - 4y$ **19.** $-x^3 - 2x^4$
21. $-y^5 + 2y^4 - 4y^3$ **23.** $30n + 10$ cents **25.** $n = -11$ **27.** $y = 72$
29. $a = 1$ **31.** $n = 2$ **33.** $a = 3$

Page 147

1. $x^2 + 3x + 2$ **3.** $n^2 + 10n + 24$
5. $x^2 + 8x + 7$ **7.** $x^2 + 2xy + y^2$
9. $x^2 + xy + 4x + 4y$ **11.** $y^2 - 7y + 12$

13. $x^2 + x - 2$ **15.** $y^2 - y - 20$
17. $16x^2 + 8x + 1$ **19.** $a^2 - b^2$
21. $2a^2 - a + 2ab - b$
23. $36x^2 - 60x + 25$
25. $n^3 + 3n^2 + 3n + 1$
27. $x^3 - 6x^2 + 12x - 8$
29. $m^3 - 4m^2 + 8m - 8$
31. $a^3 + 3a^2b + 3ab^2 + b^3$
33. $n^4 + 4n^3 + 6n^2 + 4n + 1$
35. $10a^4 - 3a^3b + 21a^2b^2 + 3ab^3 + 4b^4$
37. $n^5 - 4n^4 + 4n^3 - n^2 + n - 1$

Pages 150–151

1. $x^2 + 7x + 12$ **3.** $x^2 + 8x + 12$
5. $x^2 - 3x + 2$ **7.** $x^2 - 7x + 12$
9. $y^2 - 3y - 4$ **11.** $n^2 + n - 2$
13. $y^2 + 4y - 12$ **15.** $2x^2 + 9x + 4$
17. $a^2 - b^2$ **19.** $36a^2 - 49$
21. $9x^2 - 16$ **23.** $6x^2 + 13x - 5$
25. $21x^2 + 13xy + 2y^2$
27. $4x^2 - 23xy + 15y^2$
29. $24x^2 + 8xy - 2y^2$
31. $x^2 + 10x + 24$ **33.** $2a^2 + 5a - 12$
35. $6y^2 + 11y + 3$ **37.** $n^2 - 3\frac{1}{2}n + 1\frac{1}{2}$
39. $6x^2$

Puzzles and Things, page 151

If Melba and Fran are both wearing black hats, then Brent would know he must be wearing red (since there are only 2 black hats). Since Brent does not know, either Melba or Fran (or both) must be wearing red. If Melba is wearing a black hat, Fran would thus know she must be wearing red. Since Fran does not know, Melba must be wearing a red hat.

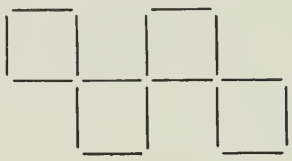
Page 153

1. $x^2 + 8x + 16$ **3.** $r^2 + 4r + 4$
5. $x^2 + 6x + 9$ **7.** $x^2 - 4x + 4$
9. $x^2 - 8x + 16$ **11.** $y^2 - 14y + 49$
13. $x^2 + 18x + 81$ **15.** $m^2 + 14m + 49$
17. $n^2 - 10n + 25$ **19.** $9a^2 + 12a + 4$
21. $16x^2 - 16x + 4$ **23.** $9x^2 + 6xy + y^2$
25. $100a^2 - 20ab + b^2$
27. $16x^2 - 24xy + 9y^2$
29. $2a^2 + 4a + 2$ **31.** $3x^2 - 12x + 12$
33. $-2x^2 + 8x - 8$
35. $8x^2 + 8xy + 2y^2$
37. $(a + b)^2 = a^2 + ab + ab + b^2$

Page 155

1. 1 **3.** a **5.** $-5x$ **7.** c^5 **9.** $7n$
11. -6 **13.** $-4rs$ **15.** $3x^2y$ **17.** $-7x^2$
19. $-7b$ **21.** $2cd^2$ **23.** $-7ab$ **25.** $2x$
27. $4a$ **29.** $4ab$

Puzzles and Things, page 155



Page 157

1. a^3 **3.** $\frac{1}{x^4}$ **5.** $9x^3$ **7.** $\frac{-2}{n^3}$ **9.** $-3a$
11. $-4x^3y^2$ **13.** $2x^3$ **15.** $\frac{6m^2}{n}$
17. $-4x^5y$ **19.** $3x^8y^3z$ **21.** $\frac{-7a^4}{c^2}$
23. $3r^3t^2$ **25.** $-6x$ **27.** $4abc^2$ **29.** $-7a^2b$
31. $27a^3c^4$

Puzzles and Things, page 157

Since the number doubles every minute, and the basket is full after 1 hour, the basket was half full in 1 hour − 1 minute, or 59 minutes.

Page 159

1. $a + 3b$ **3.** $2x - y$ **5.** $4x - 2y$
7. $8m + 9n$ **9.** $a + 9$ **11.** $b^2 - ab$
13. $a^2 + ab + b^2$ **15.** $3mn^2 - 1 + 2m$
17. $x - 3x^2y + 2$ **19.** $2rs + 3s^2 - 6r^2$
21. $-4a^2b + a^2b^2 - 2a^3$ **23.** $-2 - 6n$
25. $3x^2 - 8xy + 9y^2$
27. $-3xb + 4x^3 - 6b^2$

Page 161

1. $x + 2$ **3.** $y + 4$ **5.** $x + 2$ **7.** $x - 4$
9. $a - 5$ **11.** $x - 6$ **13.** $x - 2$

Puzzles and Things, page 161

Yes; "Nothing is better than a banana split" really means "There is nothing better than a banana split." If there is nothing better than a banana split, then crackers cannot be better than a banana split.

Page 162

1. $\frac{1}{2}$ **3.** $\frac{1}{4}$ **5.** $\frac{1}{2}$ **7.** $\frac{4}{5}$ **9.** $\frac{4}{3}$ **11.** $\frac{2}{3}$
13. $\frac{3}{4}$ **15.** $\frac{2}{3}$ **17.** $\frac{4}{7}$ **19.** $\frac{5}{7}$ **21.** $\frac{1}{2}$
23. $\frac{1}{2}$ **25.** $\frac{10}{13}$ **27.** $\frac{2}{5}$ **29.** $\frac{5}{9}$ **31.** $\frac{1}{2}$
33. $\frac{7}{12}$ **35.** $\frac{3}{13}$ **37.** $\frac{1}{2}$ **39.** $\frac{1}{4}$ **41.** $\frac{1}{3}$

Page 165

1. Green Mouthwash: \$.12/fl. oz.; Fresh Mouthwash: \$.115/fl. oz. **3.** Dazzle shampoo: about \$.21/fl. oz.; Dew Drops shampoo: about \$.20/fl. oz.

Pages 166–167

1. $2x - 2$ **3.** $7n + 2$ **5.** $5x^2 + 7y^2$
7. $-2ax$ **9.** $3n + 8$ **11.** $4n^2$
13. $6a^2 + 8b^2$ **15.** 5 **17.** $2x - 12y$
19. $-a^2 + 8$ **21.** $-x^2 - 2y^2$
23. $4m - 5n$ **25.** $7x - 2y$ **27.** $-7x$
29. a^6 **31.** a^4 **33.** $28x^4$ **35.** $-56y^5$
37. a^6 **39.** $-a^6$ **41.** m^4n^8 **43.** $-x^6y^9$
45. $30 + 5n$ **47.** $a^2 + 4a$
49. $18x - 27y$ **51.** $n^2 - 5n + 6$
53. $6x^2 - 5xy - y^2$ **55.** $10a - 31a - 14$
57. $n^2 - 9n + 18$ **59.** $30n^2 + 16n + 2$
61. $3y^2 + 13y + 4$ **63.** $n^2 + 8n + 16$
65. $x^2 + 2x + 1$ **67.** $4x^2 + 4xy + y^2$
69. $9m^2 - 12mn - 4n^2$ **71.** $3n$
73. $-10x$ **75.** $-6ab^3$ **77.** $11n^2$
79. $6n + 1$ **81.** $2x - 1$
83. $a^2 + 5a + 6$ **85.** $4ab - 3$
87. $3x + 4y + 5$

CHAPTER 6

Page 171

1. 2, 3, 7 **3.** 2, 2, 3, 3 **5.** 2, 5, 31
7. 2, 2, 2, 3, 5 **9.** 2, 2, 2, 2, 3, 13 **11.** 3
13. 7 **15.** 1 **17.** 11 **19.** 16 **21.** 12
23. 14 **25.** 25 **27.** 4 **29.** 8 **31.** 4
33. 20 **35.** 6 **37.** 100

Page 173

1. $3(3 + x)$ **3.** $2(x - 5)$ **5.** $x(3x - 1)$
7. $2x(x - 3)$ **9.** $7n(4n - 1)$
11. $4x(x - 2)$ **13.** $5mn(5 - mn)$
15. $9x^2(1 - 3y)$ **17.** $3(x^2 - 2x + 7)$
19. $11(5y^2 + 2y + 4)$
21. $4(a^2 + 3ab - 4b^2)$
23. $6(x^2 + x + 4xy + 7)$
25. $25(-2a^2 + b^2 + 3ab)$
27. $8xy(7x^2y^2 - 9xy - 8)$
29. $(x + 2)(a - b)$ **31.** $(n + y)(m - x)$
33. $(2 - a)(a + b)$ **35.** $(x^2 - 4)(y + 2)$

Page 175

1. $(4 - \pi)r^2$ **3.** $(a^2 - b^2)\pi$ **5.** 72
7. $x^2(1 - \pi) + 5(x + 3)$

Pages 178–179

1. 1, 5 **3.** 2, 4 **5.** 3, 4 **7.** 3, 5 **9.** 3, 6
11. 3, 7 **13.** 3, 8 **15.** 6, 6
17. $(a + 6)(a + 1)$ **19.** $(x + 6)(x + 3)$
21. $(r + 3)(r + 7)$ **23.** $(n + 1)(n + 16)$
25. $(a + 1)(a + 4)$ **27.** $(x + 1)(x + 10)$
29. $(n + 4)(n + 2)$ **31.** $(x + 7)(x + 2)$
33. $(y + 1)(y + 11)$ **35.** $(b + 3)(b + 9)$
37. $(x + 4)(x + 8)$ **39.** $(x + 3)(x + 10)$
41. $(a + 2)(a + 10)$ **43.** $(x + 25)(x + 2)$
45. $(m + 25)(m + 3)$ **47.** $(a + 21)(a + 3)$

Puzzles and Things, page 179

No, it is not possible.

Page 181

1. $-3, -2$ **3.** $-6, -3$ **5.** $-6, -4$
7. $(x - 5)(x - 2)$ **9.** $(x - 3)(x - 2)$
11. $(y - 5)(y - 1)$ **13.** $(x - 3)(x - 8)$
15. $(n + 6)(n + 3)$ **17.** $(n - 4)(n - 1)$
19. $(x + 4)(x + 2)$ **21.** $(y - 7)(y - 4)$
23. $(n + 5)(n + 5)$ **25.** $(y - 6)(y - 5)$
27. $(x - 7)(x - 7)$ **29.** $(m - 5)(m - 8)$

Page 183

1. yes **3.** yes **5.** no **7.** $(a + 5)^2$
9. $(x + 4)^2$ **11.** $(x - 2)^2$ **13.** $(y + 7)^2$
15. $(n + 9)^2$ **17.** $(x - 9)(x - 10)$
19. $(a - b)^2$ **21.** $(1 - 10x)^2$

Page 186

1. $-5, 2$ **3.** $-4, 1$ **5.** $-2, 1$ **7.** $-10, 3$
9. $7, -4$ **11.** $14, -1$
13. $(x + 7)(x - 3)$ **15.** $(x + 9)(x - 2)$
17. $(b - 3)(b + 4)$ **19.** $(n - 6)(n + 3)$
21. $(x - 5)(x + 4)$ **23.** $(y - 1)(y + 15)$
25. $(b + 6)(b - 4)$ **27.** $(b - 10)(b + 3)$
29. $(x + 14)(x - 2)$ **31.** $(y + 8)(y - 4)$

33. $(y + 9)(y - 3)$
35. $(m - 4)(m + 16)$ **37.** $(n - 9)(n + 7)$
39. $(y - 6)(y + 7)$

Mixed Practice Exercises, page 187

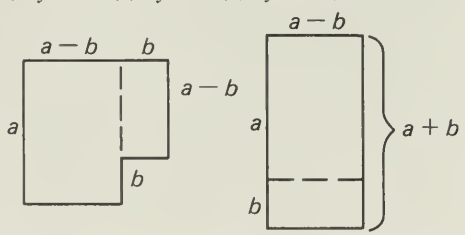
1. $(x + 9)(x + 1)$ **3.** $(n - 6)(n - 2)$
5. $(y - 3)(y + 4)$ **7.** $(y - 7)(y + 3)$
9. $(x + 3)(x + 7)$ **11.** $(b - 1)(b + 5)$
13. $(b - 7)(b - 1)$ **15.** $(y - 4)(y + 5)$
17. $(x + 7)(x + 5)$ **19.** $(n - 7)(n - 8)$
21. $(x + 9)(x + 6)$ **23.** $(n + 9)(n + 5)$
25. $(n - 10)(n + 5)$
27. $(y + 26)(y - 2)$ **29.** $4x - 4$
31. $4y - 4$

Page 189

1. $n^2 - 49$ **3.** $a^2 - 100$ **5.** $m^2 - n^2$
7. $x^2 - y^2$ **9.** $1 - 4x^2$ **11.** $9x^2 - 4$
13. $25y^2 - 4$ **15.** $81m^2 - n^2$
17. $25x^2 - 36$ **19.** $25x^2 - 9y^2$
21. $100x^2 - 25y^2$ **23.** $a^4 - 4$ **25.** 399
27. 2496 **29.** 896 **31.** 8091 **33.** 864
35. 3575 **37.** 6384 **39.** 2496

Page 191

1. yes **3.** no **5.** yes **7.** no
9. $(x - 4)(x + 4)$ **11.** $(n - 3)(n + 3)$
13. $(a - 3b)(a + 3b)$
15. $(x - 2y)(x + 2y)$
17. $(b - 8c)(b + 8c)$
19. $(2a - 5b)(2a + 5b)$
21. $(x - 11)(x + 11)$
23. $(8x - 3y)(8x + 3y)$
25. $(2 + x)(2 - x)$ **27.** $(8 - x)(8 + x)$
29. $(x - y)(x + y)$
31. $(2x^3 - 3)(2x^3 + 3)$
33. $(12 - 11x)(12 + 11x)$
35. $(6bc - 2a)(6bc + 2a)$
37. $(y^2 + 9)(y - 3)(y + 3)$
39. $(4a^2 + 1)(2a + 1)(2a - 1)$
41. $(16 + x^2y^2)(4 + xy)(4 - xy)$
43. $(x^4y^2 + 1)(x^2y + 1)(x^2y - 1)$
45.



Page 193

1. $3(y + 4)(y + 2)$ **3.** $2(y - 5)(y - 3)$
5. $5(a - 6)(a + 2)$ **7.** $4(x - 3)(x + 5)$
9. $2(x - 4)^2$ **11.** $8(x - 2)(x + 2)$
13. $4(x - 3)(x + 3)$
15. $6(x - 2y)(x + 2y)$
17. $-1(x - 3)(x - 1)$
19. $2(x - 9)(x + 9)$
21. $-4(a - 2)(a + 1)$
23. $2(a - 10b)(a + 10b)$

Page 195

1. $(3x + 1)(x + 1)$ **3.** $(2a + 3)(a + 3)$
5. $(2b + 3)(b + 1)$ **7.** $(3a - 5)(a - 1)$
9. $(3x - 4)(x - 2)$ **11.** $(3y + 1)(y - 2)$
13. $(2x - 1)(x + 3)$
15. $(2x + 3)(x - 3)$ **17.** $(4a + 9)(a - 1)$
19. $(3b + 5)(2b - 5)$
21. $(4a - 5)(a + 1)$
23. $(2x - 5)(2x + 1)$
25. $(4r + 3)(2r - 3)$
27. $(4x + 3)(2x - 1)$
29. $2(2c + 1)(c + 1)$
31. $3(3y + 2)(y + 1)$
33. $3(2x + 1)(x + 2)$
35. $(5y - 2)(2y - 5)$
37. $(3x - 11)(2x + 11)$
39. $(3x + 4)(4x - 3)$
41. $(5n - 1)(3n + 4)$

Page 197

1. 186,487 kg

Pages 198–199

1. 3 **3.** 3 **5.** 14 **7.** 6 **9.** $3y$ **11.** x^2
13. 6 **15.** 4 **17.** $2(n^2 + 2)$
19. $ab(b + a)$ **21.** $2y^2(y + 4)$
23. $2x^2(1 - 4x)$ **25.** $6(6x + 1)$
27. $x(x + 4)$ **29.** $(n + 3)^2$ **31.** $(x - 5)^2$
33. $(n + 4)(n + 5)$ **35.** $(y - 8)(y - 1)$
37. $(n - 11)(n - 2)$ **39.** $(n + 7)(n + 5)$
41. $(a - 9)^2$ **43.** $(m + 4)^2$
45. $(n + 25)(n + 3)$ **47.** $(y - 5)(y - 8)$
49. $(y + 6)(y + 3)$ **51.** $(x + 6)(x + 5)$
53. $(n + 4)(n + 16)$ **55.** $(x + 3)(x + 9)$
57. $(x - 4)(x - 20)$
59. $(x + 16)(x + 2)$ **61.** $(y - 9)(y - 7)$
63. $(n - 9)(n + 2)$ **65.** $(m + 8)(m - 2)$
67. $(y - 12)(y + 2)$ **69.** $(y - 6)^2$

71. $(a + 5)(a - 2)$ **73.** $(b + 6)(b - 1)$
75. $y^2 - 16$ **77.** $1 - 4a^2$ **79.** $16n^2 - 1$
81. $9x^2 - y^2$ **83.** $x^2y^2 - 1$
85. $16x^2 - 9y^2$ **87.** $(a - b)(a + b)$
89. $(5 - n)(5 + n)$ **91.** $(2x - y)(2x + y)$
93. $(4a - b)(4a + b)$
95. $(7a - b)(7a + b)$
97. $(1 - ab)(1 + ab)$ **99.** $3(a + 1)^2$
101. $4(x - 2)(x + 2)$
103. $3(a + 3)(a + 2)$
105. $3(x - 4)(x + 3)$
107. $2(n + 7)(n - 3)$
109. $a(b - 2c)(b + 2c)$
111. $2(a + 4)(a + 2)$

Page 200

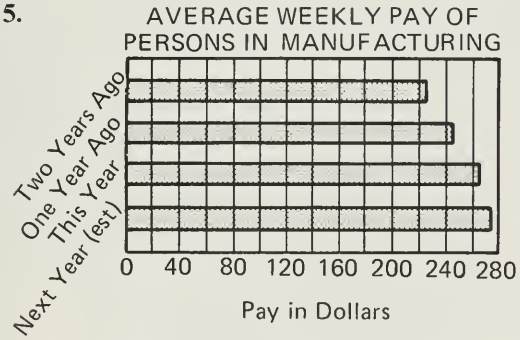
1. $x = 14$ **3.** $m = 9$ **5.** $x = -5$
7. $x = -1$ **9.** $x = -3$ **11.** $x = 6$
13. $a = -5$ **15.** $a = 1$ **17.** $2a^2 + 15$
19. $4x + 1$ **21.** $3a^2 - 3b^2$
23. $5x^2 + x - 4$ **25.** $4m^2 - 2n^2$
27. $-2m^2 + 3m - 3$ **29.** $a - 3b$
31. a^2b^4 **33.** $2x^3 + 3x^2 - 4x$
35. $n^2 - 14n + 48$ **37.** $3b^2 + b - 14$
39. $6y^2 - 7y - 3$ **41.** $25a^2 - b^2$
43. $a(a + 6b^2)$ **45.** $3mn(1 + 4m)$
47. $(a - 2)(a - 1)$ **49.** $(n - 3)(n - 4)$
51. $(x + 8)(x - 2)$ **53.** $4(x - 3)(x + 2)$

CHAPTER 7

Pages 206–207

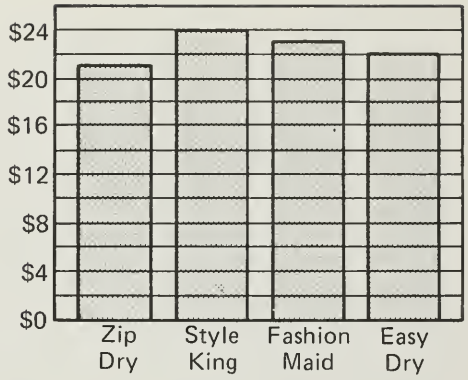
1. July, August
3. 2,500

5.



7. Basketball, 23 **9.** Answers will vary.

11.



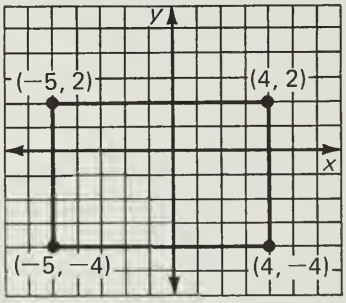
Pages 209–211

1. $1\frac{1}{2}$ **3.** 1985 **5.** Answers may vary. About 425 million. **7.** about 75 **9.** 40
11. about 10; about 65 **13.** 21 **15.** 26
17. 1951 **19.** 370 million **21.** 250
23. about 3 years ago **25.** \$1000; \$1500
27. \$.04

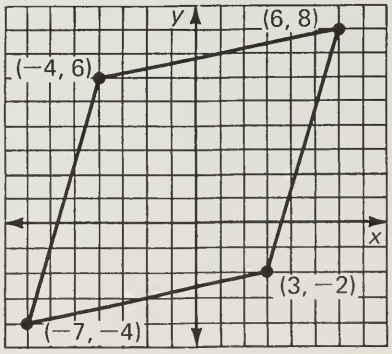
Pages 214–215

1. $(-5,4)$ **3.** $(-3,-5)$ **5.** $(0,4)$
7. $(2,-4)$ **9.** $(-2,3)$ **11.** M **13.** R
15. N **17.** Q **19.** 2 **21.** $(2,4)$ **23.** $(5,1)$

25.



27.



Pages 218–219

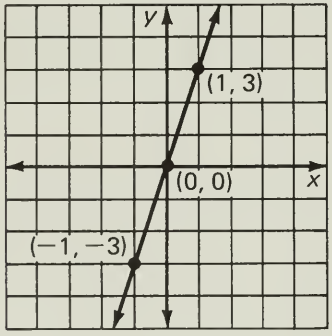
1. a, c **3.** c **5.** b **7.** 6; 10; 14; 16
9. 3; 5; −1; 8 **11.** 2; 1; 0; 4
13. 6; 3; 9; 0 **15.** $y = 9 - x$
17. $y = 6 + x$ **19.** $y = 5 - 4x$
21. $y = 9 - 4x$ **23.** $y = 9x - 7$
25. $2x - 5 = y$

Note: Answers to exercises 27–37 may vary.

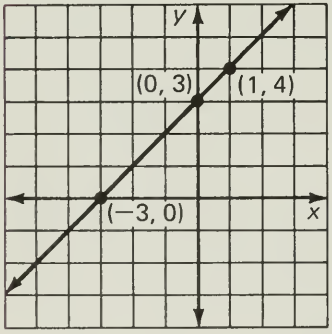
27. (1,0), (2,1), (−1,−2)
29. (1,−1), (2,−2), (−1,1)
31. (0,6), (1,7), (−1,5)
33. (0,4), (2,8), (−2,0)
35. (0,7), (1,11), (−1,3)
37. (0,2), (1,0), (−1,4) **39.** $y = 4x$
41. $y = 4x + 1$ **43.** *a*, *b* **45.** *a*, *b*

Page 221

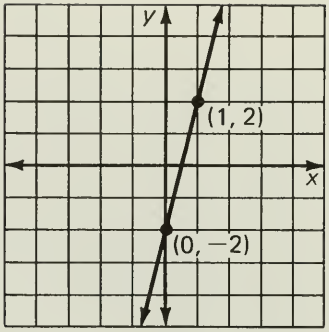
1.



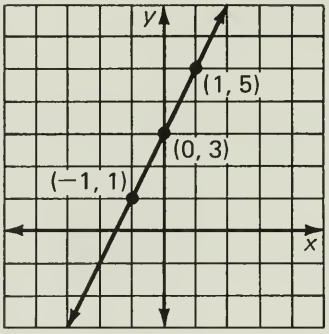
3.



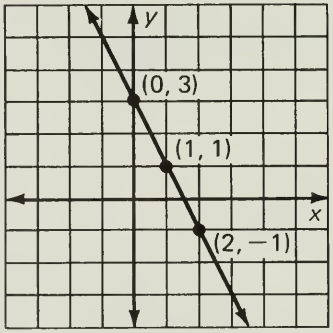
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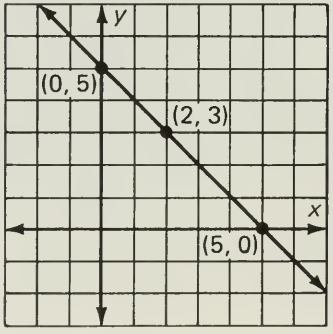
7.



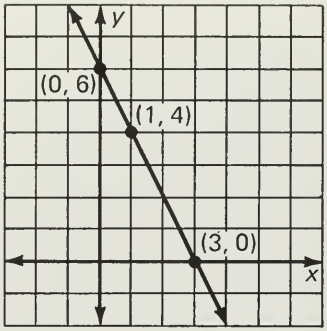
9.



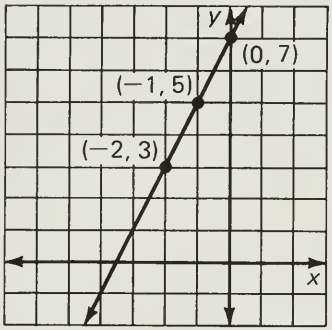
11.



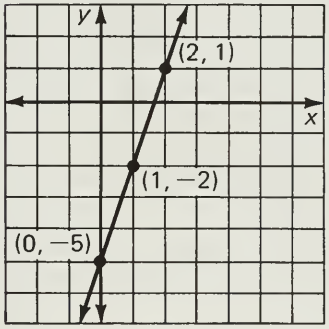
13.



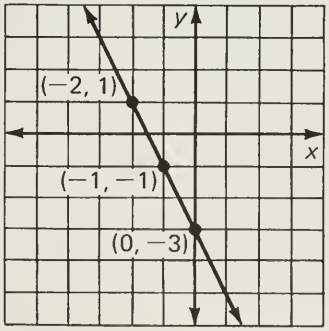
15.



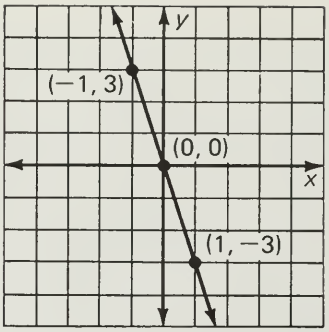
17.



19.



21.



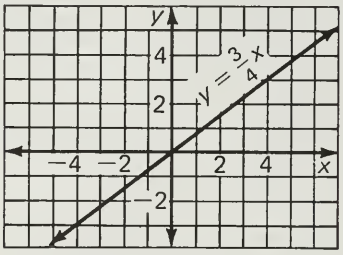
23. $y = -x + 2$

Page 223

1. $\frac{2}{3}$ **3.** $\frac{3}{2}$ **5.** $-\frac{1}{3}$ **7.** 3 **9.** 2 **11.** -1

13. -3 **15.** $\frac{1}{3}$ **17.** $-\frac{1}{5}$

19.



21. 0 **23.** 0 **25.** 0

Pages 226–227

1. $y = -4$; $x = 5$ **3.** $x = -9$

5. $n = 10$; $C = 36, 54, 216$

7. 80; 120; 160 **9.** $-17°C$ **11.** 40

13. about 15°C

Page 228

1. $\frac{3}{12}$ **3.** $\frac{3}{9}$ **5.** $\frac{3}{18}$ **7.** $\frac{18}{48}$ **9.** $\frac{20}{35}$

11. $\frac{4}{12}, \frac{3}{12}$ **13.** $\frac{2}{8}, \frac{1}{8}$ **15.** $\frac{5}{20}, \frac{4}{20}$

17. $\frac{3}{18}, \frac{8}{18}$ **19.** $\frac{6}{14}, \frac{1}{14}$ **21.** $\frac{21}{28}, \frac{12}{28}$

23. $\frac{21}{30}, \frac{20}{30}$ **25.** $\frac{32}{72}, \frac{27}{72}$ **27.** $\frac{11}{30}$

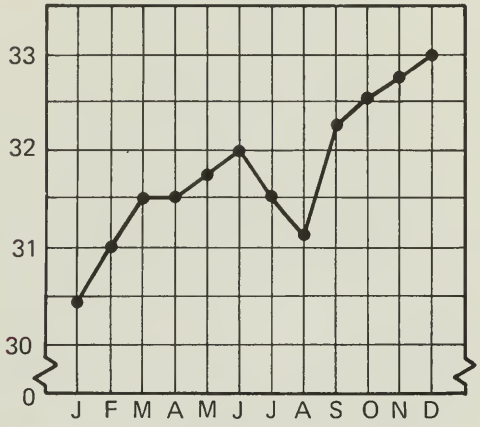
29. $\frac{4}{9}$ **31.** $\frac{1}{6}$ **33.** $\frac{1}{20}$ **35.** $\frac{4}{21}$

37. $\frac{41}{42}$ **39.** $\frac{43}{35}$ **41.** $\frac{5}{8}$

43. $\frac{16}{33}$ **45.** $\frac{1}{24}$

Page 231

1. NEWSPAPER SALES FOR ONE YEAR



3.



5 Years Ago

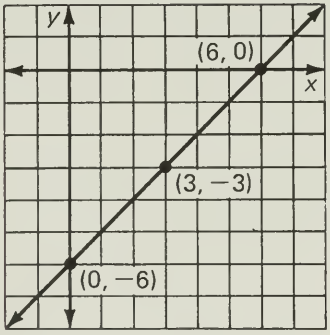


Today

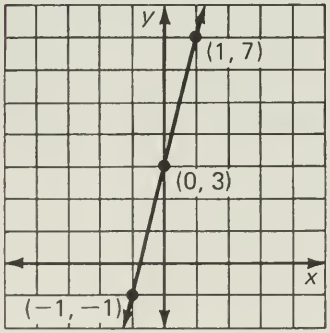
Pages 232–233

1. about 7.5 cm **3.** July **5.** decreased
7. 1966 **9.** 0; −4; −8; 8
11. 10; 9; 8; 11

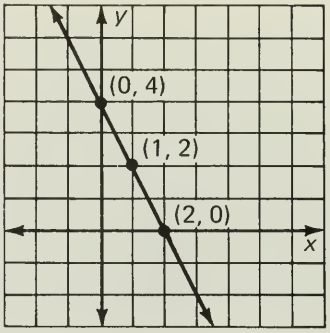
13.



15.



17.



19. 3 **21.** $\frac{3}{4}$
23. $n = 8$; $C = \$2.95, \$5.90, \$8.85, \$14.75$
25. the number of bottles you buy
27. 1 free bottle
29. $n = 110$; $A = 5, 7.50, 10, 25, 55$

CHAPTER 8

Pages 238–239

1. (1,2) **3.** (2, −1) **5.** Answers may vary. (0,12), (6,6) **7.** Answers may vary. (0,1), (1, −1) **9.** (1,2) **11.** (−1, −2)
13. (−3,0) **15.** (−2,1) **17.** (−1, −1)
19. (2, −1) **21.** $\left(-1\frac{2}{5}, 4\frac{4}{5}\right)$
23. $\left(-3\frac{1}{3}, 1\frac{1}{3}\right)$ **25.** $\left(2, -2\frac{1}{2}\right)$

Pages 242–243

1. 4 **3.** 1 **5.** 3 **7.** 4 **9.** 2 **11.** −7
13. no **15.** one **17.** one **19.** one
21. all **23.** all **25.** no **27.** one
29. a. no **b.** yes **c.** no **31. a.** yes **b.** no **c.** no

Page 245

1. (3,6) **3.** (5,2) **5.** (9,13) **7.** (1,4)
9. (3, −1) **11.** (5,2) **13.** (10,2) **15.** (3,2)
17. $A = 2h^2$ **19.** $V = 6h^3$
21. $x = 19$; $y = 31$; $z = 12$

Pages 247–248

1. (6, −4) **3.** (1, −4) **5.** (2,1) **7.** (4,2)
9. (−2,2) **11.** (3, −2) **13.** (2,6)
15. (−1,3) **17.** (7,2) **19.** (−2,4)
21. (3, −2) **23.** (2, −6) **25.** (2, −1)
27. (1, −1) **29.** (−1, −1) **31.** (3,5)

Page 249

1. $\left(1\frac{1}{3}, 1\frac{2}{3}\right)$ **3.** $(-2,-1)$ **5.** $(9,3)$ **7.** $(-20,11)$ **9.** $(4,3)$ **11.** $(3,4)$ **13.** $(3,5)$ **15.** $(3,3)$ **17.** $\left(5, \frac{2}{3}\right)$ **19.** $(-1,5)$ **21.** $(2,-2)$ **23.** $(6,-8)$

Pages 252–253

1. 17, 4 **3.** 24, 19 **5.** Bugs: 35 kg; Porky: 65 kg **7.** Bud: 170; Jan: 158 **9.** length: 18; width: 6 **11.** 5¢

Puzzles and Things, page 253

Let b = the number of balls, c = the number of cups, and p = the number of pennies. Then $b + c = 12p$ and $4b + 2p = c$; $b = 12p - c$, so $4(12p - c) + 2p = c$; $48p - 4c + 2p = c$; $50p = 5c$; $10p = c$. 10 pennies

Page 255

1. (6,3) **3.** $(-2,3)$ **5.** (2,4) **7.** $(-1,-1)$ **9.** (3,0) **11.** $(2,-3)$ **13.** $(-2,6)$ **15.** (3,2) **17.** $(-5,-3)$ **19.** $(1,-1)$ **21.** $(-1,1)$

Pages 256–257

1. hotdog: 50¢; cola: 40¢ **3.** adult: \$3; student: \$2 **5.** ice cream cone: 30¢; milkshake: 65¢ **7.** $3x$; $2y$; 48; $x = 8$; $y = 12$

Page 259

1. 5 km/h **3.** 100 km/h

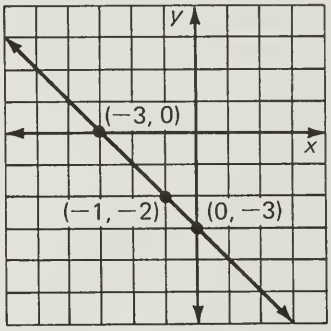
Pages 262–263

1. (2,4) **3.** $(-2,1)$ **5.** $(2,-4)$ **7.** 9 **9.** $\frac{1}{2}$ **11.** 1 **13.** -5 **15.** $\frac{1}{3}$ **17.** one **19.** one **21.** one **23.** $(-2,8)$ **25.** (2,10) **27.** $(-5,25)$ **29.** (1,4) **31.** $(-4,1)$ **33.** (1,1) **35.** $(5,-1)$ **37.** $(2,-1)$ **39.** (1,1) **41.** (3,2) **43.** $(-2,-3)$ **45.** (1,6) **47.** (9,3) **49.** 15, -1 **51.** 6, 2 **53.** pizza: \$2; root beer: \$.35 **55.** adult: \$2; student: \$1.50

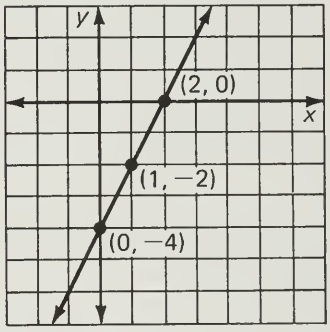
Page 264

1. 16 **3.** 3 **5.** 4 **7.** -6 **9.** 1 **11.** 2 **13.** $-14a^2b$ **15.** $4m + 1$ **17.** $-2n + 9$ **19.** $b^2 - 4b - 10$ **21.** $-2x^2 - 3$ **23.** $-4m^2$ **25.** m^4n^2 **27.** $y^3 - 2y^2 + 4y$ **29.** $3m^2 + 7m + 2$ **31.** $25x^2 - 64$ **33.** $10m^2 - 13mn - 3n^2$ **35.** $2m(m - 3)$ **37.** $(y + 1)(y + 2)$ **39.** $(a + 2)^2$ **41.** $(x - 3)^2$ **43.** $(a + 2)(a - 3)$ **45.** $(4a + 3b)(4a - 3b)$

47.



49.



51. $(4,-1)$

CHAPTER 9

Page 269

1. $\frac{2}{3}$ **3.** $\frac{1}{6a}$ **5.** $-\frac{1}{2}$ **7.** $\frac{1}{2}$ **9.** $-\frac{4x}{3}$ **11.** $\frac{4}{3r}$ **13.** $\frac{ab}{3}$ **15.** $-\frac{a^2}{5}$ **17.** $\frac{2}{a}$ **19.** $\frac{2a}{x}$ **21.** $\frac{2x^2}{5y}$ **23.** $\frac{5m}{4}$ **25.** $\frac{pq}{8r}$ **27.** $-2yz$ **29.** $\frac{2x}{7w}$ **31.** $-\frac{5b^2}{ac}$ **33.** $\frac{t^2}{12rs^2}$ **35.** $-\frac{11c^3}{ab}$ **37.** $-\frac{17x}{3yz}$ **39.** 0 **41.** 2 **43.** 2

Page 271

1. $\frac{1}{3}$ **3.** $\frac{3}{7}$ **5.** x **7.** $\frac{1}{3y-2}$ **9.** $\frac{2}{c-3}$

11. $\frac{2}{x-5}$ **13.** $\frac{n-3}{n}$ **15.** $\frac{2x}{y}$

17. $\frac{5}{k+2}$ **19.** $\frac{6}{u-v}$ **21.** $\frac{4x}{y+2x}$

23. $\frac{x+3}{x-3}$ **25.** $x-5$ **27.** $\frac{x}{2x+y}$

29. $\frac{x-3}{x+3}$ **31.** $\frac{y-5}{y-3}$ **33.** $\frac{z-10}{z+6}$

Page 273

1. $-1(2x)$ **3.** $-1(a+b)$

5. $-1(3y-2)$ **7.** $-1(8m-3)$

9. $-1(v-25)$ **11.** $-1(4+y^2)$ **13.** -1

15. -1 **17.** -1 **19.** -1 **21.** -2

23. -1 **25.** $-a-1$ **27.** $-\frac{1}{r+1}$

29. $-\frac{1}{c+2}$ **31.** $-x-5$ **33.** $-\frac{4}{t+3}$

35. $-\frac{p+3}{2}$ **37.** $-\frac{r-4}{3}$

Page 275

1. $\frac{5}{8}$ **3.** $\frac{5}{12}$ **5.** $\frac{3}{4}$ **7.** $\frac{2}{1}$ **9.** $\frac{4}{3}$ **11.** $\frac{7}{2}$

13. $\frac{100\text{ cm}}{1\text{ cm}} = \frac{100}{1}$

15. $\frac{36\text{ months}}{3\text{ months}} = \frac{12}{1}$ **17.** $\frac{50\text{ cm}}{300\text{ cm}} = \frac{1}{6}$

Page 277

1. 16 boys **3.** 21 games **5.** 25 m

7. \$8,000 **9.** 420 mL vinegar; 315 mL oil

11. 25,050 votes; 20,040 votes; 10,020 votes

Page 279

1. $x = 9$ **3.** $x = 12$ **5.** $x = 2$ **7.** $x = 8$

9. $a = 2$ **11.** $x = 10$ **13.** $x = 8$

15. $a = 5$ **17.** $x = 2$ **19.** $a = -4$

21. $n = -5$ **23.** $y = -5$ **25.** $x = 3$

27. $m = -1$ **29.** $y = -1$ **31.** $x = 12$

33. $c = 7$

Pages 281–283

1. \$1.55 **3.** \$2.25 **5.** 5 hours **7.** 21 g

9. 21 cm **11.** 8 km **13.** 1,800 cm

15. \$300 **17.** \$2,700

Page 285

1. $\frac{1}{12}$ **3.** $\frac{8}{21}$ **5.** $\frac{y^2}{2}$ **7.** $\frac{2}{3}$ **9.** $\frac{2y}{5}$

11. $\frac{1}{2y^2}$ **13.** $6r$ **15.** $\frac{5a^2}{4b}$ **17.** $\frac{x-1}{2}$

19. $\frac{x-4}{2}$ **21.** $5a-5$ **23.** $\frac{a}{ab-b}$

25. $\frac{a+b}{2}$ **27.** $\frac{r}{2}$ **29.** $-\frac{1}{3x+6}$

31. $\frac{x+2}{2}$ **33.** $\frac{2x-10}{x-1}$ **35.** $\frac{x+2}{x+4}$

37. $-7n-7$ **39.** $\frac{2}{x^2+3x+2}$

41. $\frac{m-2}{m+2}$

Page 287

1. $\frac{5}{2}$ **3.** $\frac{6}{5}$ **5.** $\frac{1}{ab}$ **7.** $\frac{21m}{8}$ **9.** $6b$

11. $\frac{ab}{8}$ **13.** 2 **15.** $\frac{a^2+1}{a}$ **17.** $\frac{2}{y+3}$

19. 1 **21.** $\frac{x+4}{2x}$ **23.** $\frac{a-3}{a+3}$

25. $\frac{3y+6}{2y^2-10y+12}$ **27.** $\frac{y-3}{2}$

29. $d-5$ **31.** $\frac{y+2}{y-6}$

Page 289

1. $\frac{7}{5}$ **3.** $\frac{9}{2a}$ **5.** $\frac{2n}{3}$ **7.** $\frac{x}{2}$ **9.** $-y$

11. $-x$ **13.** $\frac{x+1}{y}$ **15.** $\frac{x+1}{3x}$

17. $\frac{5}{a+b}$ **19.** $\frac{2}{y-2}$ **21.** 2 **23.** 1

25. $\frac{a+b}{ab}$ **27.** $\frac{-2c+4}{7c}$

Page 291

1. 8 **3.** 16 **5.** $4x$ **7.** $3y$ **9.** $6x$ **11.** 9

13. $6a$ **15.** $4y$ **17.** $15y$ **19.** $8m$ **21.** $5x$

23. $2b$

Puzzles and Things, page 291

Some other ways are as follows:



Page 293

1. $\frac{5}{6}$ **3.** $\frac{3y}{8}$ **5.** $\frac{7x}{6}$ **7.** $\frac{7x}{6}$ **9.** $\frac{5}{2a}$

11. $-\frac{13}{6y}$ **13.** $\frac{3x}{12}$ **15.** $\frac{11}{2x}$ **17.** $\frac{23}{4n}$

19. $\frac{7x}{24}$ **21.** $\frac{x-15}{5x}$ **23.** $\frac{8+x}{2x}$

25. $\frac{4y+1}{6xy}$ **27.** $\frac{2y+3x}{xy}$ **29.** $\frac{x}{4}$

31. $\frac{4x-1}{4}$ **33.** $\frac{21x-10}{6x}$

Page 295

1. $\frac{x+7}{8}$ **3.** $\frac{12t-7}{12}$ **5.** $\frac{11x-15}{12}$

7. $\frac{a-3}{6}$ **9.** $-\frac{7x+9}{12}$ **11.** $\frac{3x+1}{4}$

13. $\frac{x+8}{3}$ **15.** $\frac{4y+20}{3}$ **17.** $\frac{5x+5}{3}$

19. $\frac{5r}{16}$ **21.** 0

Puzzles and Things, page 295

Mr. Jackson borrowed one horse from a neighbor so that he then had 18 horses. The ratios $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{9}$ can be expressed as $\frac{9}{18}$, $\frac{6}{18}$, and $\frac{2}{18}$ respectively. Floyd then received 9 horses, Denise 6 horses, Harriet 2 horses, and the borrowed horse was returned to the neighbor.

Page 297

1. $x = 30$ **3.** $x = 12$ **5.** $x = 2$
7. $n = 10$ **9.** $y = -60$ **11.** $a = -9$
13. $n = 2$ **15.** $x = 3$ **17.** $x = -18$
19. $x = -7$ **21.** $x = 1$ **23.** $x = -5$

Puzzles and Things, page 297

Let x = his age at death. Then you can write the equation

$\frac{1}{6}x + \frac{1}{12}x + \frac{1}{7}x + 5 + \frac{1}{2}x + 4 = x.$

When you solve the equation you find that $x = 84$. Diophantus lived to be 84 years old.

Pages 299–301

1. 6 hours **3.** $1\frac{1}{3}$ hours **5.** $2\frac{2}{9}$ hours
7. $3\frac{1}{13}$ hours **9.** 12 hours **11.** 12 hours

Page 303

1. $\frac{8k+5}{k(k+1)}$ **3.** $\frac{-3(x+2)}{t(t+5)}$

5. $\frac{-8a}{(a+4)(a-4)}$ **7.** $\frac{y^2+3y-12}{(y+3)(y-2)}$

9. $\frac{x^2+10x-21}{(x+3)(x-3)}$ **11.** $\frac{b(3b+11)}{(b+6)(b-1)}$

13. $\frac{7x-7y-x^2-xy}{(x+y)(x-y)}$

15. $\frac{4a-4b+2a^2+2ab}{(a+b)(a-b)}$

17. $\frac{8+5c+5d}{(c+d)^2}$ **19.** $\frac{1+3x+3y}{(x+y)(x-y)}$

21. $\frac{3(a-5)}{(a+2)(a-2)}$ **23.** $x = \frac{1}{5}$

Puzzles and Things, page 303

If $ab = a^2$, then $b = a$ and $a - b = 0$. Thus, we cannot divide by $a - b$, which was done to get from step 3 to step 4.

Page 304

1. $<$ **3.** $<$ **5.** $<$ **7.** $>$ **9.** $>$ **11.** $<$
13. 6.7 **15.** 9.3 **17.** 4.3 **19.** 9.21
21. 0.08

Pages 306–307

1. \$.43 **3.** \$.66 **5.** \$.37 **7.** \$1.09
9. \$1.65 **11.** \$30 **13.** \$1.60 **15.** \$.80
17. \$2.50 **19.** \$2.60 **21.** \$25 **23.** \$.45
25. \$1.20 **27.** \$.35 **29.** \$.75

Pages 308–309

1. $\frac{1}{3}$ **3.** $\frac{x}{2}$ **5.** $2n$ **7.** $\frac{x}{6}$ **9.** $2y^2$

11. $a - 5$ **13.** $\frac{x-y}{x+y}$ **15.** $n + 2$

17. $\frac{3\,mn}{1-2\,mn}$ **19.** -1 **21.** $-\frac{1}{2}$

23. 6 cans **25.** 4 **27.** 10 **29.** 1 **31.** 4

33. \$5.40 **35.** $\frac{1}{21}$ **37.** 6 **39.** $\frac{1}{15x-15}$

41. $\frac{x-2}{5}$ **43.** 9 **45.** $\frac{1}{5ab}$ **47.** $\frac{x-y}{4}$

49. $\frac{3}{5}$ **51.** $2x$ **53.** $\frac{4+n}{m+n}$ **55.** $\frac{4}{x}$

57. $\frac{3a}{a+2}$ 59. $\frac{2n}{15}$ 61. $\frac{5a}{12}$ 63. $\frac{x-9}{3x}$

65. $\frac{y-x}{xy}$ 67. 2 69. 15 71. 7

73. $1\frac{1}{5}$ hours

CHAPTER 10

Page 313

1. 0.66 3. 2.93 5. 2.64 7. 5.12
9. 7.765 11. 12.817 13. 14.53
15. 20.511 17. 11.056 19. 12.6
21. 0.82 23. 24.30 25. 610 27. 4.1208
29. 2.5 31. 0.423526 33. 0.06018
35. 0.004182 37. 0.206244 39. 1.55

Page 315

1. 6 3. 8 5. 46 7. 1.1 9. 1.7 11. 12.6
13. 15 15. 571.4 17. 22.47 19. 25.0
21. 32.5 23. 238.46

Page 317

1. 0.125 3. 0.6 5. 0.375 7. 0.285714...
9. 0.571428... 11. 0.11... 13. 0.5454...
15. 0.4 17. 0.720 19. 0.654

Page 319

1. $x = 50$ 3. $x = 4$ 5. $x = 8.63...$
7. $x = 2.6$ 9. $x = 5$ 11. $n = 13.33...$
13. $n = 2$ 15. $x = 6600$ 17. $x = 86.25$
19. $x = 71.6$ 21. $n = 7.5$ 23. $x = 5$
25. $s = 5$

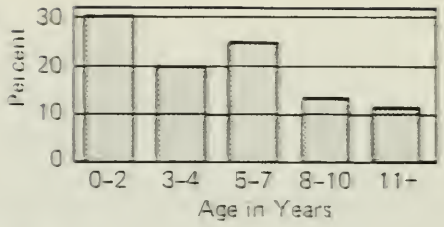
Page 321

1. 1.7, 2.1 3. 43 cm, 90.3 cm 5. 68.2 km

Pages 324–325

1. 40% 3. 65% 5. 50% 7. 7% 9. 9%
11. $4\frac{1}{2}\%$ 13. $21\frac{2}{5}\%$ 15. $12\frac{1}{2}\%$
17. 20% 19. 75% 21. 0.64 23. 0.95
25. 0.20 27. 0.67 29. 0.47 31. 0.75
33. 0.08 35. 0.09 37. 0.065 39. 0.055
41. $\frac{1}{3}$ 43. $\frac{1}{4}$ 45. $\frac{1}{2}$ 47. $\frac{1}{10}$ 49. $\frac{3}{5}$
51. 55%; 45%; 35%
53. 10%; 15%; 20%; 25% 55. 32%
57. about 10% 59. about 40% 61. 5

63.



Page 327

1. 36 3. 4.50 5. 8.40 7. 28 9. 90
11. 37.5 13. $5.40 15. $2.40 17. $4.76
19. $50.00

Pages 330–331

1. 22 3. 40% 5. 2.65 7. 55 9. 950
11. $.15 13. 0.1% 15. 25%
17. 2425 students 19. 40% 21. 10%

Page 333

1. $25 3. $62.50 5. about 14 years
7. $5

Page 335

1. $400 3. $8000 at 7% and $4000 at 9%
5. $5000 in stocks and $3000 in bonds

Pages 337–339

1. 40 kg at $.70 per kilogram and 60 kg at $.95 per kilogram 3. 400 student tickets
5. 200 liters water 7. 62.5 liters graphite

Page 341

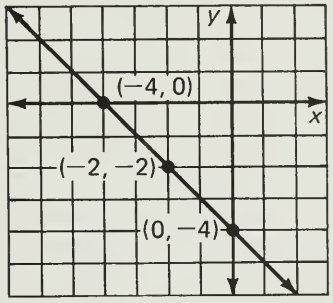
1. daily 3. If there is less than $10 in the account, the interest to be paid daily would be less than $\frac{1}{2}$ cent. 5. $4.71

Pages 342–343

1. 5.70 3. 51.15 5. 4.9 7. 78.08
9. 12.24 11. 23 13. 0.812 15. 16.1
17. 5.2 19. 52.4 21. 316.0 23. 0.1
25. 0.375 27. 0.833... 29. 0.44...
31. $x = 110$ 33. $x = 30$ 35. $x = 16.5$
37. 1.9 cm 39. 32% 41. 2.5% 43. 30%
45. 20% 47. $33\frac{1}{3}\%$ 49. 0.85 51. 0.70
53. 0.065 55. 0.0315 57. 0.075 59. $\frac{1}{10}$
61. $\frac{1}{2}$ 63. $\frac{1}{3}$ 67. $\frac{9}{10}$ 71. 30%
73. $33\frac{1}{3}\%$ 75. $n = 150$ 77. $1,200 at 4%; $800 at 9% 79. 5 liters

Page 344

1. $-30x^3$ **3.** $3x - 2y$ **5.** $0.45m$
7. $\frac{m}{7n^2}$ **9.** $x - 2$ **11.** $y = 10$
13. $x = -6$ **15.** $y = 5$ **17.** $m = 2$
19. $x = 5$ **21.** $a = 14$ **23.** $x = 8$
25. $x = 2, y = 4$ **27.** $a = 9, b = -1$
29. 40
31.



33. $b(b + 2)$ **35.** $2n(n + 3)$
37. $(y + 10)(y + 2)$ **39.** $(x - 5)(x + 2)$
41. $(b + 6)(b + 1)$ **43.** $(x - y)(x + y)$
45. $(a + 3)(a + 3)$ **47.** $(2a - b)(2a + b)$
49. $(3x - 5y)(3x + 5y)$

CHAPTER 11

Page 349

1. 2 **3.** 5 **5.** 4 **7.** -2 **9.** 1 **11.** -7
13. -6 **15.** 8 **17.** -8 **19.** 6 **21.** 8
23. 3 **25.** -10 **27.** 20 cm **29.** 5 **31.** 13

Puzzles and Things, page 349

$5 = (\sqrt{4} \times \sqrt{4}) + (4 \div 4)$;
$6 = \sqrt{4} + (4 \times 4) \div 4$;
$7 = 4 + 4 - (4 \div 4)$;
$8 = 4 + (4 \times 4) \div 4$;
$9 = 4 + 4 + (4 \div 4)$;
$10 = 4 + 4 + 4 - \sqrt{4}$;
$11 = 44 \div (\sqrt{4} + \sqrt{4})$;
$12 = \sqrt{4} \times (4 + 4) - 4$;
$13 = (44 \div 4) + \sqrt{4}$;
$14 = 4 + 4 + 4 + \sqrt{4}$;
$15 = (4 \times 4) - (4 \div 4)$;
$16 = 4 + 4 + 4 + 4$

Page 351

1. 2.828 **3.** 6.325 **5.** 4.243 **7.** 4.472
9. 8.888 **11.** 9.434 **13.** 5.916 **15.** 8.660
17. -6.164 **19.** -7.071 **21.** 7.550
23. ± 4 **25.** ± 8.185 **27.** 3.7 **29.** 7.6
31. 7.1 **33.** 4.4 **35.** 6.1 **37.** -4.2
39. -8.4 **41.** 8.4 cm **43.** 4.5 cm
45. 27.7 cm **47.** 24.7 cm **49.** 19.6 cm
51. 25.3 cm

Page 353

1. rational **3.** rational **5.** rational
7. rational **9.** irrational **11.** irrational
13. rational **15.** rational **17.** rational
19. rational **21.** rational **23.** rational
25. rational **27.** 314 cm^2 **29.** 28.26 cm^2
31. $\frac{3}{2}$ **33.** $\frac{41}{35}$ **35.** yes

Page 355

1. 12 **3.** 15 **5.** $10\sqrt{10}$ **7.** $20\sqrt{5}$
9. $8\sqrt{2}$ **11.** $6\sqrt{5}$ **13.** $6\sqrt{3}$ **15.** $8\sqrt{3}$
17. 33 **19.** 42 **21.** $4x$ **23.** $8x\sqrt{y}$
25. $5a^2\sqrt{3}$ **27.** $5x\sqrt{5x}$ **29.** Let $x = -2$; $\sqrt{(-2)^2} = \sqrt{4} = 2 \neq -2$

Page 357

1. ± 8 **3.** ± 4 **5.** ± 3 **7.** ± 11 **9.** ± 3
11. ± 6 **13.** ± 6 **15.** ± 5 **17.** $\pm 2\sqrt{3}$
19. $\pm\sqrt{6}$ **21.** $\pm\sqrt{6}$ **23.** $\pm\sqrt{7}$
25. $\pm 3\sqrt{2}$ **27.** ± 4 **29.** ± 2 **31.** ± 6
33. ± 6.3 **35.** ± 6.2 **37.** ± 8.7 **39.** ± 6.7

Puzzles and Things, page 357

If you examine the figure closely, you'll see that the longer side of the large rectangle is equal to the length of the diagonal of the square. They are both $x\sqrt{2}$, because the ratio of the length to the width of the large rectangle must be $\sqrt{2}$.

Pages 360–361

1. $2\sqrt{5}$ **3.** $2\sqrt{10}$ **5.** $2\sqrt{15}$ **7.** 5.9 m
9. 14.4 cm **11.** 48.5 cm

Page 363

1. $\frac{2}{5}$ **3.** $\frac{11}{2}$ **5.** $\frac{1}{4}$ **7.** $\frac{3}{4}$ **9.** $\frac{3}{14}$ **11.** $\frac{1}{12}$
13. $\frac{x}{6}$ **15.** $\frac{2y}{3}$ **17.** $\frac{x\sqrt{3}}{4}$ **19.** $\frac{a\sqrt{7}}{8}$
21. 0.87 **23.** 1.54 **25.** 0.89 **27.** 1.53

Page 365

1. 3 **3.** 7 **5.** $3\sqrt{2}$ **7.** 4 **9.** $4\sqrt{3}$
11. 12 **13.** $15\sqrt{2}$ **15.** 6 **17.** x **19.** $8a$
21. $8x$ **23.** $\sqrt{5}$ **25.** $\sqrt{7}$ **27.** 2
29. $10\sqrt{2}$ **31.** 8 **33.** a **35.** $\sqrt{2n}$

37. $\frac{3a}{b}$ **39.** $\frac{1}{6}$ **41.** 1 **43.** $\sqrt{5}$ **45.** $6x^2$

47. $-12x^2$ **49.** $\frac{x}{2}$ **51.** $3x\sqrt{2}$

53. $10a^4\sqrt{15}$ **55.** $-x^2$

Page 367

1. $\frac{\sqrt{2}}{2}$ **3.** $\frac{\sqrt{3}}{2}$ **5.** $\frac{\sqrt{3}}{3}$ **7.** $\frac{\sqrt{10}}{4}$

9. $\frac{\sqrt{70}}{10}$ **11.** $\frac{2\sqrt{7}}{7}$ **13.** $\frac{3x\sqrt{5}}{5}$

15. $\frac{a\sqrt{2}}{2}$ **17.** $\frac{-x^2\sqrt{y}}{y}$ **19.** 4 **21.** 3.5

23. 3.79 **25.** 0.33 **27.** 0.61

Page 369

1. $8\sqrt{2}$ **3.** $13\sqrt{5}$ **5.** $4\sqrt{5}$ **7.** $\sqrt{3}$

9. $4\sqrt{2}$ **11.** $3\sqrt{2}$ **13.** $-6\sqrt{3}$

15. $4\sqrt{5} - \sqrt{3}$ **17.** $10\sqrt{5} - 2\sqrt{2}$

19. $-2\sqrt{3} + \sqrt{6}$ **21.** $5\sqrt{5} - 4\sqrt{7}$

23. $(3 + 3b)\sqrt{b}$ **25.** 0 **27.** $-7\sqrt{6}$

29. $3\sqrt{3} - \frac{\sqrt{6}}{3}$ **31.** $\frac{7\sqrt{5}}{5} + 5\sqrt{3}$

Page 371

1. -1 **3.** -13 **5.** -5 **7.** 2 **9.** 7

11. 44 **13.** $\frac{\sqrt{3} - 1}{2}$ **15.** $\frac{2\sqrt{2} - 6}{-7}$

17. $\frac{\sqrt{6} + 6}{-5}$ **19.** $\frac{3\sqrt{10} - 2\sqrt{15}}{3}$

21. $\frac{7 + \sqrt{7}}{3}$ **23.** $-12\sqrt{2} - 6\sqrt{10}$

25. $2 - \sqrt{3}$ **27.** $\frac{1 - 2\sqrt{7}}{-9}$

29. $7 - 4\sqrt{3}$ **31.** $6 - 2\sqrt{5}$

33. $54 + 14\sqrt{5}$ **35.** $26 - 8\sqrt{10}$

37. $57 - 12\sqrt{15}$ **39.** $92 - 24\sqrt{10}$

41. $61 + 28\sqrt{3}$

Page 372

1. increase: \$.52 **3.** decrease: \$.75

5. increase: \$2.24 **7.** 50% **9.** 50%

11. 20% **13.** 25% **15.** 40% **17.** 50%

19. 20% **21.** 30% **23.** 25%

Page 374

1. \$10.00 **3.** 18% **5.** \$.77

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1. \$20.00

Pages 376–377

1. 5 **3.** 9 **5.** 3 **7.** -7 **9.** 12 **11.** 4.4

13. 7.7 **15.** 2.6 **17.** 4.6 **19.** 9.2

21. rational **23.** irrational **25.** rational

27. rational **29.** rational **31.** $3\sqrt{2}$

33. $2\sqrt{7}$ **35.** $3\sqrt{5}$ **37.** $2\sqrt{35}$ **39.** ± 3

41. ± 6 **43.** $\pm 4\sqrt{3}$ **45.** $\sqrt{74}$

47. $2\sqrt{26}$ **49.** $2\sqrt{13}$ **51.** $\frac{1}{6}$ **53.** $\frac{\sqrt{3}}{9}$

55. $\frac{1}{2}$ **57.** $\frac{x\sqrt{14}}{8}$ **59.** 10 **61.** $10\sqrt{5}$

63. $\sqrt{5}$ **65.** $\sqrt{2r}$ **67.** $\frac{\sqrt{5}}{5}$ **69.** $\frac{a\sqrt{5}}{5}$

71. $\frac{\sqrt{10x}}{5}$ **73.** $\frac{4\sqrt{2\pi}}{\pi}$ **75.** $2\sqrt{3}$

77. $-2\sqrt{5}$ **79.** $3\sqrt{2} - 2\sqrt{3}$

81. $9\sqrt{3} - 2$ **83.** $\sqrt{7} - 3\sqrt{21} + 7$

85. $-26\sqrt{2} + 3\sqrt{3}$

CHAPTER 12

Page 381

1. 0, -3 **3.** 0, -9 **5.** 0, 1 **7.** 0, 9

9. 3, 1 **11.** -4, -3 **13.** -3, -8

15. 14, 2 **17.** -5, -6 **19.** 8, -8

21. -12, 12 **23.** -100, 100 **25.** 12, -5

27. 3, -7 **29.** 6, -4

Page 383

1. $x(x - 2)$ **3.** $2x(x - 2)$

5. $(x + 3)(x - 1)$ **7.** $(x + 2)(x + 4)$

9. $(x - 6)(x + 3)$ **11.** 0 or 3

13. 0 or -3 **15.** 0 or 4 **17.** 8 or 1

19. 7 or 1 **21.** 3 or 7 **23.** -7 or 3

25. 4 or -2 **27.** 7 or -2 **29.** -5

31. 7 or 4 **33.** -3 or 3 **35.** 11 or 2

37. 11 or -5 **39.** 5 or 8 **41.** 4 or -10

Page 385

1. 0 or 3 **3.** 0 or -4 **5.** -8 or 4

7. 4 or -2 **9.** 3 **11.** 10 or 2

13. -7 or 6 **15.** 9 or -2 **17.** 7 or 5

19. -6 or 2 **21.** 7 or -7 **23.** -7 or 4

25. -4 or 3 **27.** 4 or -3 **29.** -7 or 7

31. $-1\frac{1}{2}$ or 1 **33.** $2\frac{1}{2}$ or -2

35. $-4\frac{1}{2}$ or 3

Page 387

1. ± 2 **3.** ± 4 **5.** ± 8 **7.** ± 11 **9.** ± 20
11. $\pm\sqrt{5}$ **13.** $\pm 2\sqrt{2}$ **15.** ± 1
17. $\pm 2\sqrt{3}$ **19.** ± 2 **21.** No Solution
23. ± 2 **25.** ± 7 **27.** $\pm 2\sqrt{6}$
29. No Solution **31.** $\pm\sqrt{15}$ **33.** ± 3
35. $\pm\sqrt{10}$ **37.** $\pm\frac{1}{3}$ **39.** ± 6
41. 5, 20, 80; 5, 10

Page 389

1. 9, -3 **3.** 9, -1 **5.** 2, -18
7. 11, -5 **9.** 18, -4 **11.** 2, -1
13. 5, -2 **15.** 1, -4 **17.** 3, -6 **19.** 2
21. 2, 0 **23.** $2+\sqrt{7}$, $2-\sqrt{7}$
25. $-1+\sqrt{2}$, $-1-\sqrt{2}$
27. $5+2\sqrt{2}$, $5-2\sqrt{2}$ **29.** 10, 4
31. 1, -5 **33.** -1, -5 **35.** 1, -3

Page 392

1. $-\frac{1}{2}$, -1 **3.** $\frac{1}{3}$, 2 **5.** $\frac{1}{4}$, -2
7. $-4, \frac{1}{2}$ **9.** $\frac{1}{3}$, 2 **11.** $-\frac{2}{3}$, 1
13. 1, -1 **15.** $\frac{1}{2}$, $-\frac{1}{2}$
17. $\frac{-1+\sqrt{17}}{4}$, $\frac{-1-\sqrt{17}}{4}$
19. $-\frac{2}{5}$, -1 **21.** $1+\sqrt{6}$, $1-\sqrt{6}$
23. $\frac{-1+\sqrt{57}}{4}$, $\frac{-1-\sqrt{57}}{4}$
25. $\frac{1+\sqrt{41}}{4}$, $\frac{1-\sqrt{41}}{4}$
27. $3+\sqrt{6}$, $3-\sqrt{6}$
29. $\frac{7+\sqrt{33}}{4}$, $\frac{7-\sqrt{33}}{4}$ **31.** 0, 5
33. 5, -1 **35.** 2, 5 **37.** 2, 4 **39.** -3, 4

Page 393

1. 0, 2 **3.** 0, 4 **5.** 0, 2 **7.** -2, -1
9. -4, 2 **11.** 6, -5 **13.** ± 3
15. $\pm\sqrt{35}$ **17.** $\pm\sqrt{19}$ **19.** $\pm 3\sqrt{2}$
21. 5, -3 **23.** $3+\sqrt{17}$, $3-\sqrt{17}$
25. -1, 3 **27.** -4, 2 **29.** -1, $-\frac{1}{3}$
31. $-1, 1\frac{2}{3}$ **33.** 1, 2
35. $\frac{1+\sqrt{33}}{4}$, $\frac{1-\sqrt{33}}{4}$

Page 395

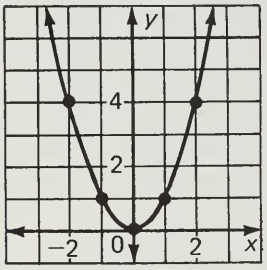
1. Length is 8 cm; width is 5 cm. **3.** -5 and -7 or 5 and 7 **5.** 2 **7.** 5 **9.** 12
11. Length is 12 cm; width is 4 cm. **13.** Al bikes 9 km/h; Vince bikes 12 km/h.

Pages 398–399

1.a.

$y = x^2$	
x	y
-2	4
-1	1
0	0
1	1
2	4

b.

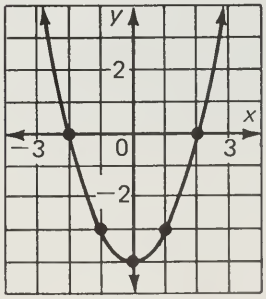


c. (0,0)

3.a.

$y = x^2 - 4$	
x	y
-2	0
-1	-3
0	-4
1	-3
2	0

b.

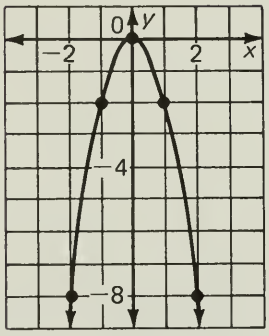


c. $(0,-4)$

5.a.

$y = -2x^2$	
x	y
-2	-8
-1	-2
0	0
1	-2
2	-8

b.

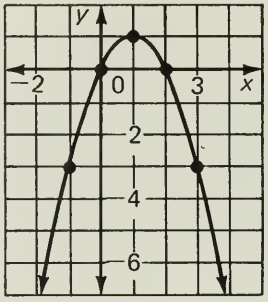


c. (0,0)

9.a.

$y = 2x - x^2$	
x	y
-1	-3
0	0
1	1
2	0
3	-3

b.

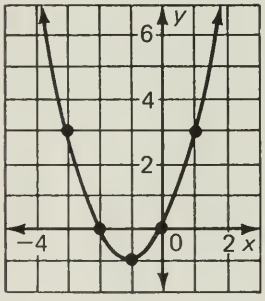


c. (1,1)

7.a.

$y = x^2 + 2x$	
x	y
-2	0
-1	-1
0	0
1	3
2	8

b.

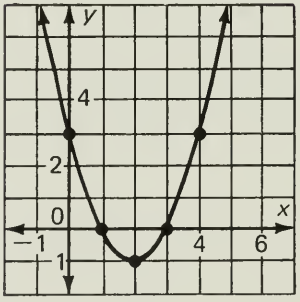


c. $(-1,1)$

11.a.

$y = x^2 - 4x + 3$	
x	y
0	3
1	0
2	-1
3	0
4	3

b.

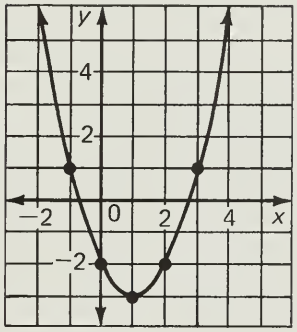


c. $(2,-1)$

13.a.

$y = x^2 - 2x - 2$	
x	y
-1	1
0	-2
1	-3
2	-2
3	1

b.

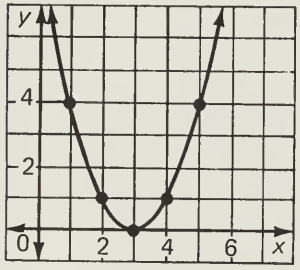


c. (1, −3)

15.a.

$y = x^2 - 6x + 9$	
x	y
1	4
2	1
3	0
4	1
5	4

b.

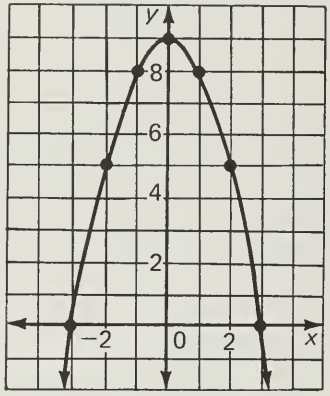


c. (3,0)

17.a.

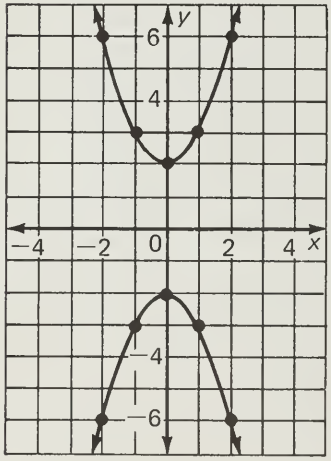
$y = (3 - x)(3 + x)$	
x	y
-3	0
-2	5
-1	8
0	9
1	8
2	5
3	0

b.



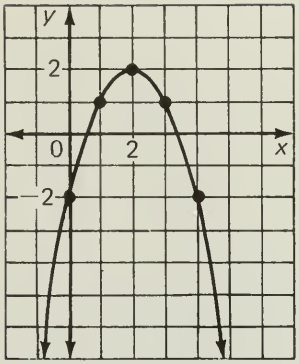
c. (0,9)

19.



21. 1 and 3 **23.** two

25.



27. of $y = x^2 - 2x + 4$, (1,3); of $y = x^2 - 2x + 2$, (1,1); of $y = x^2 - 2x$, (1,−1) **29.** 2 times **31.** 1 time

Pages 402–403

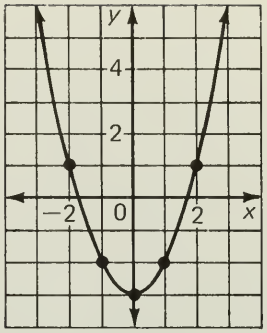
1. 0, 2 **3.** 0, 7 **5.** 0, −6 **7.** −1, −5
9. −5, 3 **11.** 6, 2 **13.** 0, 3 **15.** 0, −5
17. 0, −2 **19.** −3, −2 **21.** 4, 2
23. −6, 3 **25.** 0, 2 **27.** 0, −1
29. −5, −2 **31.** 2, 8 **33.** −7, 3
35. 2, 7 **37.** ± 3 **39.** ± 6 **41.** $\pm\sqrt{21}$
43. $\pm 3\sqrt{3}$ **45.** $\pm\sqrt{6}$ **47.** ± 6
49. 5, −3 **51.** 4, −12 **53.** 2, −12
55. 1, −3 **57.** −1, −5 **59.** 1, 5
61. 2, −5 **63.** $\frac{1}{2}$, −3 **65.** −3, −5
67. $-\frac{1}{2}$, 1 **69.** −1, $-1\frac{1}{2}$
71. $\frac{-7 + \sqrt{17}}{8}, \frac{-7 - \sqrt{17}}{8}$
73. length is 9 cm; width is 4 cm.

75.a.

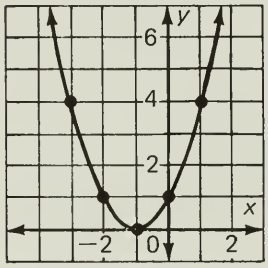
$y = x^2 + 2$	
x	y
−3	11
−2	6
−1	3
0	2
1	3
2	6
3	11

b. −2, 2

77.



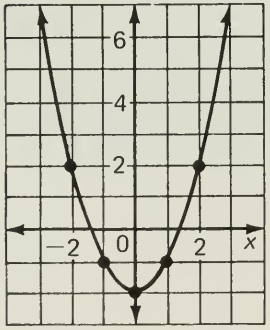
79.



Page 404

1. $28a^3$ **3.** $0.87m$ **5.** $-\frac{n}{4}$ **7.** $a(a + 7)$
9. $3m(m + 4)$ **11.** $(y - 6)(y + 3)$
13. $(y + 6)^2$ **15.** $(x - 8)(x + 3)$
17. $(m + 10)(m - 4)$ **19.** $x = 4$
21. $a = -4$ **23.** $a = -1$ **25.** $x = 3$
27. $y = 30$ **29.** $a = 21$
31. $x = 3, y = -1$ **33.** $x = 3, y = -1$
35. $66\frac{2}{3}\%$ **37.** 0 or −3 **39.** 0 or 5
41. 3 or −2 **43.** ± 5 **45.** $\frac{1}{2}$, −2
47. $1, \frac{1}{2}$

49.



EXTRA PRACTICE EXERCISES

Page 406
(pages 2–7) **1.** 42 **3.** 63 **5.** 10 **7.** 15 **9.** 10 **11.** 2 **13.** 27 **15.** 6 **17.** 15 **19.** 25 **21.** 24 **23.** 4

Page 406
(pages 8–11) **1.** $2n$ **3.** $3n$ **5.** $2x$ **7.** $10m$ **9.** $9a$ **11.** $3a$ **13.** $10x$ **15.** $3a - b$ **17.** $5x - 5$ **19.** $11m + 1$ **21.** $11y + 1$ **23.** $16x + 2y$ **25.** $n + 2r$

Page 407
(pages 12–15) **1.** 9 **3.** 4 **5.** 4 **7.** 27 **9.** y^3 **11.** a^3 **13.** rs^2 **15.** xy^3 **17.** $3a^2$ **19.** $10n^2$ **21.** $8r$ **23.** $16n$ **25.** $8r^2$ **27.** $56y^2$ **29.** $8ab$ **31.** $10a^2$

Page 407
(pages 16–17) **1.** $3x + 9$ **3.** $5a - 5$ **5.** $8n + 32$ **7.** $4x + 2$ **9.** $x^2 - 2x$ **11.** $n^2 + 2n$ **13.** $12n - 48$ **15.** $6a - 3$ **17.** $6a - 9b$ **19.** $6a^2 + ab$ **21.** $2n + 7$ **23.** $3n + 11$ **25.** 8 **27.** $a + 13$ **29.** $y + 14$ **31.** $10a + 14$

Page 408
(pages 18–21) **1.** 0 **3.** 0 **5.** 0 **7.** 6 **9.** 0 **11.** impossible **13.** 0 **15.** 3 **17.** 1 **19.** 1 **21.** $2x$ **23.** $6a$ **25.** n **27.** x

Page 408
(pages 22–25) **1.** 1 **3.** 3 **5.** 2 **7.** 1, 2 **9.** 15, 16 **11.** 3, 4, 10

Page 409
(pages 34–37) **1.** 6 **3.** 6 **5.** 11 **7.** 13 **9.** 4 **11.** 1 **13.** 15 **15.** 6 **17.** 25 **19.** 13 **21.** 8 **23.** 7 **25.** 6 **27.** 24 **29.** 7 **31.** 24

Page 409
(pages 38–41) **1.** 6 **3.** 3 **5.** 5 **7.** 4 **9.** 12 **11.** 12 **13.** 6 **15.** 24 **17.** 9 **19.** 6 **21.** 35 **23.** 64

Page 410
(pages 42–45) **1.** 1 **3.** 6 **5.** 2 **7.** 10 **9.** 10 **11.** 7 **13.** 1 **15.** 12 **17.** 7 **19.** 9

Page 410
(pages 48–49) **1.** 3 **3.** 2 **5.** 4 **7.** 5 **9.** 2 **11.** 3 **13.** 8 **15.** 8 **17.** 6

Page 410
(pages 54–57) **1.** 8 **3.** $10

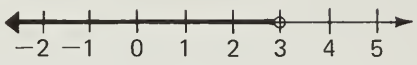
Page 411
(pages 58–59) **1.** 10 **3.** 1 **5.** 10 **7.** 4 **9.** 2 **11.** 3 **13.** 1 **15.** 2

Page 411
(pages 60–61) **1.** 12 **3.** 2 **5.** 2 **7.** 6 **9.** 1 **11.** 7 **13.** 9 **15.** 7

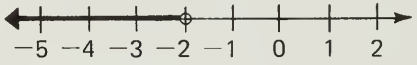
Page 411
(pages 62–63) **1.** Ed: $5; Bob: $15; Al: $45 **3.** 21 years old

Page 412
(pages 72–75) **1.** $>$ **3.** $>$ **5.** $>$ **7.** $>$ **9.** 3 **11.** -2

13.



15.



Page 412
(pages 76–80) **1.** -4 **3.** 0 **5.** 0 **7.** 12 **9.** -7 **11.** 14 **13.** -11

Page 412
(pages 82–83) **1.** $-3n$ **3.** $-10b$ **5.** $-6y$ **7.** $a + 1$ **9.** 3 **11.** $x + 2$ **13.** $3x^2 - y$ **15.** $5n - n^2$

Page 413
(pages 84–87) **1.** 48 **3.** 56 **5.** 25 **7.** -25 **9.** $-21n$ **11.** $4a - 2b$

Page 413
(pages 88–89) **1.** -9 **3.** 7 **5.** -7 **7.** -3 **9.** -8 **11.** -4

Page 413
(pages 90–91) **1.** 3 **3.** -3 **5.** 0 **7.** 5 **9.** -4 **11.** -4 **13.** 1 **15.** -2 **17.** 6 **19.** -6 **21.** -4 **23.** 7 **25.** 40 **27.** 18

Page 414
(pages 102–103) **1.** 28 cm **3.** 20 m **5.** $4a$ **7.** $20x$ **9.** $x = 4$

Page 414
(pages 106–107) **1.** 28 **3.** $4x^2$ **5.** $47.40

Page 415
(pages 108–111) **1.** $V = Bh$ **3.** 216 cm^3

Page 415
(pages 112–121) **1.** hK **3.** 40 km

Page 415
(pages 122–125) **1.** $n - y$ **3.** $4y - 12$
5. $\frac{y}{n}$ **7.** $15 - 10y$ **9.** $\frac{y}{3}$ **11.** $k + y$

Page 416
(pages 136–139) **1.** $4n + 4$ **3.** $2a^2$
5. $5z$ **7.** $-3x - 2y$ **9.** $13a + 3b$
11. $3x^2$ **13.** a **15.** 8 **17.** $-2n^2$
19. $-4a - 8$ **21.** $x + 5$ **23.** $5a + 3b$
25. $5n + 5$ km

Pages 416–417
(pages 140–143) **1.** y^4 **3.** $2n^3$ **5.** $-x^2y$
7. n^7 **9.** n^{12} **11.** y^{10} **13.** a^2b^2 **15.** m^4n^4
17. $-x^3y^3$ **19.** $-27x^6$ **21.** $-a^3b^4$
23. $512r^3s^6$ **25.** $-81m^4n^4$
27. $8000m^{12}n^{12}$

Page 417
(pages 144–147) **1.** $3n + 15$
3. $6a^2 + 6b^2$ **5.** $ax - ay$
7. $-6a - 12b - 18c$
9. $4x^3y + 4x^2y^2 + 12xy^3$
11. $-3x^4 + 5x^3 - x^2 + x$
13. $20 - 9a + a^2$ **15.** $x^2 + 12x + 32$
17. $25x^2 - 30x + 8$
19. $a^3 + a^2 - a - 1$
21. $m^3 - mn^2 - nm^2 + n^3$ **23.** $a^4 - 1$

Pages 417–418
(pages 148–153) **1.** $n^2 + 7n + 12$
3. $m^2 - 7m + 10$ **5.** $25y^2 - 1$
7. $8x^2 + 2xy - y^2$ **9.** $15x^2 - 8xy + y^2$

Page 419
(pages 170–173) **1.**
5. 2, 2, 5, 7 **7.** 4 **9.**
15. $4(5n - 1)$ **17.** 2
19. $7x(2x + 1)$ **21.**
23. $3(n^2 - 3n + 4)$
27. $6xy(1 + 7xy - 1$
29. $12b(a^2 - 3ab + 4$
31. $15m^2n^4(1 + 3mn$

Pages 419–420
(pages 176–181) **1.**
3. $(n + 6)(n + 10)$ **5**
7. $(b + 9)(b + 10)$ **9**
11. $(n + 17)(n + 2)$
15. $(x + 14)(x + 2)$
19. $(y - 9)(y - 5)$
23. $(n - 10)(n - 8)$
27. $(x - 100)(x - 1)$
29. $(m - 30)(m - 2)$
31. $(x + 16)(x + 4)$

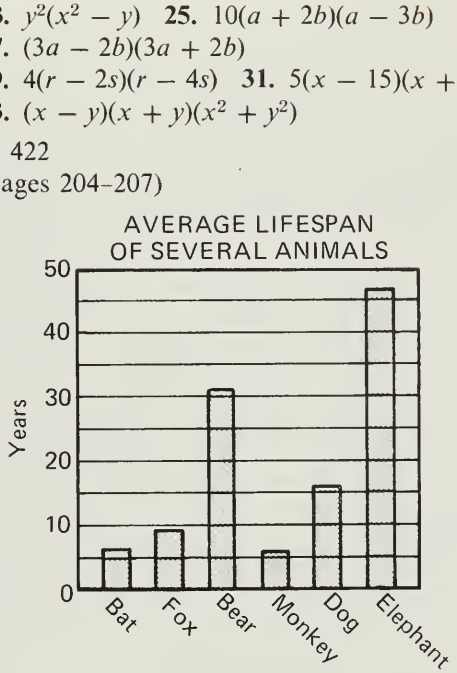
Page 420
(pages 182–187) **1.**
3. $(n - 7)^2$ **5.** $(n +$
7. $(n + 4)(n + 1)$ **9.**
11. $(mn - 2)^2$ **13.**
15. $(y - 4)(y + 10)$
17. $(m - 10)(m + 5)$
21. $(n - 4)(n + 1)$

Pages 420–421
(pages 188–191) **1.**
3. $1 - 25x^2$ **5.** $4x^2$
9. $16n^2 - 9$ **11.** 81
13. $(x - 5)(x + 5)$
17. $9(x - 2y)(x + 2y$

15. $5(a - b)(a + b)$ **17.** $4(x - 3)^2$
19. $4(n - 6)(n + 4)$ **21.** $x(1 - 3x)$
23. $y^2(x^2 - y)$ **25.** $10(a + 2b)(a - 3b)$
27. $(3a - 2b)(3a + 2b)$
29. $4(r - 2s)(r - 4s)$ **31.** $5(x - 15)(x + 4)$
33. $(x - y)(x + y)(x^2 + y^2)$

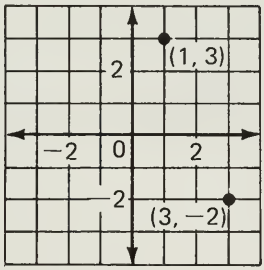
Page 422
(pages 204–207)

1.



Page 422
(pages 212–215)

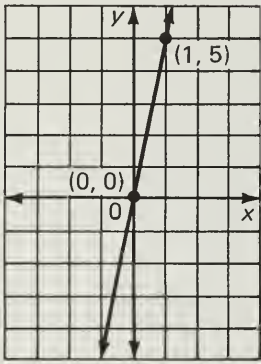
1., 3.



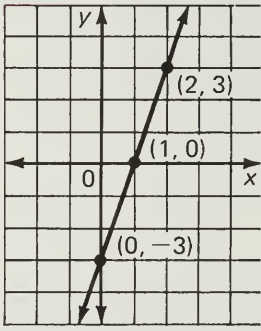
Pages 422–423
(pages 216–219) **1.** 4, 5,
3. 2, 5, 11, 14 **5.** $7 - x$
9. $x - 5$ **11.** $-10x - 2$
19. Answers may vary.
13. $(-1,-4)$; $(0,-3)$; $(1,$
15. $(-1,3)$; $(0,1)$; $(1,-1)$
17. $(-1,2)$; $(0,0)$; $(1,-2)$
19. $(-1,-2)$; $(0,0)$; $(1,2)$
23. $y = 3x - 1$

Page 423
(pages 220–223)

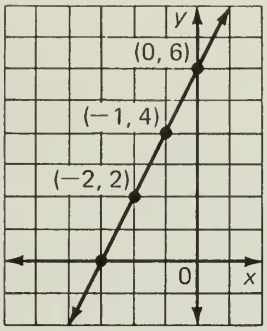
1.



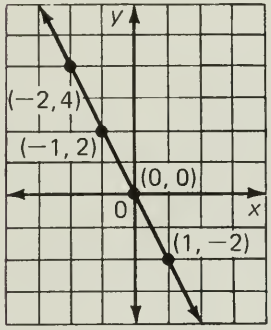
3.



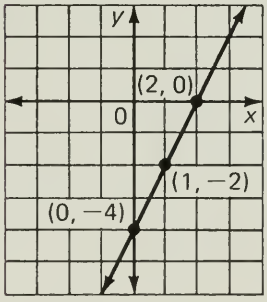
5.



7.



9.



11. -3 **13.** $\frac{1}{2}$

Page 423

(pages 224–227) **1.** $y = 3x$ **3.** 90, 120

Page 424

(pages 236–239) **1.** $(-1,1)$ **3.** $(2,1)$

5. $\left(\frac{-5}{2}, \frac{-11}{2}\right)$ **7.** $\left(\frac{3}{4}, \frac{-7}{4}\right)$

Page 424

(pages 240–243) **1.** 3 **3.** -3 **5.** no

7. all

Pages 424–425

(pages 244–249) **1.** (3,9) **3.** (0,2)

5. $(5,-1)$ **7.** $(-2,-3)$ **9.** $(-9,-34)$

11. (1,1) **13.** (45,9) **15.** $\left(\frac{3}{4}, \frac{-5}{4}\right)$

Page 425

(pages 250–257) **1.** $(-7,6)$ **3.** $(-2,-1)$

5. (2,0) **7.** $(1,-1)$ **9.** 5, 10 **11.** \$4.00

13. Jo's score is 5; Al's score is 11.

15. $(18,-12)$ **17.** (0,0) **19.** $(-8,-5)$

21. 24 pencils, 1 pen

Page 426

(pages 268–273) **1.** $\frac{1}{4}$ **3.** $\frac{1}{4}$ **5.** $\frac{1}{3}$

7. $-\frac{1}{7}$ **9.** $-\frac{1}{4xy}$ **11.** $-\frac{y^2}{3x}$ **13.** $-\frac{8}{ns}$

15. $2xy$ **17.** $\frac{3}{10}$ **19.** $\frac{1}{3}$ **21.** $-1(-x)$

23. $-1(n^2 - z^2)$ **25.** -1 **27.** $-x - y$

29. $-\frac{1}{3 + x}$ **31.** $-\frac{1}{3n + 3}$ **33.** $\frac{x - 3}{x + 2}$

Pages 426–427

(pages 274–283) **1.** $\frac{3}{2}$ **3.** $\frac{1}{4}$ **5.** 12, 9

7. 12 **9.** 24 **11.** 3 **13.** -5 **15.** 6 km

17. $\frac{1}{5}$ **19.** $\frac{1}{12}$

Page 427

(pages 284–293) **1.** $\frac{1}{16}$ **3.** $\frac{2a^2}{b^2}$ **5.** 6

7. $\frac{x - 3}{3}$ **9.** $-\frac{4n}{3}$ **11.** $\frac{3n - 1}{n - 1}$

13. $\frac{5}{4}$ or $1\frac{1}{4}$ **15.** $\frac{3x}{10}$ **17.** $\frac{y + 3x}{xy}$

19. $\frac{9a - b}{a^2b^2}$ **21.** $\frac{x - y}{x + 8}$ **23.** $3a^2 - 9a$

Page 428

(pages 296–301) **1.** -100 **3.** 2 **5.** 2

7. 6 **9.** -12 **11.** $2\frac{2}{5}$ hours or 2 hours 24 minutes **13.** -30 **15.** $7\frac{1}{2}$ hours

Page 429

(pages 312–317) **1.** 0.61 **3.** 5.30

5. 0.505 **7.** 1.64 **9.** 47.5 **11.** 62.5

13. 3.5577 **15.** 4.275 **17.** 1.2 **19.** 6.9

21. 0.5 **23.** 52.8 **25.** 0.4

27. 0.142857 . . . **29.** 0.714285

31. 0.083 . . . **33.** 0.6 . . .

Page 429
(pages 318–321) **1.** 10 **3.** 6000
5. −355.4 **7.** 20 and 30 **9.** 2000

Page 430
(pages 322–327) **1.** 0.05; $\frac{1}{20}$
3. 80%; 0.80 **5.** 0.09; $\frac{9}{100}$ **7.** 6%; $\frac{3}{50}$
9. \$0.63 **11.** \$3.07 **13.** \$3.10 **15.** \$10
17. \$140

Pages 430–431
(pages 328–331) **1.** 20% **3.** 350 **5.** 60
7. \$0.98 **9.** $2\frac{1}{2}\%$

Page 431
(pages 332–335) **1.** \$390 **3.** \$6000
5. \$150 at 9% and \$50 at 18%

Page 431
(pages 336–339) **1.** 500 student tickets

Page 432
(pages 348–353) **1.** 7 **3.** 5 **5.** 1 **7.** 0
9. 6 **11.** −3.464 **13.** −1.732
15. −8.718 **17.** 9.9 **19.** 7.9
21. rational **23.** irrational **25.** irrational

Page 432
(pages 354–355) **1.** $10\sqrt{5}$ **3.** 80
5. $7\sqrt{3}$ **7.** 27 **9.** $9x$ **11.** $x^2y\sqrt{5}$

Page 432
(pages 356–357) **1.** 7 or −7
3. 6 or −6 **5.** $2\sqrt{7}$ or $-2\sqrt{7}$ **7.** ±4.2
9. ±4.5

Page 433
(pages 358–361) **1.** 7.2 cm **3.** 7.5 cm

Page 433
(pages 362–367) **1.** $\frac{1}{3}$ **3.** $\frac{3}{10}$ **5.** $\frac{x}{5}$
7. 6 **9.** 12 **11.** $90x$ **13.** 15 **15.** $x^2\sqrt{2}$
17. 0.79 **19.** 0.26

Page 433
(pages 368–369) **1.** $-6\sqrt{3}$ **3.** $3\sqrt{2}$
5. $2\sqrt{6}$ **7.** $\sqrt{2}$ **9.** $-2\sqrt{3n}$ **11.** $\frac{5\sqrt{2}}{2}$

Page 434
(pages 380–385) **1.** 0, −5 **3.** 0, 2
5. 5, 2 **7.** 0, −1 **9.** 0, 9 **11.** −3, −8
13. 2, 1 **15.** −5, 2 **17.** 6, 1

Page 434
(pages 386–389) **1.** 0 **3.** no solution
5. −4, 8 **7.** $4 + \sqrt{2}, 4 - \sqrt{2}$
9. $\frac{2\sqrt{2} - 1}{3}, \frac{-2\sqrt{2} - 1}{3}$ **11.** $\pm 2\sqrt{2}$

Pages 434–435
(pages 390–393) **1.** 3, 2
3. $\frac{5 + \sqrt{21}}{2}, \frac{5 - \sqrt{21}}{2}$ **5.** 4, −5
7. $\frac{1 + \sqrt{21}}{2}, \frac{1 - \sqrt{21}}{2}$
9. $\frac{1 + \sqrt{33}}{4}, \frac{1 - \sqrt{33}}{4}$ **11.** $1, -\frac{1}{4}$
13. −2, 7 **15.** 4, 4 **17.** 1, 2

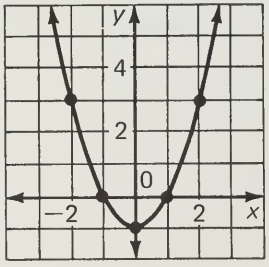
Page 435
(pages 394–395) **1.** 7 and 2 or
−7 and −2 **3.** 14, 15 or −14, −15
5. 2, 3, 4 or −2, −3, −4

Page 435
(pages 396–399)

1.a.

$y = x^2 - 1$	
x	y
0	−1
1	0
−1	0
2	3
−2	3

b.

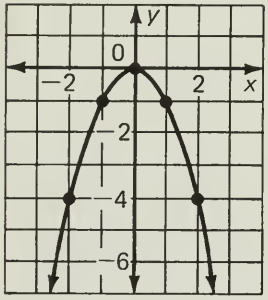


c. (0, −1)

3.a.

$y = -x^2$	
x	y
0	0
1	−1
−1	−1
2	−4
−2	−4

b.

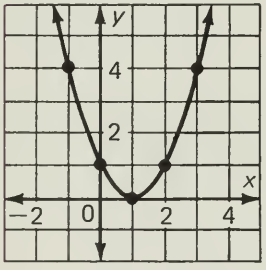


c. (0, 0)

5.a.

$y = x^2 - 2x + 1$	
x	y
1	0
0	1
−1	4
2	1
3	4

b.

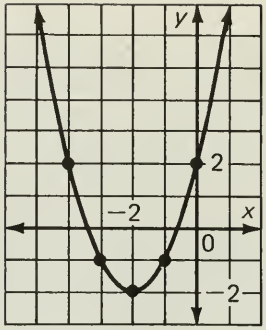


c. (1, 0)

7.a.

$y = x^2 + 4x + 2$	
x	y
−2	−2
−1	−1
0	2
−4	2
−3	−1

b.

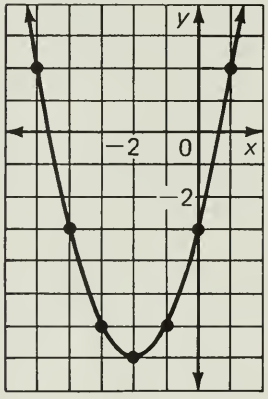


c. (−2, −2)

9.a.

$y = x^2 + 4x - 3$	
x	y
−2	−7
−1	−6
0	−3
−4	−3
−3	−6

b.



c. (−2, −7)

CUMULATIVE REVIEW, CHAPTERS 1-3

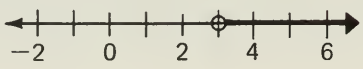
Pages 438–439

Basic Skills: **1.** 64 **3.** 12 **5.** 5 **7.** 0 **9.** 1 **11.** 5 **13.** −5 **15.** 3 **17.** −16 **19.** 3 **21.** −2 **23.** 8 **25.** −4 **27.** −6 **29.** $9b$ **31.** $9x - 2$ **33.** 4 **35.** $3a^2b^2$ **37.** x **39.** $2 - a$ **41.** $3x + 3$

Equations and Inequalities: **1.** no

3. a. 4 **b.** yes

c.



5. -1 **7.** 6 **9.** -6 **11.** 7 **13.** 9
15. -8 **17.** 8 **19.** 1 **21.** 9 **23.** $<$
25. 0; impossible

Problems: **1.** 38 **3.** Mary has 70, Don has 35. **5.** 10

CUMULATIVE REVIEW, CHAPTERS 1-6

Pages 440–441

Basic Skills: **1.** 0 **3.** $-10a$ **5.** $9x^2$
7. $4x$ **9.** $4a^4$ **11.** $-27a^3b^3$
13. $x^2 + 5x - 6$ **15.** $y^2 - y - 6$
17. $x^2 + 8x + 16$ **19.** $9x^2 - 12xy + 4y^2$
21. $4n^2$ **23.** $-9x$ **25.** $5a - a^2 - a^3$
27. $3xy(1 - 2xy)$ **29.** $(y + 1)(y + 6)$
31. $(n + 11)(n - 2)$

Equations, Inequalities, and Formulas:

1. 25 **3.** 3 **5.** 8 **7.** 4 **9.** 1 **11.** $\frac{y}{6}$

13. 3 **15.** $4x$ **17.** $\frac{D}{t}$

Problems: **1.** 16 cm **3.** $8x^2$ **5.** 12, 13

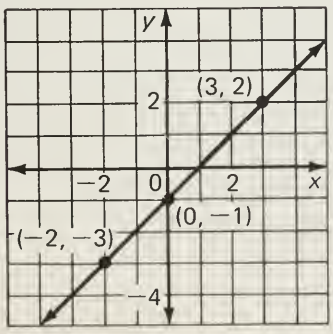
CUMULATIVE REVIEW, CHAPTERS 1-9

Pages 442–443

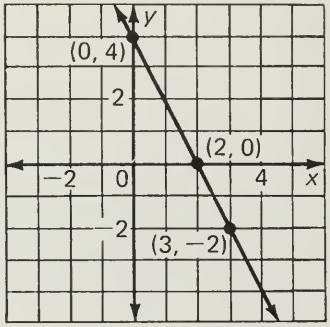
Basic Skills: **1.** impossible
3. impossible **5.** 0 **7.** $-9x^2y^2$ **9.** $4a^2b^4$
11. $270y^6$ **13.** $-x^2$ **15.** $x^2 + 6x + 8$
17. $8x^2 + 2x - 3$ **19.** $x^3 + x^2 - 6x$
21. $2(x^2 - 2y^2)$ **23.** $(n - 1)(n - 2)$
25. $(x + 4)(x + 4)$ **27.** $(2y + 3)(2y - 3)$
29. $(x + 3)(x - 2)$

Equations, Inequalities, and Graphs:
1. 3 **3.** -2 **5.** 4 **7.** $x - 3$ **9.** $5x - 10$
11. $8 + 2x$

13.



15.



17. $\frac{1}{2}$ **19.** $x = 3$; $y = 3$
21. $x = -2$; $y = 1$ **23.** $x = 6$; $y = 3$
25. $x = 2$; $y = 9$ **27.** $x = 1$; $y = 1$

Problems: **1.** $\frac{4}{3}$ hour or 1 hour and 20 minutes

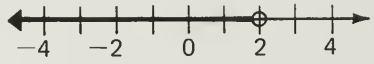
CUMULATIVE REVIEW, CHAPTERS 1-12

Pages 444–445

Basic Skills: **1.** $-2x - 10$ **3.** $5a + 1$
5. $-x$ **7.** $b - 2$ **9.** $24x - 24$
11. $n^2 - 16$ **13.** $10n^2 + 21n - 10$
15. $-2x^2 + 7x + 49$
17. $3(n + 1)(n + 2)$ **19.** $(x - 1)(x - 6)$
21. $(x - 7)(x + 2)$ **23.** $\frac{2y}{3}$ **25.** $\frac{1}{x}$
27. $6x$ **29.** $\frac{\sqrt{10}}{5}$ **31.** $-\sqrt{3}$ **33.** $\sqrt{2}$

Equations and Inequalities:

1.



3. 2 **5.** 4 or -4 **7.** -8 **9.** $\pm 5\sqrt{2}$
11. 12 **13.** -4 or 6 **15.** 2 or -7
17. **a.** 8 **b.** $\frac{24}{a}$ **c.** $\frac{c}{a}$ **19.** $x = 1$; $y = 1$
21. 1 or 6 **23.** ± 4 **25.** $\frac{1 + \sqrt{10}}{3}$ or $\frac{1 - \sqrt{10}}{3}$

Problems: **1.** 40 kg of the 65¢ kind, 60 kg of the other

CUMULATIVE REVIEW OF WORD PROBLEMS

Pages 446–448

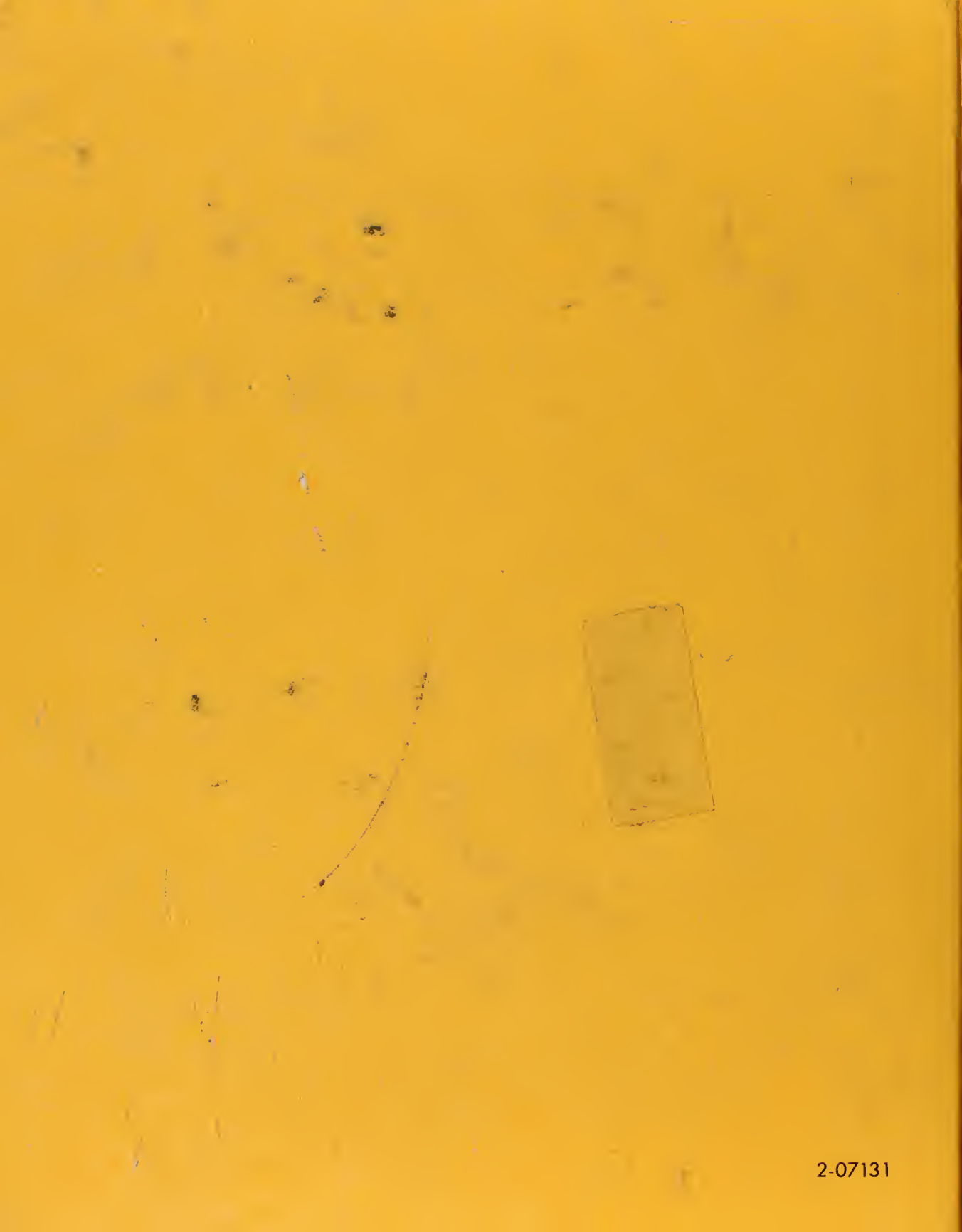
1. 10 **3.** 16 **5.** 24 **7.** 12 **9.** 30 L

11. 12 km **13.** $1\frac{1}{5}$ hours **15.** 66

17. 240 **19.** 31, 42 **21.** width: 6 cm, length: 17 cm **23.** hamburger: 75¢, root beer: 45¢ **25.** width: 10 cm, length: 15 cm **27.** 20 kg of Brazil nuts; 10 kg of cashews **29.** $7\frac{1}{2}$ hours **31.** $40.32

33. width: 4 cm, length: 16 cm

35. 12, 14



2-07131